# Performance of Class Based Queues with Self Similar Traffic

Javier Orellana, Dr. Lionel Sacks and Dr. Hermann de Meer.

University College London

**Abstract:** The aim of this paper is to show how the performance of a Class Based Queue (CBQ) degrades when the arrival traffic is self-similar. We use the mean waiting time of a packet in the queue to express the performance of the CBQ. We find a generic equation to analyse the performance of such system with self-similar traffic and, finally, we provide some numerical results using a NS-2 simulation when the traffic for the class with highest priority has the self-similarity property.

## 1 Introduction.

The real Internet traffic has the property of self similarity (SS), or sometimes also called traffic with "Power Tail" (PT) distribution. A SS distribution has a very singular feature: it is well behaved but some of its moments are infinite for any sampling period of time depending on the value of one of the parameters in the distribution, called the shaping parameter $\alpha$.

A traffic with such characteristic, depending on the shaping parameter, can make the performance of a network very poor due to the overflow of the queues. We will show this poor performance using a Class Based Queue (CBQ) node as a testbed. First, we will deduct a generic and analytical based equation to model our CBQ system, and we will see that a unique solution for such model exists but it must be obtained in an iterative manner. Second, we will model a CBQ using the NS-2 [1] simulator. Using this model with SS traffic in the arrival for the class with highest priority it will be shown that the performance of the system degrades. The metric used to watch the performance is the mean waiting time of a packet in the queue.

## 2. CBQ System.

A CBQ system is shown in Figure 1. We define the arrival traffic as an expression of the arrival rate parameter $l_i$ for the class *i*. The total incoming traffic is the aggregation of flows for all the classes. There is a finite number $R$ of classes ($1 \le i \le R$), and the higher the class number, the higher the priority. The traffic is classified and the customers put in a single queue, depending on the class, on a FCFS basis. There is a single server in the CBQ. It serves each customer with a service parameter rate of $m_i$ depending on the class.

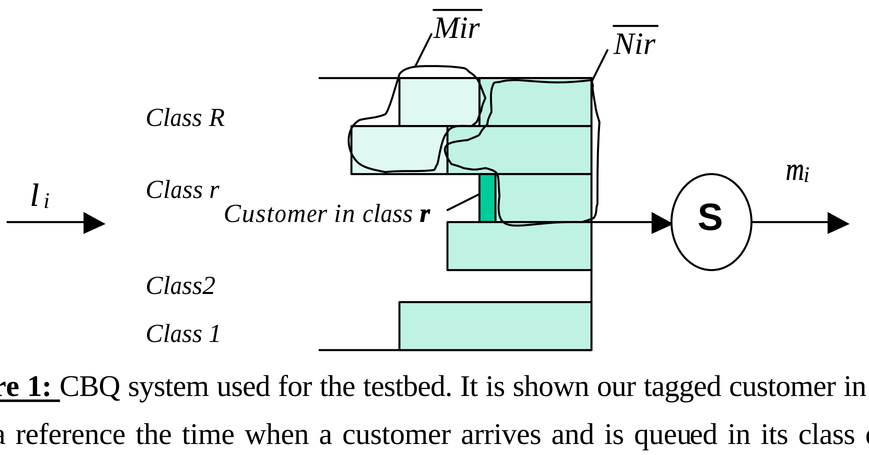


**Figure 1:** CBQ system used for the testbed. It is shown our tagged customer in the class ***r***.

Taking as a reference the time when a customer arrives and is queued in its class queue ***r***, we then define the next parameters:

- $\overline{Wr}$ : mean waiting time for our tagged customer (in the class ***r***) before being served.
- $\overline{Nir}$ : mean number of customers in the queues with higher priority than our tagged customer.

- $\overline{Mir}$ : mean number of customers of higher priority than our tagged customer arriving during the time $\overline{Wr}$ .
- $\overline{Wo}$ : mean remaining time of the element being served

and following [2]:

$$\overline{Wr} = \overline{Wo} + \sum_{i \geq r} \overline{Nir} \frac{1}{m_i} + \sum_{i > r} \overline{Mir} \frac{1}{m_i}$$

After some manipulations, and following the Cobham analysis, we find an expression for the mean waiting time for a customer depending on its class *r*:

$$\overline{Wr} = \frac{\overline{Wo}}{\left(1 - \sum_{i=r}^{R} r_i\right)\left(1 - \sum_{i=r+1}^{R} r_i\right)} \qquad (2.1)$$

Where $r_i$ is the utilization parameter for the class $i$ so $r_i = l_i / m_i$ assuming [3] that the **traffic distribution in the arrival (times between arrivals) is exponential** and also that the **service time in the server is exponential**.

### 3. G/M/1 Subsystem.

Having a generic arrival (times elapsed between arrivals) distribution and an exponential service distribution in a single FCFS subsystem, then we can't assume that $r_i = l_i / m_i$ is the utilization parameter for each class. It is possible to show [4] that the equivalent utilization parameter is the solution of the equation:

$$A^*(m_i(1-z)) = z \qquad (3.1)$$

where $A^*$ is the Laplace Stieltjes Transform (LST) of the inter-arrival time CDF. The root of this equation ($r_{oi}$) will be real and positive between 0 and 1.

### 4. Power Tail Arrival Distribution.

A particular case of a generic distribution is a PT distribution. This kind of distribution has a singular property: **it is well behaved but some of the moments are infinite for any sampling period depending on the shaping parameter** $a$ **of the distribution**. [5] introduces a PDF that behaves as an easy to operate PT distribution:

$$f_A(x) = MG(N) \sum_{n=0}^{N-1} \left(\frac{q}{g}\right)^n e^{-\frac{Mx}{g^n}} \qquad (4.1)$$

being $M = \frac{1-q}{1-qg}$; $G(N) = \frac{1-q}{1-q^N}$ where $q$ is a probability value between 0 and 1, and $g$ is a constant greater than 1. Calculating the moments for distribution we get:

$$E\{X^l\} = \frac{G(N) l!}{M^l} \frac{1-\left(qg^l\right)^N}{1-qg^l}$$

These moments converge only if $qg^l < 1$ so $l < -\frac{\ln q}{\ln g}$. Calling the shape parameter $a = -\frac{\ln q}{\ln g}$ we can say that all the moments $l \geq a$ will diverge for a given value of $a$ . So for instance, if $a = 1.1$

then the first moment (the mean) will converge but the variance and the rest of the moments will diverge. **So the bigger** a **the better the distribution will behave, and viceversa, the smaller** a **the worse the distribution will behave.**

Substituting (4.1) in (3.1) we can get the next equation (4.2) whose solutions are the utilization values ($r_{oi}$) of the CBQ system for classes *i* with SS traffic:

$$G(N)\sum_{n=0}^{N-1}\frac{q^{n}}{1+\frac{m_i}{M}(1-z)g^{n}} = z \qquad (4.2)$$

In figure 2 we show some numerical results of this equation for a given class. We can see the evolution of the roots depending on a .

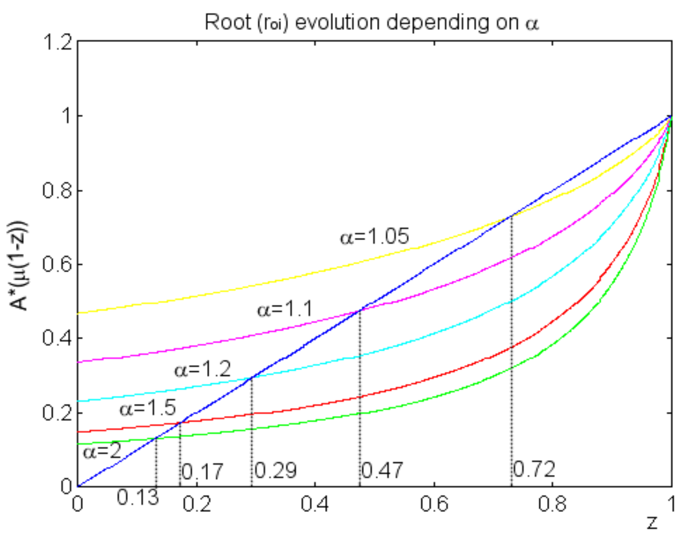


**Figure 2**: roots evolution upon a **.**

Assuming for instance in our CBQ system that the highest classes between $C \le i \le R$ have SS traffic then we can rewrite the expression (2.1) as:

$$\overline{Wr} = \frac{\overline{Wo}}{\left(1-\sum_{i=r}^{C-1} r_i - \sum_{i=C}^{R} r_{oi}\right)\left(1-\sum_{i=r+1}^{C-1} r_i - \sum_{i=C}^{R} r_{oi}\right)} \qquad (4.3)$$

In this case the system will be stable or with finite mean waiting time given a class **r** if $\sum_{i=r}^{C-1} r_i + \sum_{i=C}^{R} r_{oi} < 1$. Also analysing each term in the expression (4.3) we can relate them to different factors in the upper classes [6] like: $\sum_{i=r}^{C-1} r_i$ shows the influence of customers with Poisson distribution arrival waiting in the system when our tagged customer just arrived, $\sum_{i=r+1}^{C-1} r_i$ shows the influence of customers with Poisson distribution arrival coming while our tagged customer is waiting to be served, and $\sum_{i=C}^{R} r_{oi}$ shows the influence of customers with PT distribution arrival.

Looking at the expression (4.3) we can expect an increase in the mean waiting time $\overline{Wr}$ due to the presence of PT traffic distributions, and therefore a poorer performance of our CBQ system, specially when the shape parameter a in the PT distributions are close to 1.

## 5. Results from the Simulations.

Assuming we have a CBQ with four classes: r = 1,2,3,4 (the higher the class number the higher the priority) built in a NS-2 simulation. Four source nodes (one for each class) are injecting traffic into this CBQ link and ending into a sink node.

Some common features are: the packet are UDP and the size is constant and equal to 210bytes; the statistic distribution is related to the times between packets; the size in the four queues is infinite; the scheduling in each queue is FCFS. We have two different traffic scenarios for these four nodes:

- **Scenario A**: the four sources with ON/OFF exponential traffic distribution with an average rate parameter $\lambda$ = 19 packets/sec.
- **Scenario B**: the source with highest class priority (class 4) sending ON/OFF traffic with PT distribution (we use the Pareto distribution with the equivalent traffic rate than in the case of exponential sources and a =**1.5**) and the rest of the sources injecting ON/OFF traffic with exponential distribution and average rate parameter $\lambda$ = 19 packets/sec.

The figures 3 and 4 show the mean waiting time for each class. The class with lowest priority (Class 1) is the most affected when the scenario is changed from case A to case B.

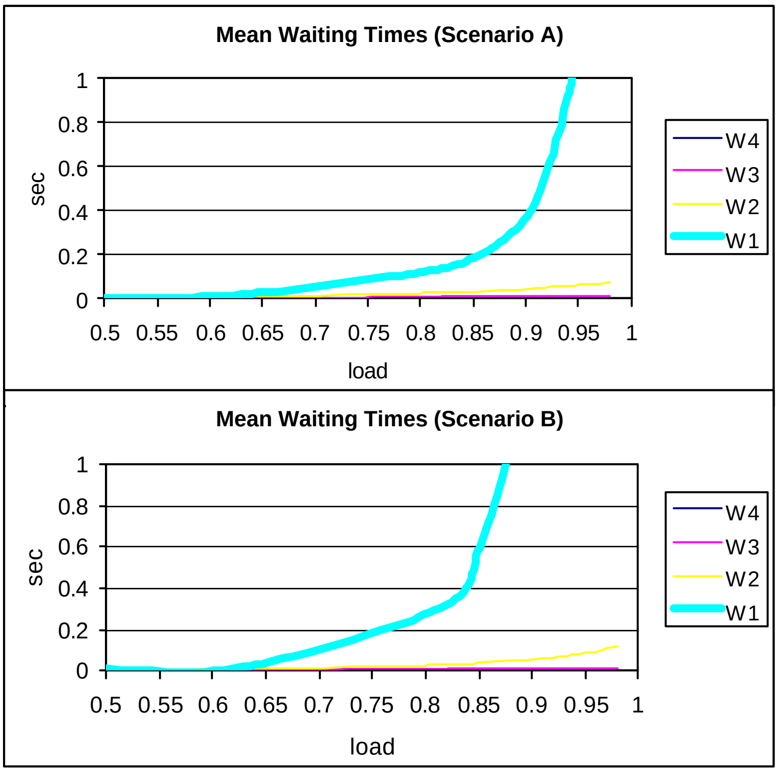


**Figure 3 & 4**: Mean Waiting Times in the queues for Scenario A and B.

## 7. Conclusions.

This paper has shown how the performance in a CBQ decreases when we have at least one class in the traffic with PT distributions. It has been shown from both theoretically and using simulations. The worst class performing, as expected, it is the class with lowest priority that could even experience infinite waiting time before being served with high load conditions (close to 0.9).

## References.

[1] S. McCanne and S. Floyd. ns--Network Simulator. http://www.isi.edu/nsnam/ns/.

[2] Gunter Bolch; Stefan Greiner; Hermann de Meer; Kishor S. Trivedi: *Queuing Networks and Markov Chains*. Wiley-Interscience., **6.15.1 System without Preemption.**

[3] Donald Gross; Carl M. Harris: *Fundamentals of Queuing Theory*. Third edition. Wiley Series in Probability and Statistics., **3.4.2 Nonpreemptive Markovian Systems with Many Priorities.**

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[5] Michael Greiner, Manfred Jobmann, Lester Lipsky: *The Importance of Power-Tail Distributions for Modeling Queueing Systems*. **3.3 Behavior of Queues based on Truncated Tails.**

[6] Kleinrock, Leonard: *Queueing Systems Vol.2 Computer Applications*. **3.6 Head-of-the-Line Priorities.**