3rd DRAFT

Thoughts on Energy BPM Chicane Design

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Goals of this exercise

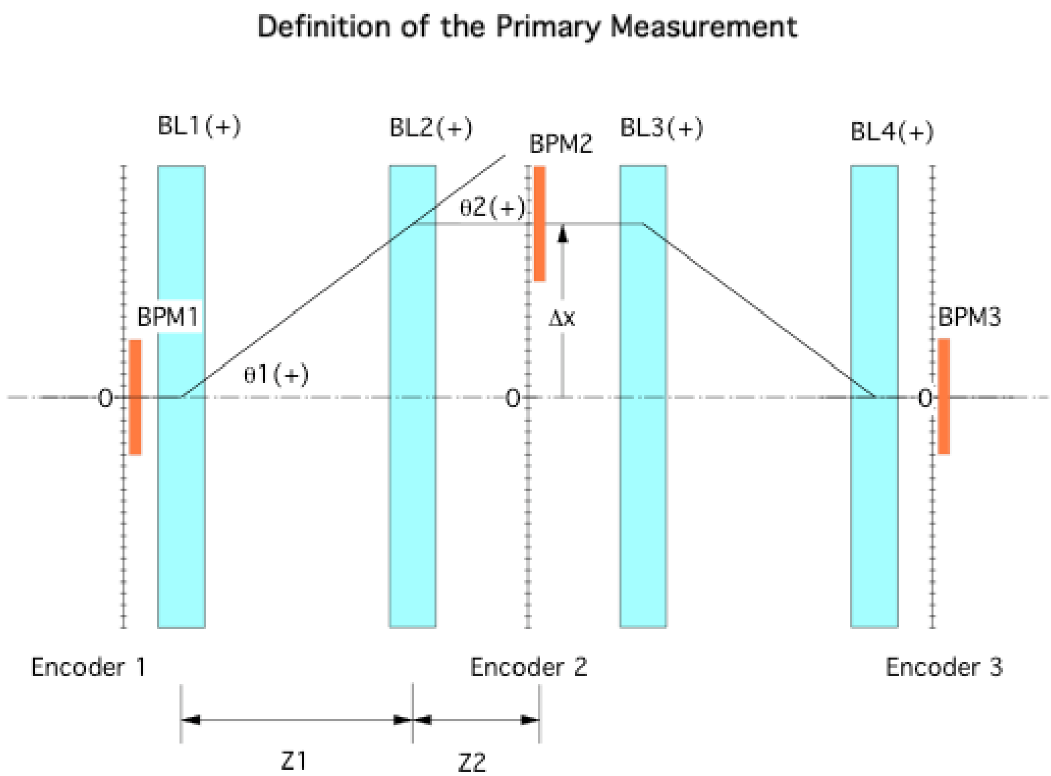
This is a collection of some of the factors affecting the performance and design choices for the energy BPM chicane. One goal here is to find where the BPM's and magnets might be placed to achieve the desired precision. Another goal is to explore the sensitivity of the error in the energy measurement to various parameters of the beam and chicane design and to possible measurement strategies. A final goal will be to try to lay out which are the more important or sensitive parameters that drive the optimization, and try to find a set of design parameters that give reasonable errors for reasonable cost. The main work will be to fiddle around with magnet positions and strengths to get the required energy measurement precision of 10^{-4} consistent with keeping the dilution of the beam emittance below 0.5% of the emittance at the IP. We will do this by scaling from the model presented by Peter Tenenbaum (PT) for which he calculated the emmitance growth. In principle the BPM resolution could enter as a parameter in the optimization, but for optimizing the design we assume for the time being that the BPM's have 1 μm resolution. This is consistent with the known performance of existing RF BPM's. Later we will look at how the BPM resolution enters the optimization. We also assume a mechanical arrangement where drifts during the measurement are constrained by design or measured and corrected to better than 1 μm. We give some ideas on how this can be achieved and explore the consequences if it is not.

Before going into details I should say that I am a novice at linear colliders and precision energy measurements, and these notes borrow heavily from hero's work by those who came before me. In particular I highly recommend the recent preprint by Mike Hildreth and the LEP Energy Working Group (CERN-PH-EP-2004-032). That work is the foundation from which we hope to extend and improve. Also I borrow heavily from the FFTB style of doing things which is summarized nicely in an overall description of the FFTB instrumentation is given in:
epaper.kek.jp/p95/ARTICLES/TAG/TAG01.PDF
Most of what I know about the linear colliders I learned from some great accelerator physics lectures in June 2003 by Nick Walker, PT, Andrei Seryi and Andy Wolski that you can find at
http://www.desy.de/~njwalker/uspas/
This is where I learned the gory details of how synchrotron radiation changes the emmitance.

I regard this as a first hack at getting down some numbers that will be modified as we go to fix errors and refine the ideas. It's a bit long but I found it useful to learn the ropes. Please criticize and make suggestions.

<u>Chicane layout and definition of the measurements</u>

We first describe the layout of the chicane we have in mind and specify the parameters which must be measured and perhaps kept steady over time to determine the beam energy. (An alternative chicane design is also examined below.) The chicane layout is a comprised of a series of 4 magnets B1-B4 of lengths L1-L4 arranged as in the figure below. There are three BPM stations with BPM1 and BPM3 positioned outside the chicane and BPM2 positioned between B2 and B3. The BPM's are moveable with x position measured by accurate encoders. The separation Z1 between B1 and B2 is longer than the separation Z2 between B2 and BPM2.



We are going to explore tradeoffs in the design that affect contributions to the total error from uncertainties in these parameters. We also explore the contributions to the total error from BPM resolution, dispersion in the chicane, and strategies for measuring and correcting for beam jitter. The various measurement strategies will involve averaging measurements over various numbers of beam bunches, which will set the time scales for drifts to be monitored or kept constant. These time scales and the precision with which various "constant" factors entering 1)-5) above can be monitored or kept really constant will also affect the total error, and help determine the optimum strategy.

Definition of the primary measurement

Ignore for the moment the variations of incident beam positions and angle from beam jitter, and consider a system with identical magnets, BPM's with gains calibrated and encoder values known for beam in their electrical centers, and beam on the axis of BPM1 and BPM3. The primary energy measurement is obtained from absolute measurements of the quantities:

1) BL1(+) – total integral BdL in B1 along the beam path with magnets in the + polarity.
2) BL2(+) – same for B2
3) Z1 – B1-B2 separation(m)
4) Z2 – B2-BPM2 separation (m)
5) Δx – encoder offset (m) of BPM2 center from straight ahead line.

From this definition of the primary measurement we can already draw some conclusions about the measurement strategies and contributions to the total error:

1) The uncertainty in BL1 is more important than the uncertainty in BL2 by the ratio Z1/Z2. Since Z1 is larger than Z2, the offset at BPM2 is dominated by BL1.
2) The B1 and B2 form the primary instruments of the chicane. The bends B3-B4 are used to get the beam back on line but do not provide an independent measure of the energy because there is only one measure of position between them. The BPM stations may have pairs of BPMs, but we use them for redundancy not for independent measurements. BPM3 can be used to constrain the beam jitter, but without an independent measure of position before B3, the B3-B4 bends do not give a measure of Δx.
3) In principle a measurement of the energy could be made for each beam bunch. In this case the measurement of Δx offset from dispersion would be from comparing the x position of the bunch extrapolated to the z position of BPM2 with the x position of the same bunch measured in BPM2. This assumes that all the "constant" factors that enter parameters 1)-5) above are know. The time scale on which they have to be kept constant is the duration of one bunch. In principle if "constant" factors drift between beam bunches and can be measured fast enough, the drifts could be corrected so that each bunch would be an independent measurement of the energy.
4) We will see below that beam betatron jitter is dominated by x position jitter not by x angle jitter. This leads to three possible strategies for dealing with betatron jitter. The first is to use only BPM2. When beam jitter is small or comparable to the BPM resolution, we could average over many bunches to average out the beam jitter. When jitter is large, as we will see it is, this is not an efficient measurement strategy. In that case we should use measurements of x(BPM1) and x(BPM3) to determine the beam jitter correction to the corresponding measurements x(BPM2). The second method is to use only BPM1, together with the z positions of BPM1 and B1 and the strength BL1, to extrapolate the measurement in BPM1 to BPM2. The unmeasured betatron angle jitter would be averaged over. The third method is to use both BPM1 and BPM3 to measure both x and angle jitter. This also requires knowing z positions of B3, B4, and BPM3 and magnet strengths BL3 and BL4. The position in BPM1 and BPM3 from betatron x jitter is completely correlated. There is only one degree of freedom (x position), and the extrapolation from BPM1 and BPM3 to the value of the beam x position at the z of BPM2 (for equal distances between BPM1-BPM2 and BPM2-BPM3) is simply the

average of BPM1 and BPM3. $$x(BPM2)=\frac{x(BPM1)+x(BPM3)}{2}+/-\frac{\Delta BPM}{\sqrt{2}}$$

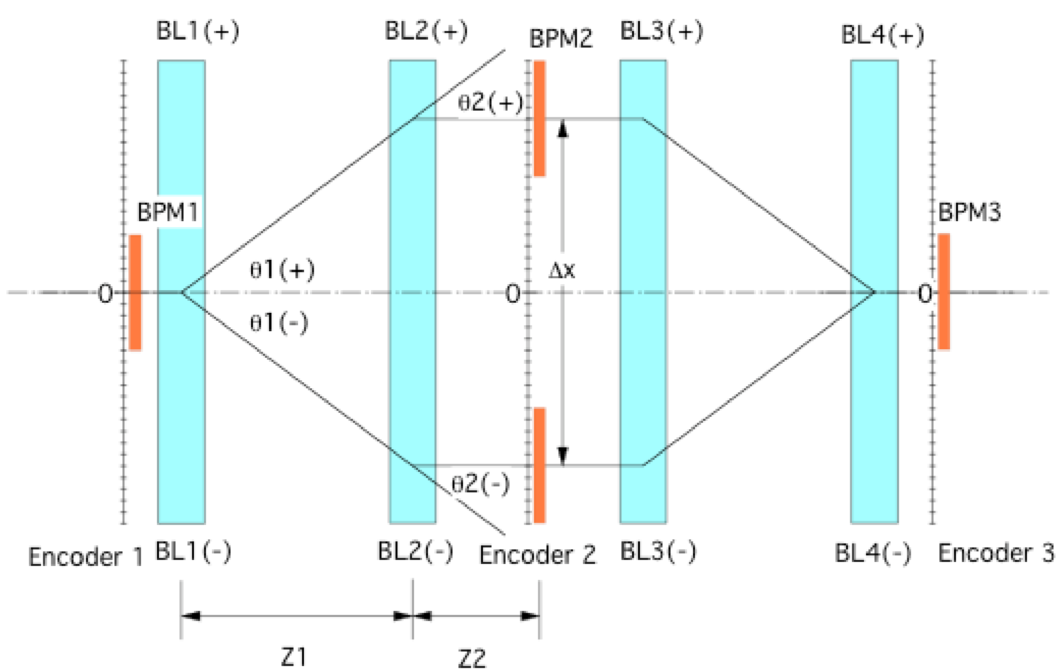
where ΔBPM is the BPM resolution

5) Correcting for betatron jitter by extrapolation of the x position measured in BPM1 and BPM3 to the z of BPM2 requires that we have a cross calibration of the x encoder positions of BPM1 and BPM3 with respect to the encoder of BPM2. In the figure we have drawn the encoders as though the zero positions were lined up in space, but all that is required is that they be cross registered in x. To obtain this calibration requires a separate measurement. Ideally, if the magnet fields could be set to exactly B = 0, then a measurement of the encoder positions that put the beam at the electrical center of each BPM would suffice. The error from this cross calibration must be factored into the total error on the measured energy.

Definition of the extended measurement

In the following discussions we are going to explore extensions of the primary measurement to include two additional features: a) averaging over some number of bunches to average out errors from BPM resolution and/or from beam jitter, b) extracting the Δx offset from measurements with B fields set to +B compared to B fields set to –B. The extended measurement is illustrated in the following figure.

Definition of the Extended Measurement



As in the case for the primary measurement in only one magnet polarity, ignore for the moment the variations of incident beam positions and angle from betatron jitter, and consider a system with identical magnets, BPM's with gains calibrated and encoder values known for beam in their electrical centers, and beam on the axis of BPM1 and BPM3. The primary energy measurement is obtained from absolute measurements of the quantities with magnets in the +B state:

1) BL1(+) – total integral BdL in B1 along the beam path with magnets in the + polarity.
2) BL2(+) – same for B2
3) Z1(+) – B1-B2 separation(m)
4) Z2(+) – B2-BPM2 separation (m)
5) Δx(+) – encoder offset (m) of BPM2 center

together with measurements of the corresponding quantities in the –B state.
Measurements are made with field in the +B configuration for some number of beam bunches extending over some number of beam pulses (at 5 Hz). Then the magnet polarities are reversed to the –B state, BPM2 is moved to the Δx(-) position and a new set of measurements of 1)-5) are made at some later time. The beam energy is then obtained from the total offset

$$\Delta x = \Delta x(+) + \Delta x(-)$$

and knowledge of all the "constant" factors in 1)-5).

The extended measurements cannot be made for single bunches. The extended result is extracted from average quantities measured over some number of bunches in the +B and –B state. This introduces additional sources of errors from the comparison of system parameters in two states at different times. Extraction of the energy from the sum Δx = Δx(+) + Δx(-) makes the result more insensitive to some parameters, in particular to the encoder cross calibration obtained with B = "zero".

From this procedure we can draw some immediate conclusions about measurement strategies and sources of error on the measured energy:

1) The general statements from the primary method still apply: a) Uncertainty in BL1 is still more important than the uncertainty in BL2 by the ratio Z1/Z2; b) The bends B3 and B4 do not provide an independent measurement from the bends B1 and B2.
2) The total error now also depends on the uncertainty of BL1 and BL2 in the –B state.
3) As for the primary method, the motion from betatron jitter can be accounted for by a) using only BPM2 and averaging over the jitter, b) use BPM1 to correct for position jitter in BPM2, c) use BPM1 and BPM3 to correct for position and angle jitter in BPM2. We will see below that the jitter is large compared to BPM resolution, so not correcting for jitter will be an inefficient strategy.
4) If BPM1 and BPM3 are used to correct for beam jitter at BPM2, then it is required to have accurate cross calibration of BPM1, BPM2, and BPM3 encoders. In addition to quantities 1)-5) above, it is also required to know accurately the z positions of B1, B3, B4, BPM1, and BPM3, and the field strengths BL3, and BL4.
5) For the extended energy measurement there are two time scales for monitoring and correcting for drifts or, better yet, for keeping the "constant" factors in 1)-5) really constant:

a) the duration of one set of beam bunches required to measure the encoder position for Δx(+) and Δx(-)
b) the time required to change from +B to –B state.

These time scales could range from several minutes to several hours. In principle, the extended measurements have twice as many constant factors as for the primary energy measurement, because all of the items1)-4) could vary with time or B state and must be measured. In practice it is likely that the lengths Z1 and Z2 will not vary significantly. Since the extended measurement requires significant lapsed time and also requires cross calibration of the BPM encoders for jitter correction, the possible drift in the x position of the BPM bases that hold the encoders must be monitored.

Now lets spec some parameters:

Beam Parameters

We assume beam parameters from a combination of the Tesla TDR accelerator design grafted onto the NLC beam delivery system. Since the new ILC is not yet designed there are no actual values for any parameters yet for this hybrid system, and it is likely that some of the important parameters of the new design will change from those in the Tesla/NLC documents. We will design a chicane to work for E_beam = 500 GeV (same as the PT model) and note how the performance will change at E_beam = 250 GeV. The Tesla TDR design has the macroscopic pulse rate of 5 Hz, with 2820 bunches per pulse, separated by 337 ns. The ILC design is now being reconsidered and there is some discussion of shortening the bunch spacing by half to 168 ns. We should pay attention to see if these parameters change will they bother the optimization of our BPM energy chicane design.

The normalized horizontal and vertical emittances at the IP from the Tesla TDR for E_beam = 250 and 500 GeV are:

E = 250 GeV	E = 500 GeV	
$\gamma = E/m_ec^2 = 0.489 \times 10^6$	$\gamma = 0.978 \times 10^6$	
$\gamma\varepsilon_x^* = 1000 \times 10^{-8}$ m-rad	$\gamma\varepsilon_x^* = 700 \times 10^{-8}$ m-rad	(scaled from E=400)
$\gamma\varepsilon_y^* = 2 \times 10^{-8}$ m-rad	$\gamma\varepsilon_y^* = 1 \times 10^{-8}$ m-rad	

Since the energy chicane will dilute the emittance primarily in the bend plane, it must bend in the horizontal so the emittance we are trying not to make worse is the large one.

For our scaling exercises comparing our models to the model by PT we have some extra room to play because he obtained the 0.5% emittance growth from the chicane compared to $\gamma\varepsilon_x^* = 300 \times 10^{-8}$ m-rad expected from the output of the damping ring in the NLC design. This gives us a factor of 8/3 = 2.66 room to spare.

Next we look at expected beam sizes and jitter at the proposed place for the chicane. We calculate typical values of the beam size and angular spread from betatron jitter using:

$$\sigma = (\varepsilon\ \beta\ )^{1/2} \qquad \text{and} \qquad \theta_{RMS} = (\varepsilon/\beta)^{1/2}$$

The typical $\beta_x^{1/2}$ in the chicane is around 50 $m^{1/2}$. This gives:

$\sigma_x = 226 \times 10^{-6}$ m and $\theta_{RMS} = 9.0 \times 10^{-8}$ rad for E_beam = 250 GeV
$\sigma_x = 159 \times 10^{-6}$ m and $\theta_{RMS} = 6.4 \times 10^{-8}$ rad for E_beam = 500 GeV

If we assume beam position jitter to be $0.25\sigma_x$ to $0.5\sigma_x$ (PT mentioned $0.25\sigma_x$), this gives beam position jitter in the chicane of 55 to 110 μm at 250 GeV and 20 to 80 μm at 500 GeV.

From this we learn several interesting things:

Spot size and position jitter are large
The beam spot sizes and betatron position jitter are large (tens to hundreds of microns) compared to the position resolution of around 1 μm we will be aiming for in the BPM's. This has several implications.

1) The BPM's must work well with spot sizes large compared to their required resolution. This is a technical issue for BPM experts to tell us about sensitivity to bunch size, bunch tilt, non Gaussian distributions, and other diseases.
2) To extract the energy measurement from position measurement in BPM2 we must correct for the incoming position jitter, which on a bunch-by-bunch basis is 100 or so times the BPM resolution. Measuring and correcting for incoming position jitter will degrade the overall error.

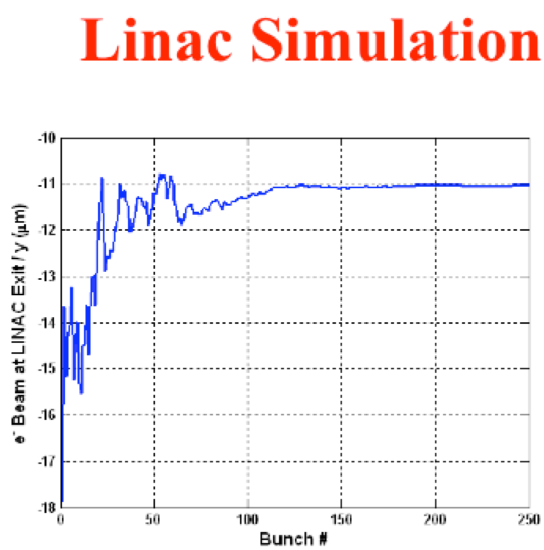
Betatron angle jitter is too small to measure in BPM pairs
The next thing we learn is that it will not be practical to measure the jitter of the angle of the beam before or after the chicane with a pair of reasonably spaced BPM's with 1 μm resolution. The distance Z between two BPMs with 1 μm resolution needed to resolve the $\theta_{RMS} = 6.4 \times 10^{-8}$ rad (adding 1 μm resolutions in quadrature) would be approximately:

$$Z = 2^{1/2} / \theta_{RMS} = 22 \text{ m}$$

which is much longer than the space we will have available. The small relative size of the betatron angle jitter has several consequences for the design.

1) The spacing between adjacent pairs of BPM's any place in the system should be determined by mechanical constraints and not be any desire to measure the beam angle. We will use pairs of BPM's at each BPM station for redundancy and to measure the relative resolutions. The pairs will be placed as close together as practical given their overall length and the need for connecting flanges and bellows.
2) The BPM's before and after the chicane can be viewed as primarily measuring the x position jitter for correction of the measurement in BPM2, leaving the much smaller position jitter in BPM2 from angle jitter to be averaged out by averaging over many beam bunches. More on this later when we discuss the measurement strategies.

Beam motion with feedback systems
Some more interesting features of the expected incident beams can be seen in the studies of the feedback systems envisioned to obtain and maintain beams in collision. To get and keep collisions there will be an elaborate feedback system using BPM’s and steering correctors throughout the linac and the BDS. Each bunch is launched into the linac with a separate kick from the damping ring and so can get a separate launch error which has to be taken care of by feedbacks. The energy chicane will be embedded in the middle of this feedback system and the beam will be moving around in position over the length of the 2820 bunch train. Some indication of this can be seen in a plot from a presentation on Fast Feedback by Glen White to Working Group 4 at the recent ILC-America workshop given here:
http://www-project.slac.stanford.edu/ilc/meetings/workshops/US-ILCWorkshop/wg4.html



This result from a set of seeds (a similar plot is shown in the Tesla TDR in Fig 3.2.9) shows how typically the beam at the end of the linac moves around by some tens of microns (this case in y), and the stable position is not obtained from the feedback until 150 or so bunches have passed. The motion in x will be similar. The details of the expected performance are not known at this time, so for the time being we will use the best guess from PT who suggests that the beam will move about 0.25 to 0.5 of σ_x.

Consequences of beam parameters for the energy chicane measurements

These features of beam motion during the pulse and action of the feedbacks have several implications for the energy measurements.

What kind of BPM’s to use?
The first is that the energy spectrometer should measure the energy of those bunches likely to be used for luminosity. That means it has to know which bunches are not colliding and be able to resolve them and remove them from the energy measurement. This puts a requirement on the time resolution of the BPM’s. Currently we are thinking of using RF BPM’s which have an

intrinsic time resolution determined by the Q of the cavity. For high Q the cavity rings for along time, and thus the output signal in a given small (few ns) time interval contains contributions from many bunches. The BPM's from SLAC experiment E158 had Q around 3000 and 1/e decay time for a single bunch of about 300 ns and they obtained resolutions of about 2 μm for 110 ns long pulse trains. The ILC bunch spacing is only 337 ns (or perhaps 176ns). Using BPM's with such high Q has the ugly consequence that at any given time the BPM signal is the sum from some number of previous bunches each weighted by their charge and offset and signal decay. The design of the RF BPM's and the data acquisition and analysis system must be optimized to permit resolution of single bunches and provide measurements and corrections for charge and decay time on each bunch.

No beam motion for BPM calibration

The second implication from the needs for feedback in the region of the energy chicane to maintain luminosity is that it is not a good idea to use movements of the beam from upstream correctors to establish the BPM gain calibrations. Frequent gain calibrations would be useful for monitoring stability of electronics. Movements of the beam outside the luminosity feedback system will interfere with luminosity and either make gain calibrations infrequent or make the energy measurements very unpopular. An alternative is to incorporate the BPM calibration offsets in the feedback algorithms, but that sounds complicated. The designs discussed below assume the BPM gain calibrations will be obtained by moving the BPM's leaving the beam fixed. Such measurements would be invisible to the luminosity (baring effects from wakefields in the BPM's) and could be made as often as the speed of BPM motion would permit.

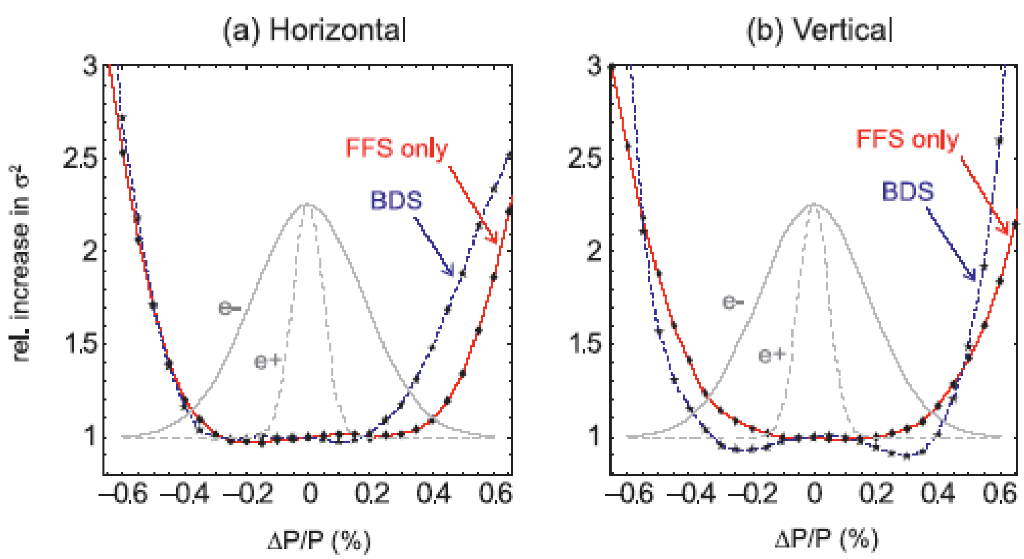


Figure 7.2.6: *Energy bandwidth for the (a) horizontal and (b) vertical plane. The bandwidth for the entire BDS and only the FFS are indicated. The energy profile of both e^- and e^+ are also shown for comparison.*

Beam energy spread and jitter

Next we look at the expected energy spread and jitter and time scales for energy variations for the various beam scenarios. The figure above from the Tesla TDR shows the single bunch energy spread in the BDS along with the energy acceptance of the Tesla BDS. The electron energy RMS width is $\sigma_E/E = 1.5x10^{-3}$ and the positron width is $\sigma_E/E = 0.6x10^{-3.}$

These widths are determined by the linacs, the undulator positron source, damping rings, bunch compressors, and other components. The total energy spread of single bunches from the linac will be $\sigma_E/E = 6x10^{-4}$ from correlated spread (energy variation correlated with z of the bunch from off peak phase and BNS damping) and uncorrelated spread from the source. The bunch-to-bunch energy jitter expected from the linac will be determined by the fast feedback used to compensate for beam loading and Lorentz forces on the cavities. The Tesla TDR says the feedback will keep residual energy variations along the bunch train to $5x10^{-4}$ so that it is not much larger than the single bunch spread. The electron energy spread is increased to 0.15% at the IP in the Tesla design by passing the electrons through the undulator to create positrons. The undulator increases the energy spread of single bunches and shifts the centroid down by 1.3% at 250 GeV. Since radiation in the undulator is a sum of many small losses the centroid shifts down and the energy spread of the bunchs increases but the jitter of the centroid does not increase. The jitter in the bunch energy centroid is still governed by the errors from feedback in the linac and is still $5x10^{-4.}$ The momentum bandwidth of the BDS is only 0.4%, so the jitter must be kept lower than that or there will be significant problems from backgrounds and ultimately for machine protection. Energy errors larger than 2% will provoke the beam abort system to dump subsequent bunches.

The energy spread and jitter for the new ILC design could (will) be different from these numbers. In particular it is possible that for early running there will be a conventional positron source, so the electron energy spread will not be increased by an undulator. For purposes of estimating errors here we will assume the single bunch spread is $\sigma_E/E = 6x10^{-4}$, and that the bunch-to-bunch deviations of the bunch centroids are kept to $5x10^{-4}$ by the feedback.

On time scales of a few pulses to many minutes we assume that the beam energy does not change outside the 2% abort window, and will be varying only due to energy jitter (with feedbacks included). It is this time scale that is relevant for the beam energy measurements. We are aiming to make measurements frequently and complete a measurement within a few minutes while the beam is in some stable configuration.

On time scales of hours to days it is likely that the beam energy will change, due to changes in klystron configurations, beam phase, or whatever. The beam energy measurements need to be done often enough to accurately sample the variations in beam energy over hours.

Effect of bunch energy spread and energy jitter on spot size and position in the chicane
Jumping ahead lets assume we have a chicane with 5 mm dispersion in BPM2. Taking values of the single bunch energy width or bunch-to-bunch energy variation to be those that exit the linac, the spread of the beam spot size in x from dispersion would be $(5x10^{-3})x(6x10^{-4}) = 3.0x10^{-6}$ m. This is much smaller than the undispersed spot size (160 to 220 μm) or the betatron x jitter (20 to 100 μm). This would be the case for the situation where electrons are not passed through the undulator to make positrons. After the undulator the electrons with energy spread $\sigma_E/E = 1.5x10^{-3}$ would give a spot size dispersion to $(5x10^{-3})x(1.5x10^{-3}) = 7.5x10^{-6}$ m. This is still much

smaller than the undispersed spot sizes. Our chicane will not make the spot sizes significantly larger than the undispersed spot sizes.

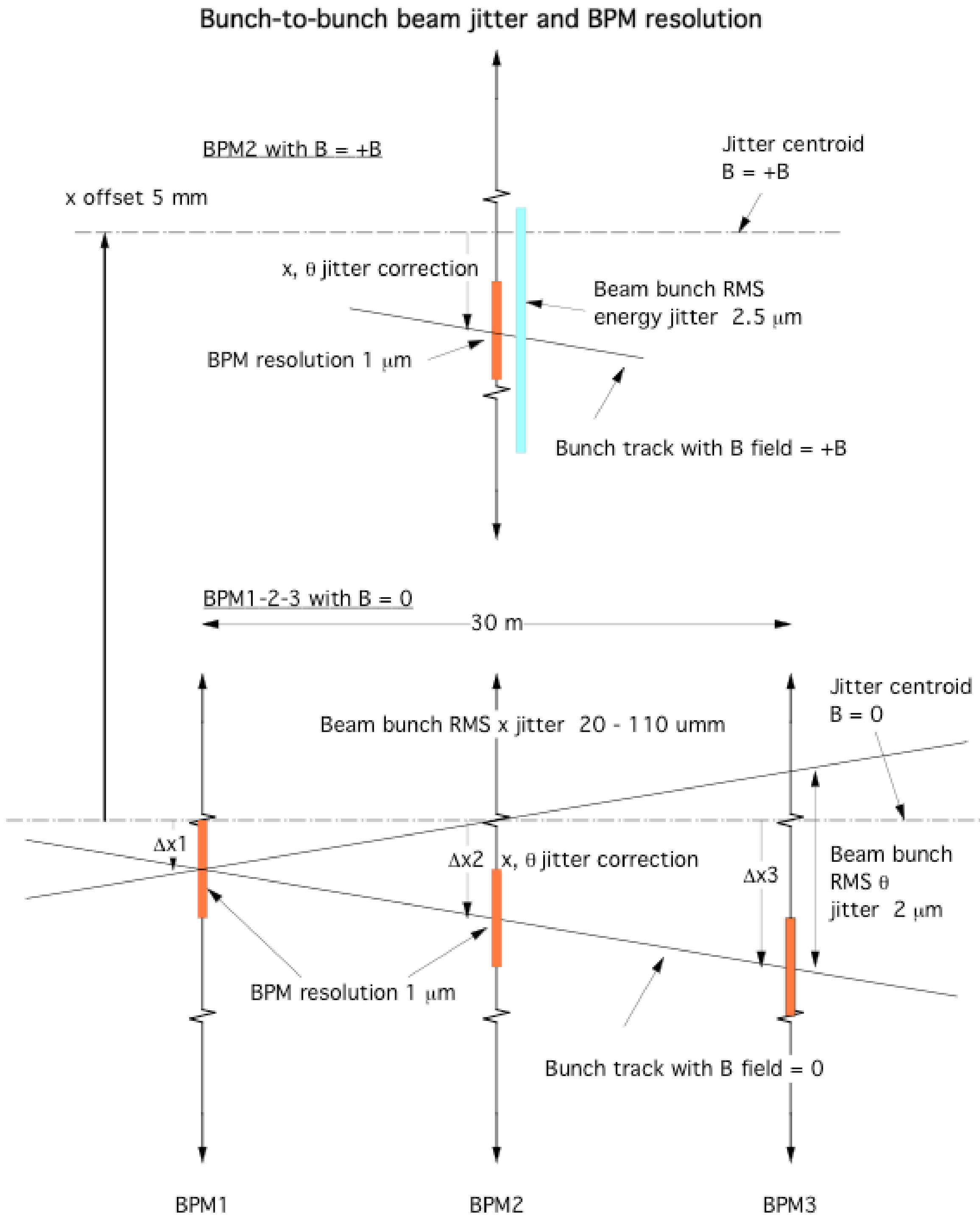
The position jitter from bunch-to-bunch energy variation of the bunch centroids will be $(5x10^{-3})x(5x10^{-4}) = 2.5x10^{-6}$ m. This is slightly larger than the position resolution of 1 μm we are aiming for in the BPM's.

Effects from bunch charge jitter
The signal from an RF BPM depends on the bunch charge q_b. For cylindrical cavities the signal from the dominant monopole mode is proportional to charge q_b but independent of position. For the rectangular cavities used to measure x and y the dipole mode signal is proportional to the product of charge and the beam offset x(BPM) or y(BPM) from the cavity electromagnetic center, $V_x \sim q_b x(BPM)$ and $V_y \sim q_b y(BPM)$.

According to the TESLA TDR the bunch charge can vary smoothly by up to 5% over the 2820 bunch train and there can be bunch-to-bunch jitter of up to 1%. The TESLA TDR describes how long term drifts in beam dynamics from variations in position, energy, and bunch charge are compensated by the feedback system in the linac. The long term charge variation and bunch-to-bunch charge jitter that is not corrected appears in the beam. It is likely that variations in positron bunch charge will be larger than variations in electron charge.

The consequences from variations in bunch charge for the BPM energy spectrometer when using RF BPM's is that the bunch charge must be measured and accounted for on each bunch. The precision to which bunch charge is measured and the signals V_x and V_y are corrected affects the measurement strategy. For example, if the bunch charge is measured to 0.1%, which is possible for good toroids, then the BPM single bunch measurement of Δx after correction for charge will contain a jitter of 0.1% of qx(BPM). The requirement for averaging out the residual jitter from bunch charge enters into the optimization of the tradeoffs between the BPM resolution, number of pulses to be averaged, and the number of BPM's used to measure Δx.

Summary of betatron and energy jitter effects on the measurements
The various effects from beam jitter are summarized in the following figure.



Energy measurement goals and possible measurement strategies

Given the expected beam parameters and BPM performance there are several possible measurement strategies. The primary issues we consider here is how to deal with beam jitter and beam motions including feedback over the bunch train. In this discussion we ignore for the time being the errors from uncertainties in absolute values or drifts in magnetic fields, BPM positions, electronics noise and other sources.

First we should note what are the physics driven goals for measuring the energy to 10^{-4} precision. We do not need to know the energy for every single bunch. What is required is that the energy be know with precision for some reasonably small length of time (minutes to hours) so that it can be correlated to the events produced with a given luminosity. Most interesting physics data will only be statistically useful after many days or even many months of data. In that case the time scale for sampling the beam energy and obtaining precise averages will be set by the long term (over hours) variations in the beam energy from the linac operations. We are envisioning a system that involves frequent cycles through:

1. Calibrations by movement of the BPM's with stable beam,
2. Measurements of baseline beam positions with magnetic fields off and BPM's aligned on straight ahead beam, and
3. Measurements with chicane fields on +B and –B and BPM2 moved to the offset beam.

The time scale for one cycle is yet to be determined but will probably be on the order of some minutes, and less than one hour.

Calibrations of BPM gain and relationship to encoder values
The BPM gains and the relationship between BPM electrical center and encoder readings for each BPM would be determined by moving the BPM's through the beam in a series of steps with beam in stable position. The BPM gain is the value of BPM output (volts) versus μm of motion. The encoders are envisioned to have precision of sub μm over a range of many mm. The electrical center of the BPM's is determined by the encoder position where the signal goes to zero. This requires measurements at a number of x points each averaged over some number of beam bunches to average out the beam jitter. This procedure requires that the bunch charge be measured, or that jitter in bunch charge be averaged out. The procedure for measuring gains and encoder calibrations, probably including iterations to get the jitter corrections, needs to be optimized to suppress the error below the BPM resolution.

Cross calibration of relative encoder settings
In order to relate BPM data from one to the other in terms of relative position in space it is necessary to cross calibrate the encoder values for some known beam. The easiest way to do this is to set the B = 0 in all magnets and use the straight beam as a reference. The procedure for doing this will depend on what are the residual fields, if any, in the magnets when they are in the "zero" state. The possible errors from unmeasured or miss understood residual fields are explored below.

Measurements of Δx
The energy measurement is to be extracted from measurements of the total offset $\Delta x = \Delta x(+) + \Delta x(-)$ at BPM2. There are several factors that influence the optimum strategy for doing this.

1. BPM resolution, assumed for now to be ~ 1 μm

2. Position jitter at BPM2 from bunch-to-bunch energy jitter, ~ 2.5 μm for 5 mm beam offset
3. Position jitter at BPM2 from betatron x jitter ~ 20 to 100 μm
4. Position jitter at BPM2 from betatron angle jitter ~ 2 μm
5. The jitter in the BPM2 signal from errors in accounting for bunch charge, which we take to be ~ 0.1% of V_x ~ q_bx(BPM) where x(BPM) is the beam offset from center in the BPM2. The value of this correction will depend on where in the available BPM dynamic range the beam is positioned. The dynamic range of the BPM is a parameter of the design influenced by performance requirements for BPM resolution, signal/noise, and ease of operation. The dynamic range needs to be large enough to cover the excursions of x(BPM) spanned by the position jitter, and it needs to be large enough for convenient operation with beam motion from feedbacks. If the BPM dynamic range is 10 times the betatron x jitter, or 1000 μm, this would give a maximum jitter of 1.0 μm from 0.1% error in the bunch charge correction at the edges of the dynamic range.

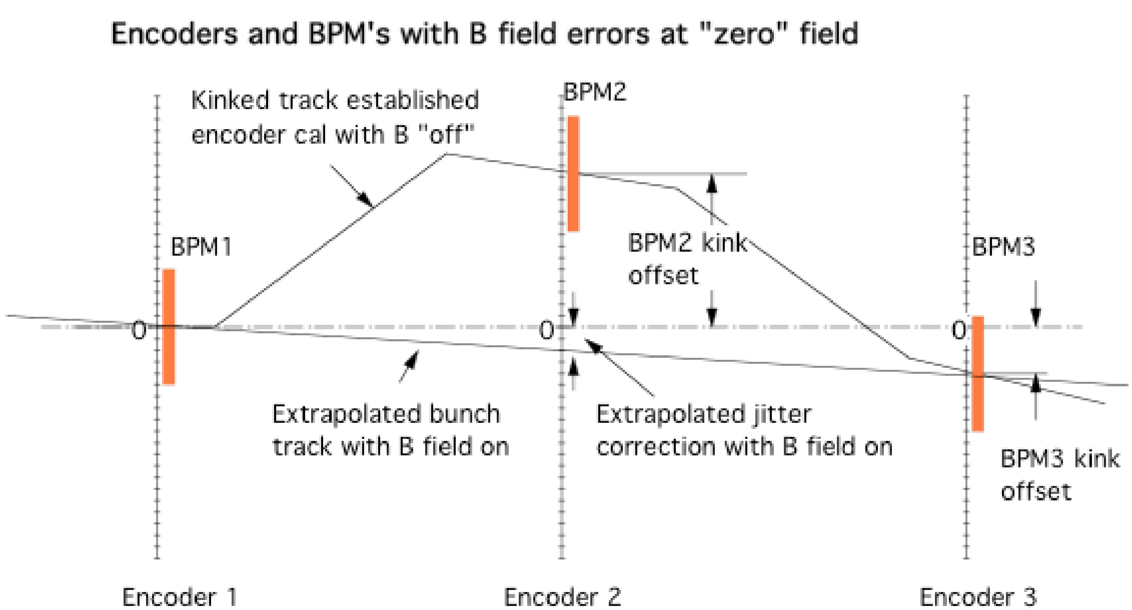
Three measurement scenarios can be considered:

1. Use only measurements of Δx(+) and Δx(-) from BPM2. Then all factors above that give jitter in the BPM2 signal would need to be averaged over in some number of bunches. This is an inefficient method because the betatron x jitter is very large, factors of up to 100 larger than the BPM resolution and other sources of jitter. Averaging down to 1 μm would consume at least 10^4 bunches, which would stretch over as many as 4 to 5 pulses of 2820 bunches.
2. Use measurement in BPM1 to correct BPM2 for x betatron jitter. This would reduce the position jitter that needs to be averaged down to the quadrature sum of jitter 1), 2), 4), and 5) above, totaling in the range of 5 to 10 μm. To achieve energy error of 10^{-4} for total beam offset of 1 cm requires averaging over about 100 bunches. This method would also bring in sources of error from knowledge of z positions of BPM1, and cross calibration of BPM1 encoder with BPM2. These additional errors would have to be kept small enough to ensure that the improvement in efficiency from correcting for x jitter did not compromise the total error.
3. Use measurements in BPM1 and BPM3 to correct for both x and angle betatron jitter. This would make a small improvement in the efficiency. This method would also bring in sources of error from knowledge of z positions of BPM3, B3, and B4, from cross calibration of BPM3 encoder with BPM1 and BPM2, and from knowledge of the field BL3 and BL4. Correcting for the relatively small betatron angle jitter would only make sense if these additional errors could be kept small enough to ensure that they did not compromise the total error.

The bottom line from constraints from beam jitter and BPM resolution is that, with correction for x position jitter, we can imagine making an average Δx(+) or Δx(-) measurement with 10^{-4} statistical error on some subset of the bunches in each beam pulse of 2820 bunches. The Δx measurements for +B field could be averaged and assembled with those for the –B field to extract the energy using the extended measurement scheme. The absolute energy precision will then depend mostly on how well we measure the parameters other than position in the BPM's and on how well we can monitor drifts between beam pulses and between +B and –B and make corrections.

Sources of systematic error from imperfect magnetic fields
The energy measurement requires knowledge of the absolute values of BL, magnetic field integrated over the path length in the magnets. The primary requirement is knowledge of the absolute value of BL1, with less precision required for BL2 because of the shorter distance between B2 and BPM2. The fields in B3 and B4 don't contribute to the energy spectrometer proper, but they are required if we want to use BPM3 to measure the jitter. In addition to needing to know the absolute values of BL1 and BL2, there are some additional requirements on the variations in the BL among the magnets. To the extent that the individual BL are not equal in all four magnets, such that the bend angles in each are not exactly the same, there are several sources of systematic error that can occur. If the BL are not equal in all magnets and the variations can change from one measurement state (B = 0, +B, -B) to another and the variations are not known, then there can be bias in energy measurement and in the jitter correction.

One source arises from possible variations in residual BL when the magnets are nominally at B = "zero", which could arise from hysteresis and from the variations in the previous history of the magnet fields from variations in operational cycles. Magnetic fields at "zero" are harder to measure accurately than the large ones at +B and –B because NMR techniques do not work. The question is, how big would be the effect on the jitter corrections if the "zero" fields were not known or poorly constrained. The size and difficulty of dealing with this effect might influence the design of the magnets, for example tilt the design toward super conducting magnets with no iron in them that could be really turned off.



A sketch of the kinked track from bend in typical residual fields in magnets at B = "zero" is given above. When the residual fields are not B = 0, the encoder cross calibrations at BPM2 and

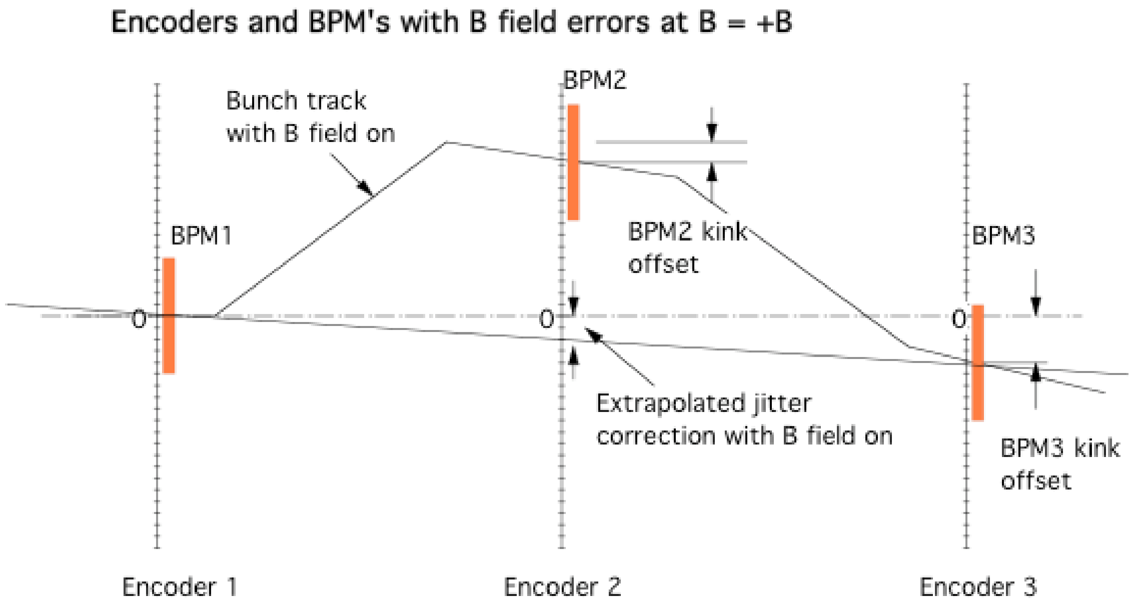
BPM3 will have offsets when normalized to the kinked beam. When the magnets are then turned to B = +B and BPM1 and BPM3 are used to extrapolate the jitter correction to BPM2 there will be a systematic offset that will give a bias in the jitter correction. If the residual fields depend on the history of the magnet cycles, as they will, then the bias correction will depend on the sequence of the data taking.

To give an estimate of the potential size of the kink angles from residual fields consider that residual field in a typical iron magnet might be 10's of gauss, and the distance from B1 to BPM2 or from B3 to BPM3 could be 15 to 20 m. The kink angles would be in the range 1 to 3 $x10^{-6}$ rad for magnets from 1 to 3 m length and the kink offsets at BPM2 and BPM3 might be as large as 20 to 50 μm. These would be large offsets compared to the jitter corrections and to the absolute precision around 1 μm we are seeking, so control of the bias in the jitter correction would have to be done carefully. This would require either measuring the fields near B = "zero", or using operating procedures in the Gaussing cycle of the magnets to ensure that the jitter bias is the same for +B and –B states, or building iron free super conducting magnets that can be certified to be truly off. There is also the possibility of characterizing kink offsets by measurements of encoder cross calibrations after various magnet cycles, leaving the beam in a steady state. These measurements would cost beam time and would be subject to their own errors from precision and reproducibility that would have to be factored in.

The kink offset in the calibration of BPM2 encoder at B = "zero" does not affect the final energy measurement if we are using the extended method with measurements and $\Delta x = \Delta x(+) + \Delta x(-)$. In that case all we need is the total distance in encoder clicks for BPM2 between the +B and –B states. It does not matter that the zero of the encoder is offset from the straight beam. This assumes that the same encoder calibration at B ="zero" is used for analysis of data for +B and –B states. This puts a limitation on the data taking cycle. Insensitivity of the final energy from kinks at B = 'zero" is a primary advantage of the extended method, making it relatively insensitive to encoder cross calibration.

Another source of systematic bias arises if the BL are not all the same in all magnets in the +B and –B states. The effects are illustrated in the figure below.

This case is different from the B ="zero" case in several respects. First of all the large fields can be accurately measured using NMR, so the central values should be well known. It will be necessary to carefully measure the magnets so the absolute values of integral BL versus excitation and for various paths are known accurately. The kink offsets illustrated in the figure above represents errors in bend angles that are not understood. They would affect the BPM2 measurement and the extrapolated jitter correction as illustrated. The kink offset of BPM2 would give a bias in the measured Δx that would directly bias the energy measurement. The kink offset of BPM3 would bias the extrapolated jitter correction. These biases would not bother the total energy measurement using the extended method if they were the same sign for the +B and –B states. If they were of opposite sign, the bias offsets would add to the total Δx and bias the energy measurement.



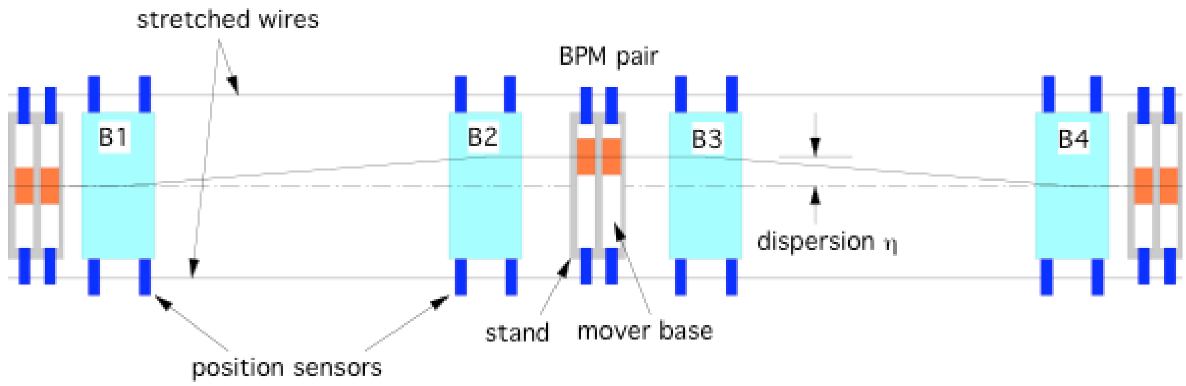
We can make a crude estimate of how large such errors might be. If we assume that BL is known in every magnet to 1×10^{-4}, that the bend angle in each is 300 μrad, then the kink angles could be as large as 3×10^{-8} rad. If the distance between B1 and BPM2 and between B3 and BPM3 is about 15 m, and the kink angles are all in the same direction, the kink offsets at BPM2 might be 0.5 μm and at BPM3 could be 1.0 to 1.5 μm. These are large enough to be a problem. The message here is that we need to analyze the precision required for BL to ensure that the differences between +B and –B states do not introduce errors of the type illustrated here.

There are other strategies for ameliorating these errors. It would be possible to measure the offsets in BPM3 with respect to BPM1 for the +B and –B states using measurements with beam in a steady state to look for kinks cause by total BL not equal to zero. If the offset for +B and –B were different, it would be possible to adjust one or more B value to make the offset zero. This would put a constraint on the total BL but would not measure the individual components, so it would still be possible to have no kink offset at BPM3 yet still have offsets in BPM2 that would introduce bias in the energy measurement.

Sources of systematic error from imperfect z positions of the magnets and BPM's
In a similar way that non equal BL causes kink offsets, so would non equal (or miss measured) z positions for the magnets and BPM's. The z position errors would not be distinguishable from BL differences by constraining offsets in BPM3 relative to BPM1 to be zero for +B and –B. Errors in knowledge of z positions would add uncertainty in Δx measured in BPM2. Keeping this error small would put constraints on the alignment tolerances of the z positions of the magnets and the BPM's.

Chicane overall layout

The following figure gives the basic layout of the chicane we have in mind (not to scale).



The chicane must fit in some overall length Z_total in the BDS, yet to be determined. It would be comprised of sets of pairs of BPM's mounted on x-y movers of the roller-cam type that are mounted on sturdy stands, with one set before the first magnet, and one after the last magnet to measure beam jitter, and one set between the middle two magnets to measure beam offset. We use pairs of BPM's at each BPM station for redundancy and so measurements of relative resolution can be made. The angle of the beam cannot be usefully constrained in one closely spaced pair of BPM's, but it could be somewhat constrained in the approximately 40 m length of the chicane. The BPM movers move only in x and y with no rotation. The figure indicates beam offset in the +x direction with the middle BPM's offset, but we would design the chicane to permit movement in +/- x to double the range to reduce the total error for a given dispersion. Also indicated are some stretch wire position monitors like those used at LEP and FFTB to monitor transverse motions. The position sensors would be attached to the magnets and to the base of the BPM movers to monitor long term drifts in relative x positions. It might also be useful to put stretch wires and sensors to separately monitor movements of the BPM stands. The BPM positions in x and y with respect to their mover bases would be measured with precision position encoders.

The basic operating procedure was outlined above. First, the BPM gain calibrations (signal output voltage per μm offset) and the relationship between each BPM center and its encoder value would be measured by leaving the beam in a fixed state and moving each PBM separately. We would NOT calibrate by moving the beam. Gain calibrations could be relatively fast and might be done often to monitor for drifts. Relative positions of the BPM's needed for jitter corrections would be established with respect to "straight" beam with magnets off. Then in some short period of time (minutes) the magnets would be energized to +B and –B, and the middle BPM's would be moved to find the beam in the offset positions. The beam energy would be extracted from measurements in all BPM's to account for beam jitter. The relevant time scale for errors on the absolute energy from drifts would be the time it takes to cycle from magnets off to magnets on, which could be short (minutes). The stretched wire sensors would be able to measure relative offsets to better then 1 μm, which is comparable to the position resolution of the BPM's. By design the emittance growth and offsets of the beam from the chicane would be invisible to the luminosity, so this procedure could be done as often as the speed of the movements would permit.

The ultimate precision of the energy will depend on lots of things we haven't worried about yet, mainly the values of the magnetic fields in the "off" and "on" positions and any possible changes in stray fields in minutes during the measurement.

The purpose of rest of this exercise is to find out what are the constraints on magnet and BPM locations from practical considerations, and from the requirement to minimize the emittance growth while achieving a dispersion of 5mm. For that we take the z location, the lengths L and strengths B of the magnets, and the overall length L_total as the variables.

Realistic magnets and BPM stations

Before we can establish parameters of the chicane layout we need first to put some numbers on the proposed arrangement of the BPM's and the space constraints required by realistic magnets, taking into account coils, flanges, bellows and other practical stuff.

Realistic magnets
We are going to play with magnet lengths L and strengths B where the L refers to the region of uniform field B. Realistic magnets also require extra z space on the beamline for coils, mirror plates, water couplings etc. We don't have a magnet design yet, but looking at some typical magnets we will assume that each magnet in our plan will require 20 cm on each end beyond the field length L.

Realistic BPM's
The RF BPM's we are considering would include three independent cavities: two rectangular x and y cavities , and one cylindrical phase measuring cavity . These, together with their flanges and bellows for motion, will consume approximately 50 cm of z space.

Sketch of a proposed BPM station
The BPM's will be positioned in pairs on stands, each with its own mover and readout systems with bellows to allow independent movement in x and y. A sketch of the main features is shown in the following figure.

(scale drawing of BPM station to be provided)

The main components are:

1) A main support stand of some robust material such as the high strength concrete used at FFTB, or Jurassic limestone like at LEP, or granite, or some other suitable material. Two BPM's would share a common support block stand. A main criterion for the stands would be that they not move much with changes in temperature, and that they not have large amplitude modes for vibrations.
2) An alignment mechanism between the stand block and the bottom of the BPM mover for adjustment and alignment for fixed pitch, yaw, and z location at installation time. The main criteria for these, aside from robust alignment capabilities, is that they not have

large thermal coefficients of expansion and that they not permit large amplitude vibrations.

3) A mover of the FFTB roller cam type with motion for x and y and roll on top of the installation alignment stage. The rang of x motion of the mover for the BPM2 must cover the desired range of beam offset for energy measurements (up to perhaps +/-5 mm). The range of the y motion of all the movers and the range of the x motion for the BPM1 and BPM3 will be used primarily for gain calibrations and to center them on the beam. In principle different designs for movers with different x ranges for the middle BPM's and outer BPM stations might work, but given the desire for standard equipment, might not be desirable.
4) One BPM mounted on each roller cam mover with bellows between neighboring components.
5) Position readout devices (position encoders) that measure the relative x and y motion between the base of the roller cam mover and the base of the BPM. There are several candidates for the position encoders. More details are given below.
6) Stretched wire position sensors to monitor long term drifts in transverse motion of the mover bases, magnets, and possibly the BPM stands. More details are given below.

Encoders

There are several promising technologies for precision position encoders capable of sub μm precision over many mm range.

LVDT's

(this section to be written)

Optical encoders

Following the hint from Mike Brown in his presentation on possible encoders for use the NLC movers, we might consider optical encoders such as those produced by Heidenhain. (The details for these can be found at http://www.heidenhain.com/product.html) . These are beautiful devices with exquisite precision over the range in mm we need. A primary consideration is how susceptible are they to radiation damage. They have solidstate electronic parts (optical diodes and photovoltaic light sensors) in them that would be sensitive to radiation. We need to get estimates from our ILC collaborators of how much radiation is likely to be absorbed by such devices a few cm from the beam pipe in the chicane region over a period of some years. It is possible that they would survive. We have used such optical encoders in SLAC ESA that survived harsh radiation for two decades.

Here are some pretty pictures of possible precision encoders. I'm especially interested in the two coordinate ones. They would considerably simplify the design of the mechanical layout of the encoders in the mover.

One dimensional x optical encoders

Exposed Linear Encoders

LIP

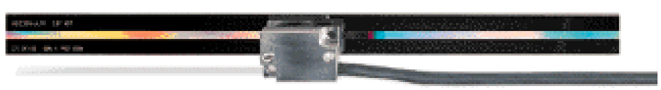
LIP linear encoders from HEIDENHAIN are available in accuracy grades from ± 1 µm to ± 0.1 µm. Through the choice of scale material it is possible to use a measuring standard whose thermal behavior is comparable to that of the measured objects.

A grating period of 4 µm produces an output signal period of 2 µm. This results in a measuring step of only 0.5 µm even without interpolation.

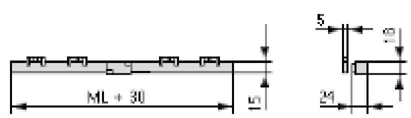
The high uniformity of the scanning signals permits measuring steps as fine as one nanometer.

	LIP 481 LIP 471	LIP 581	LIP 382 LIP 372
Measuring standard	DIADUR phase grating		
Output signals	LIP 481: ∿ 1 V_{PP} LIP 471: ⊓⊔ TTL	∿ 1 V_{PP}	LIP 382: ∿ 1 V_{PP} LIP 372: ⊓⊔ TTL
Signal periods	LIP 481: 2 µm LIP 471: 0.4 µm/0.2 µm	4 µm	LIP 382: 0.128 µm LIP 372: 0.004 µm
Accuracy grades	± 1 µm; ± 0.5 µm	± 1 µm	± 0.5 µm
Recommended measuring steps	1 µm to 0.005 µm	1 µm to 0.05 µm	0.001 µm
Measuring lengths ML	10 to 420 mm	70 to 1440 mm	70 to 270 mm
Reference mark	one	one or distance-coded	None

LIP 400 Series

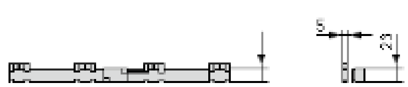


- Small dimensions
- Measuring step to 0.005 µm
- Scale available with various thermal expansion coefficients

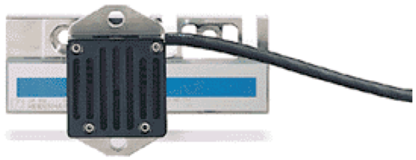


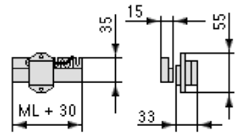
LIP 500 Series

- For relative large measuring lengths up to 1440 mm
- Measuring step to 0.05 µm

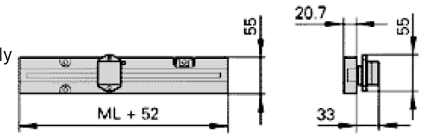


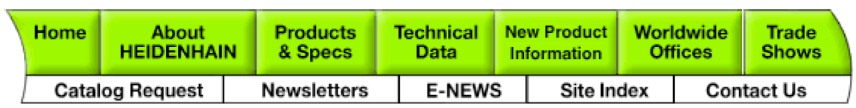
LIP 300 Series





- For very high resolution with measuring steps to 1 nanometer
- Very high repeatability through an extremely fine signal period
- Defined thermal behavior



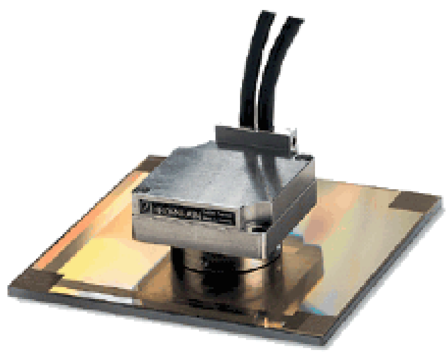


Two dimensional x-y optical encoders

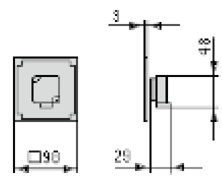
Two-Coordinate Encoder
PP

	PP 281 R
Measuring standard	Two-coordinate DIADUR phase grating on glass
Output signals	∿ 1 Vpp
Signal periods	4 µm
Accuracy grades	± 1 µm
Recommended measuring steps	0.01 µm
Measuring range	68 mm x 68 mm; other ranges available on request
Reference mark	One per measuring direction

PP 281 R



PP 281 R: ∿ 1 Vpp



The **PP 281R** incremental two-coordinate encoder can measure positions in a plane.

Two reference marks, one for each measuring direction, serve to reproduce a datum.

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Eddy current sensors
(this section to be written)

Stretched wire position monitors

The idea for using these comes from the experience with the LEP energy spectrometer and the FFTB system. Those experiments quote precisions of sub 1 µm and talk about good drift stability over long times. We need to study them more to see if they would work over the somewhat longer z distances we need (40 m or so). Stretched wire systems have many advantages in simplicity, robustness, resistance to radiation damage, and a track record for good performance. They seem superior for these reasons to any optical system using lasers and mirrors, but this needs to be studied.

First try Chicane from PT

We are going to scale the parameters of the chicane suggested by PT to estimate emittance growth.

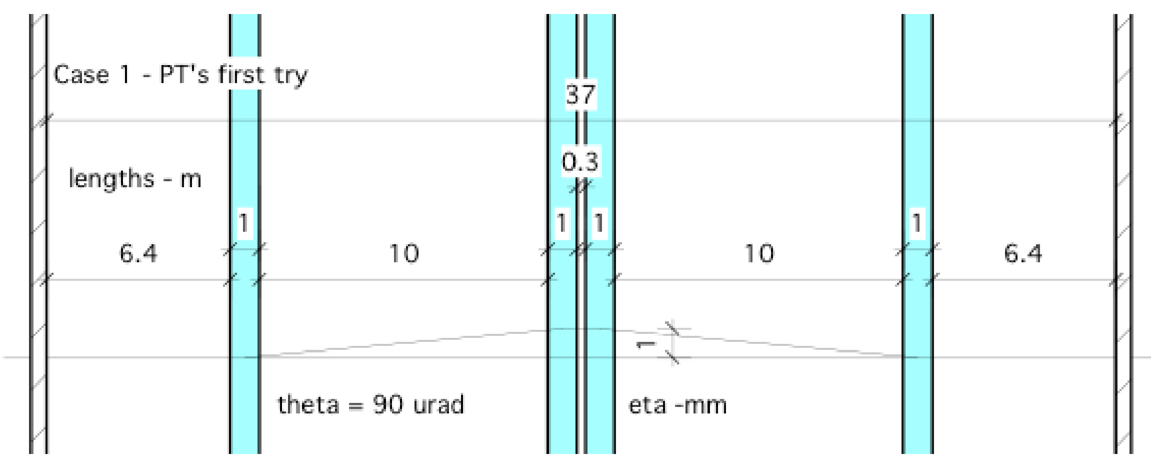
The PT chicane parameters are:
Total drift space available – 37 meters in which he put:

6.35 m drift
1 meter bend with 90 μrad bend angle
10 meter drift
1 meter anti-bend with 90 μrad bend angle
30 cm drift
1 meter anti-bend with 90 μrad bend angle
10 meter drift
1 meter bend with 90 μrad bend angle
6.35 meter drift

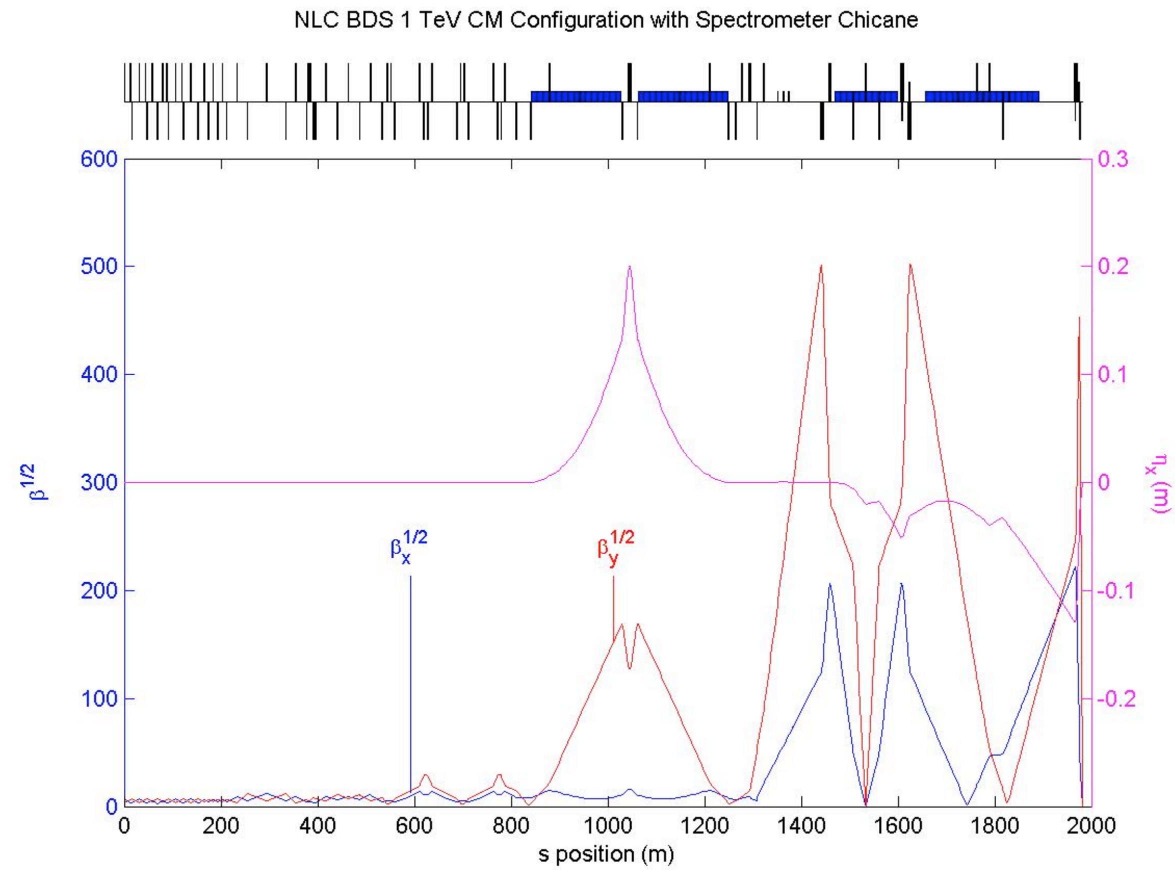
For E_beam = 500 GeV the emittance growth was calculated to be 0.017 μm. For 1 m magnets to bend 500 GeV by 90 μrad bend angle requires B field of 1.5 kGauss.

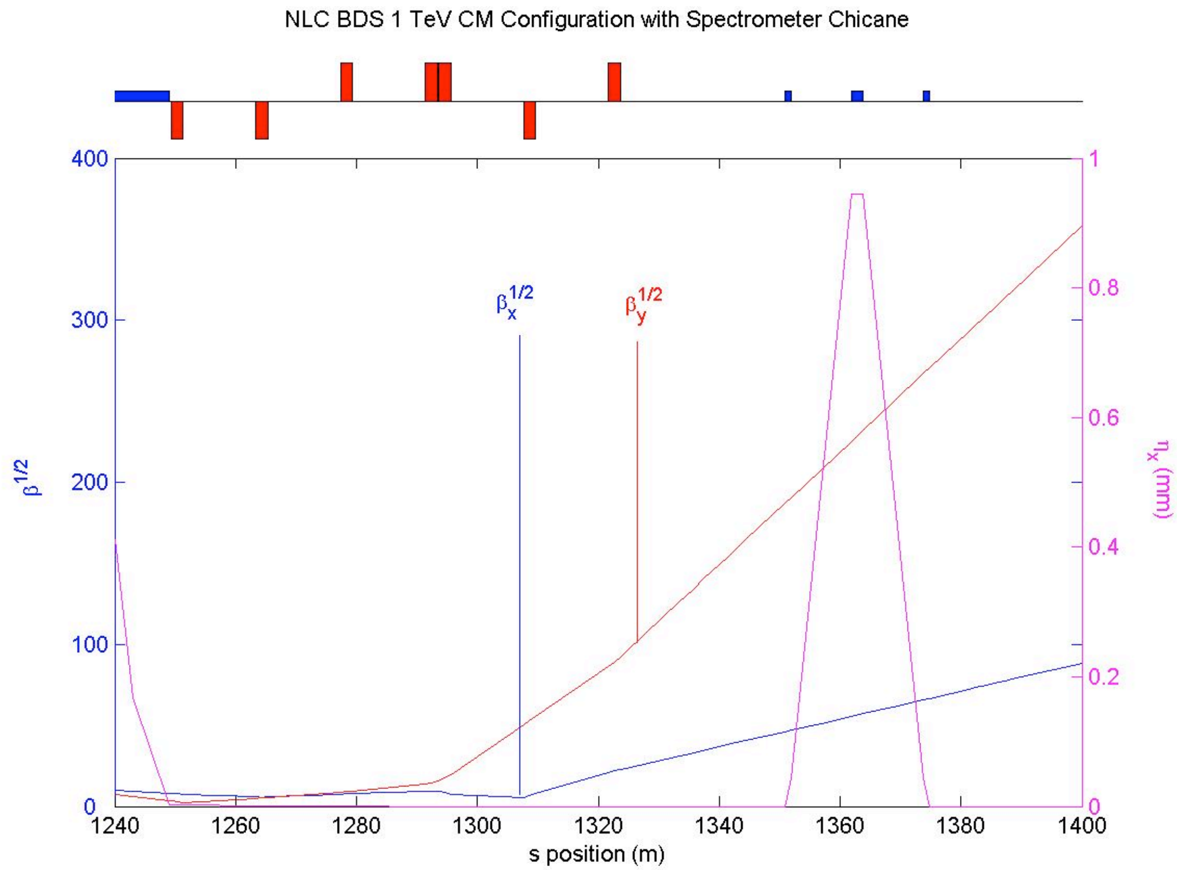
The PT chicane has dispersion η= 0.9 mm (call it 1 mm) in the middle where the BPM's would be placed. To achieve 10^{-4} precision (0.1 μm) in measurements of the beam offset with 1 μm resolution in the BPM's per measurement, it would be necessary to average over 100 measurements, which could be 100 bunches or 100 strings of a few bunches.

The following sketch shows a scaled figure of the chicane with the horizontal scale in meters and the vertical scale in mm.



The twiss parameters (β,η) in the BDS region selected by PT are shown in the following figs.


NLC BDS 1 TeV CM Configuration with Spectrometer Chicane
$\beta^{1/2}$
$\beta_x^{1/2}$
$\beta_y^{1/2}$
η_x (m)
s position (m)
0
200
400
600
800
1000
1200
1400
1600
1800
2000
100
300
500
0.3
0.2
0.1
-0.1
-0.2



The chicane creates a bump in the dispersion centered around 1362 m in these coordinates.

The question now is how to adjust the chicane dimensions to see if we can get larger dispersion without compromising on emittance growth. Its too hard to calculate emittance growth by hand so we will scale from the value of 0.017 calculated by PT using the laws for emittance growth from synchrotron radiation.

Emittance growth from Synchrotron Radiation in bend magnets

For formulas describing the emittance growth from SR in bend magnets we turn to the nice lecture notes given in http://www.desy.de/~njwalker/uspas/. Effects of SR are described there in the lecture by Andrei Seryi in Unit 6 on Beam Delivery Systems, slides 21-24. A more detailed description of the physics of SR effect on emittance, including the quantum effects, is given in the lecture on Damping Rings by Andy Wolski. The first order expression for emittance growth (ignoring derivatives of η and β over the path length dS) from radiation over a path element dS during horizontal bend in x is

$$\frac{d\varepsilon_x}{dS} \approx \frac{\gamma^5 \eta^2}{R^3 \beta_x}$$

where:

η is the dispersion at the location where the SR is emitted,
$\gamma = E/m_ec^2$ is the relativistic energy factor,
β is the horizontal beta function where the radiation is emitted,
R is the radius of curvature of the bend.

This formula accounts for the classical effect of energy loss on the emittance while the particle is on a bend path in the field. Contrary to what you might think, the SR does not damage the emittance by giving an angle kick to the emitting electrons. The emittance growth is caused by the energy loss. In the classical picture, the SR is emitted in a narrow cone along the direction of the instantaneous path. The particle does not change position or angle, it only losses energy. When that happens the equilibrium orbit on which it was traveling moves out from under it to a new position, which is offset from the original orbit by

$$\Delta x = \eta \Delta E / E$$

At that point the particle starts a new betatron oscillation around the new equilibrium orbit with amplitude Δx. If we compare this oscillation amplitude with the beam size

$$\sigma = (\varepsilon_x \beta_x)^{1/2}$$

then the emittance growth is

$$\Delta \varepsilon_x = \frac{\Delta x^2}{\beta}$$

The energy loss ΔE due to SR depends on the energy of the electron and the radius of curvature in the bend

$$\frac{d(\Delta E / E)^2}{dS} \approx \frac{\gamma^5}{R^3}$$

In the total formula for $d\varepsilon_\xi/dS$ there are two components:

1) A factor $\frac{\gamma^5}{R^3}$ which gives the amount of SR emitted
2) A factor $\frac{\eta^2}{\beta}$ which gives the size scale of the damage to the emittance

Clearly if more SR is emitted (higher energy γ or smaller R) then the $d\varepsilon_\xi/dS$ is larger. But it is also important where, that is at what local values of η and β, the radiation occurs. If the radiation loss is at a place where η is large, then the amplitude of the offending oscillation is proportional to η^2, and the emittance growth will be large. If the SR is emitted where β is large, the amplitude of the offending oscillation is small compared to β, and the emittance growth is not so big. We can use these features to optimize our chicane design to obtain large η in the middle where we need it while keeping the total emittance growth small.

Emittance growth in the PT First Try chicane

Before we begin our exercise to try to increase the dispersion in the middle of the chicane, lets examine the emittance growth in the PT chicane. From the formulas above we see that most of the emittance growth occurs in the middle two magnets B2-B3. The amount of SR produced in each magnet from the factors γ^5/R^3 is identical (same energy, same bend L and B), but the damage $\Delta\varepsilon_x$ is dominated by the radiation in the B2-B3 magnets because that's where η is large. The η^2 values at entrance and exit of the magnets are:

<u>End Magnets</u>

$\eta^2 = 0$ en trance to B1, exit of B4
$\eta^2 = 2.0\text{x}10^{-9}\ \text{m}^2$ exit of B1, entrance of B4

$\eta^2 = 7.7\text{x}10^{-10}\ \text{m}^2$ average in B1 and B4

<u>Middle Magnets</u>

$\eta^2 = 8.0\text{x}10^{-7}\ \text{m}^2$ entrance to B2, exit of B3
$\eta^2 = 8.1\text{x}10^{-7}\ \text{m}^2$ exit of B2, entrance of B3

$\eta^2 = 8.05\text{x}10^{-7}\ \text{m}^2$ average in B2 and B3

So the ratio of the amount of emittance damage in the B1+B4 magnets to that in B2+B3 is

$$\eta^2(\text{B1+B4}) / \eta^2(\text{B2+B3}) = 9.56\text{x}10^{-4}.$$

In absolute terms the emittance growth was

$\Delta\varepsilon_x = 1.62 \text{ x } 10^{-11}$ m-rad in B1+B4
$\Delta\varepsilon_x = 1.7 \text{ x } 10^{-8}$ m-rad in B2+B3.

<u>Scaling from the PT chicane</u>

In the PT chicane design with equal magnet lengths, the η in the B1 and B4 magnets is created in those magnets and is relatively small, whereas the η in B2 and B3 is mostly created outside the magnets by accumulation of η created in B1 over the distance of the separation between B1 and B2-B3. From the point of view of chicane design to improve energy resolution we want to create larger η in B2-B3 where the middle BPM's will measure beam offsets. This can be done by some combination of increasing the bend angle in B1 or by increasing the distance between B1 and B2-B3. From the point of view of emittance growth, having larger η in B2-B3 however it is created is bad because that is were all the growth occurs. To increase the dispersion in B2-B3 and keep the emittance growth acceptable we are going to have to modify magnet lengths and strengths to change the bend radius and use the $1/R^3$ dependence to decrease the SR. We see immediately that this is most effectively done by increasing the bend radius of the B2-B3 magnets, because that's where the damage occurs. We also see from this analysis that it is relatively easy to increase the η in B2-B3 by increasing the bend angle in B1and B4 without paying a heavy penalty in emittance growth because the emittance damage there is a factor of 9.56×10^{-4} smaller for a given bend radius than it is in B2-B3.

Given these numbers our strategy for increasing η at BPM2 is clear:

1) increase the bend angle in B1 and B4, and also perhaps
2) increase the distance between B1 and B2-B2, and between B2-B3 and B4 to accumulate η without SR,
3) increase the length L and decrease the field B in B2-B3 to ameliorate the increased emittance growth from increased η in B2-B3, and the increase in reverse bend angle.

The variation of β over the length of the chicane, and especially over the length of B2-B3 where it really matters, plays a much smaller role, and for the time being we ignore variations in β. The task now is to see if we can find a set of parameters that satisfy our requirements that also satisfy the requirements from space constraints determined by the overall length available and the space required for real magnets and BPM stations.

For fixed beam energy γ the emittance growth can be adjusted by changing the $1/R^3$ factor by adjusting the length L and field strength B of the magnets to adjust the bending radius R or the bend angle θ using the standard relations:

$$p = 0.3BR \quad \text{(GeV, kG, m)}$$
$$\theta = L/R.$$

As a start, lets see by how much we would have to modify B2-B3 magnet lengths to ameliorate the emittance growth from the desired increase to η = 5 mm. Starting from the PT model with η = 0.9 mm and increasing η by increasing the accumulation distance between B1 and B2 without changing the bend angles, we get an increase in $\Delta\varepsilon_x = (5/0.9)^2 = 30.8$ m-rad from increased η in B2+B3. We can eat a factor of $\Delta\varepsilon_{x(}$(Tesla)/ $\Delta\varepsilon_{x(}$(NLC) = 800/300 = 2.66 for the expected emittance at the IP that we are trying not to destroy compared to that used by PT. This leaves an overall factor of 30.8/2.66 = 11.6 increase in $\Delta\varepsilon_x$ to be removed by softening the bends B2-B3 according to $1/R^3$. If we lengthen the B2-B3 by

$$(11.6)^{1/3} = 2.25$$

we recover the original emittance. If our strategy also includes increasing the bend angles and thus increasing the $1/R^3$ factor for SR production, then we have more emittance growth to worry about.

Criteria for the optimum dispersion

For the chicane we are considering, the energy measurement is essentially derived from measurement of the beam offset in BPM2 a distance z after a bend magnet (mostly B1 in our case). In that case the dispersion η (mm) at BPM2, which gives the deviation $\Delta x = \eta \Delta E/E$ for energy deviation ΔE from the central orbit at E, is essentially equal to the beam offset $\Delta x = z\Delta\theta$ from the straight ahead direction from bending in B1. So we can loosely speak of either the dispersion at BPM2 or the offset at BPM2 as the same quantity.

The question then is: what is the optimum dispersion or offset at BPM2? The answer to this is coupled to the expected values for

1) BPM resolution,
2) uncertainty from bunch charge measurement,
3) residual position jitter in BPM2 from any uncorrected betatron jitter,
4) bunch-to-bunch energy jitter,
5) the number of bunches averaged over in one $\Delta(+)$ or $\Delta(-)$ data point.

In all this discussion we are ignoring for the time being the uncertainties in "constant" factors like the magnetic fields, BPM positions, electronics signals, and the like, and assume that they are being held constant and are known to the required precision of 10^{-4}. We take as a starting point that BPM resolutions are fixed at about 1 μm because we know that this can probably be readily achieved.

The amount of dispersion sets the absolute scale of precision in the measurement at BPM2 required to obtain a given error in the energy. For example, if the total offset between $\Delta(+)$ and $\Delta(-)$ is 1 cm, then measurement of $\Delta x = \Delta(+) + \Delta(-)$ to ~ 1 μm is required for 10^{-4} error. The measurement to 10^{-4} energy precision necessarily involves averaging over some number of bunches because the bunch-to-bunch jitter is larger than 10^{-4}. The extended measurement scheme, which involves comparisons of $\Delta(+)$ with $\Delta(-)$ in different magnet states at different times, also requires that the separate measurements be obtained from averages over bunches. The number of bunches required is determined by the relative size of the quadrature sum of bunch-to-bunch position uncertainties 1)-4) at BPM2 listed above compared to the desired position precision set by the total beam offset. If the beam offset is larger, then the number of bunches required to average down to the position uncertainty corresponding to 10^{-4} energy error is less.

We are going to assume that one goal of the design will be that it is possible to make a measurement of $\Delta(+)$ or $\Delta(-)$ within the 2820 bunches of a single pulse, including losses from unused bunches at the beginning. This sets some limits on factors which drive up the number of bunches required for 10^{-4} error. If the dispersion is too small, or if any of the factors 1)-4) above in BPM2 is too large, then the measurement cannot be made within one 2820 bunch train.

Another goal of the design is that it be robust against degradation in the performance of any of the parts. From that point of view large beam offset at BPM2 can be regarded as a capital investment in a robust parameter, essentially BL in magnets and geometry for motion of BPM2, that will give room for other parameters in the system to degrade without seriously compromising the total system performance.

A primary reason for increasing the dispersion is to make the BPM resolution less critical. If for some reasons the BPM resolutions were to degrade due to noise in electronics or other problems, and if the averaging required is driven by the BPM resolution, we would be forced to average over more bunches to reduce the error. This would not seriously impact the final answer as long as the dispersion was large enough that the precision could still be obtained in an average within one beam pulse.

One way to specify the dispersion would to be to make the expected bunch-to-bunch jitter from energy and uncorrected beatron jitter similar to the BPM resolution. If we increase the dispersion of the PT chicane from ~1 mm up to several mm we can get these sources of jitter closer to balanced. This requires increasing the dispersion by a factor of about 3 or 4. To be a bit conservative and give room for the BPM resolution to degrade, we could pick in round numbers that the dispersion should be increased by a factor of 5 to 5 mm at BPM2.

We could also consider what would be the effect of leaving the dispersion at ~1 mm and trying to achieve the desired precision by improving the BPM resolution. In that case the absolute precision in position required at BPM2 would be 0.1 μm. The BPM resolution would have to be reduced to 0.1 μm to get the desired precision (ignoring the errors from bunch charge measurements which depend on x(BPM) but not resolution). Also the precision of cross calibration of encoders needed for jitter corrections, and the stability of the system as monitored by wire sensors or other means, would have to reach 0.1 μm level. Reaching precision and stability of 0.1 μm on all the components would be significantly more difficult than reaching 1 μm.

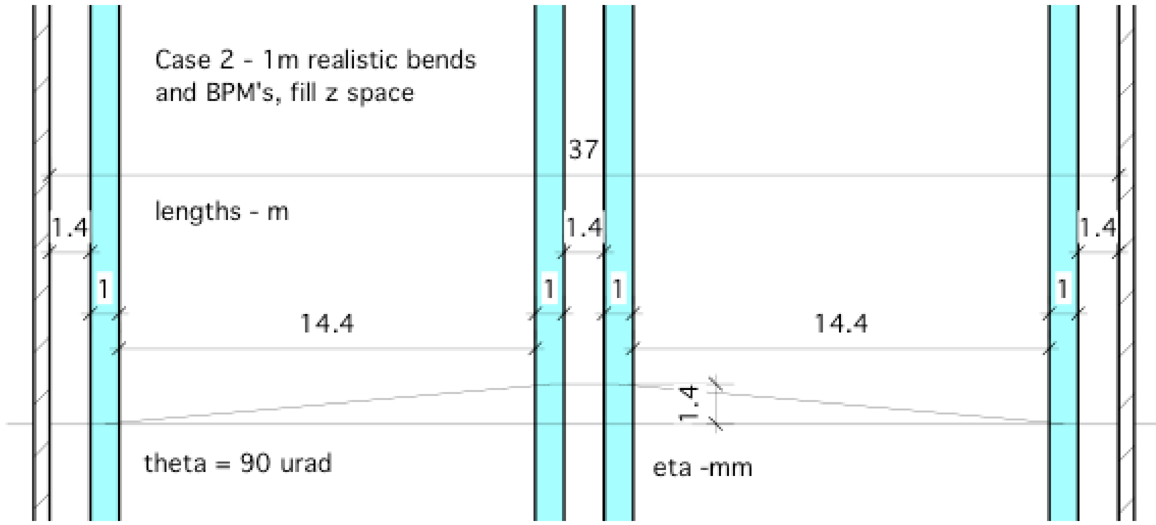
For now we take as a goal that we want to increase the dispersion, or equivalently the beam offset at BPM2, to 5 mm. After looking at how this might be done, we may revisit this choice as we understand how it affects other things, like the strength and shapes of the magnetic fields, the overall length of the system, sensitivity to position drifts, etc.

Candidate chicane models by scaling the PT model

Case 2 – keep the PT magnets, extend the length

To begin exploring the length and strength parameters we first sketch out a trial chicane that uses realistic magnets and BPM stations keeping the 1 m long magnets of the PT design, but extending the length to fill the currently available 37 m drift space so we can increase η by extending the distance between B1 and B2-B3. This will not satisfy our design requirements, but it gives you some idea how much η can be increased given the 1 m magnet lengths fit into the 37 m drift space.

Case 2 is sketched in the following figure, where the horizontal dimensions are in m and the dispersion is marked in mm.



For realistic magnets and BPM stations the space required for one pair of BPM's plus the coils of the adjacent magnets is 1.0 m (BPM's) + 2x(0.2 m) (coils) = 1.4 m. Adjusting the magnets to fill the 37 m drift space, keeping the same 90 μrad bend angles results in an increase in η to only 1.4 mm in the middle BPM's. This is not the 5 mm we are looking for, and it would be accompanied with an increase in emittance of $(1.4/0.9)^2 = 2.4$.

So we have to do better.

Case 3 – B1,B4 = 1 m, B2,B3 = 3 m, fill 37 m drift space

For this case we leave B1 and B4 at 1 m length and increase B2 and B3 lengths to 3 m. This is longer than the increase to 2.25 m needed to suppress the emittance growth from increasing the dispersion to 5 mm at fixed bend angle, but it's not ridiculous. Using1.4 m for realistic magnets and BPM's, and adjusting the package to fill the 37 m, we end up with a bend angle in B1 and B4 of 350 μrad, which is 3.89 times the original 90 μrad. The emittance growth in B1-B4 would increase by:

$(350/90)^3 = 58.8$ from increasing the bend angle (more SR)

$(350/90)^2 = 15.1$ from increasing the average η in the magnets (worse damage).

for a total increase factor of 887. The fraction of the emittance growth produced in B1-B4 is only 9.5×10^{-4} of that produced in B2-B3, so the emittance in B1-B4 would increase from 1.62×10^{-11} to 1.43×10^{-8}. This is clearly acceptable.

Meanwhile the emittance growth in B2-B3 would change by:

$(350/90)^3 = 58.8$ from increasing the bend angle (more SR)

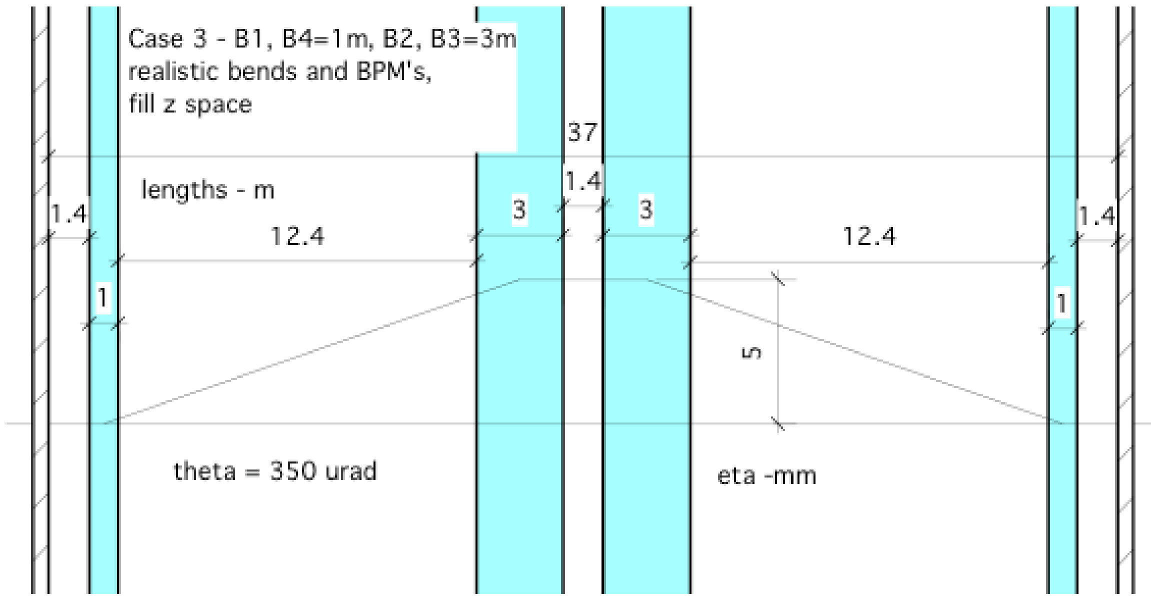
$(1/3)^3 = 0.037$ from lengthening the magnets (less SR)

$(5.0/0.9) = 30.8$ from increase in average η (worse damage)

for a total increase factor of 67.0. After taking into account the factor of 2.66 we have for the IP emittance we are shooting at compared to PT, we still have an overall growth of a factor of 25.2 larger the PT model which was 0.5% of the IP value. This is clearly unacceptable.

For this scenario the magnetic fields are:

B1, B4 = 5.83 kG
B2-B3 = 1.94 kG.



Case 4 – longer system length and longer B2-B3

Case 3 is a scenario we cannot live with. The two ways to fix it are a) to lengthen the system and make the bend angles smaller to reduce the growth (mostly in B2-B3), or b) increase the lengths of B2-B3 even more. It doesn't pay to increase lengths of B1-B4 because they are not causing the problem. To fix the problem by making the system longer requires lengthening the distance between B1 to B2 and B3 to B4 by a factor (1/25) = 2.92. This would probably be unacceptable from constraints in the BDS optics and from other problems it would create, e.g. measuring and monitoring positions of components spread over such a long length. Perhaps there is more real estate to be had in the BDS optics and we may want to negotiate for some more space from our BDS beam designers.

Meanwhile lets see what we could obtain if we were to tweek both the lengths of B2-B3 and the length of the system. If we partition the factor of 2.92 we need (call it 3) into a factor for the system length and one for the increase in B2-B3 length:

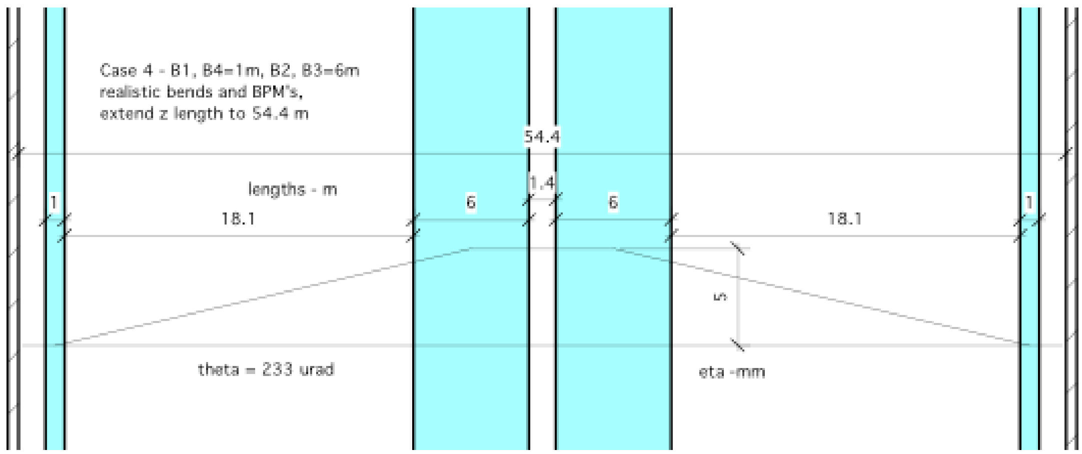
$$f_{L-Syst} \times f_{L-B2-B3} = 3$$

we could imagine increasing the spacing between the centers of B1 to B2 and B3 to B4 magnets by perhaps a factor approximately 1.5 to 21.6 m decreasing the bend angles to 233 μrad, while also increasing the length of B2-B3 by a factor of about 2 to L = 6 m. I know this is getting scary, but the LEP magnets were 5 m long so it's not completely ridiculous.

Lets forget about the small change this would make in the emittance growth in B1-B4 and look at the change in B2-B3. The emittance growth in B2-B3 would change from the PT models by:

$(233/90)^3$ = 17.35 from increasing the bend angle (more SR)
$(1/6)^3$ = 0.0046 from lengthening the magnets (less SR)
(5.0/0.9) = 30.8 from increase in average η (worse damage)
for a total increase factor of 2.5.

This is getting close to something we can live with for emittance growth. The question now is can we live with it for a lot of other reasons?



We could also reconsider the maximum dispersion we design for and reduce the system length by reducing the dispersion, say to 4 mm instead of 5 mm. But this is getting silly. You get the idea. We need to iterate on the optimum tradeoffs and understand better what are the real constraints in the BDS optics. Then we need someone to calculate the real increase in the emittance for the trial designs using MAD or other tools.

Spectrometer performance and optimization at 500 GeV CM

We considered the parameters of the chicane at 1 TeV, 500 GeV beam energy, because that is what the ultimate design calls for and we wanted to scale from the values presented by PT. We also want to look at how the performance optimization would be affected by lowering the beam energy to 250 GeV.

(this section to be written)

What magnet design is best?

There are many issues to explore that affect the choice of magnet design. Whatever magnets we use, it will be required to do the hard work of measuring their fields for various excitations in the volume to be occupied by curving beam tracks. In some sense it doesn't matter what magnets we use as long as they are well measured and they maintain their field quality for a long life after they have been characterized before they are installed. However there may be some magnet configurations that are less prone to error than others. Much more work is required here. Some issues would be:

1) Should the magnets be iron free super conducting to guarantee B = 0 when off?
2) Is there a problem having relatively short (1m) magnets which have a larger fraction of their BL from the end fringe fields?
3) What is the longest magnet that is practical to consider? Is 6 m too long?
4) Is there any reason why not to use magnets of different lengths?
5) What will be the difficulties in maintaining precision for magnets that must be ramped up and down over relatively short periods of time. Iron magnets with standard coils will heat up and cool down, perhaps causing motions.
6) Will the settling times of the fields after ramping be a problem?
7) How will the magnets age over 10 or 20 years?

Various subtle effects to investigate

In addition to the parameters investigated above, there are other more subtle issues that might affect the performance and design choices of the energy chicane. A partial list includes:

x-y coupling from magnets
The BDS optics in the preliminary region proposed for the chicane has fairly large β_y which leads to large vertical beam sizes. We plan to include vertical motion of the BPM's in the design, and would calibrate the y BPM gains and encoders and center them on the beam in y. If there is any significant x-y coupling in the chicane magnets, which could arise from non zero roll of the bend magnets or from higher order terms in the fringe fields, then the jitter in y would cause motion in x that may affect the measurements. This needs to be evaluated.

Weird effects from non Gaussian beam distributions
If the beam distributions are non Gaussian, will this affect the energy measurements in any way?

Bunch tilt and bunch angle in the BPM's
Related to the issue of z center mentioned above, are their any weird effects in the BPM x response from bunches that have x-z correlations or from crossing the BPM at an angle?

Variation of β_y within the chicane
The betatron functions will be varying over the length of the chicane. Do we care?

What should we be testing in the beam test experiments?

(to be written)

What is the best BPM design for our application?

We need a good examination by the experts of the parameter tradoff's for RF BPM's that could resolve single bunches spaced by 168 or 337 ns.

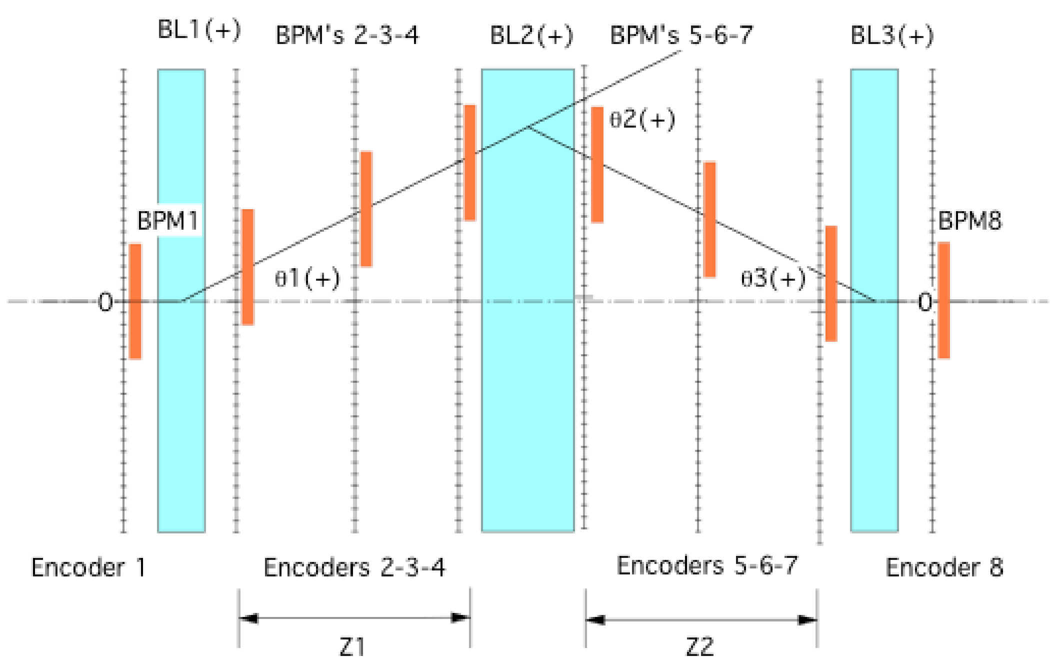
Alternative chicane design – angle measurement version

In this section we look at an alternative chicane design that would use measurement of the bend angle in the middle magnet(s) to determine the energy, rather than measuring the Δx offset. This is the scheme sketched in presentations of the BPM energy spectrometer in preliminary work, and is the idea that probably first comes to mind when one thinks of measuring beam energy from bending in a magnet. The question are, how does the angle measurement scheme compare in performance to the offset measuring version, and what are the issues in design and operation that determine its performance? Our conclusion will be that the angle measuring method is not as good as the offset measuring version. Lets look at the issues.

Chicane layout and definition of the measurements – angle version

The layout of the angle measuring version, sketched in the figure below would be comprised of a series of three magnets (B2 may actually be two magnets), with BPM stations before (BPM1) and after (BPM8) the chicane, and three stations of single BPM's on each leg of the beam before and after the bend B2. As before, the BPM's are mounted on precision movers that move in x and y with x and y positions measured with accurate encoders. We suppose each station might contain only one BPM, not a pair of them, because this plan already has lots of BPM's to consider. It is likely that BPM performance would not be impaired by beam transiting them at a small angle, so it is not necessary to rotate the BPM's when positioning them on the deflected beam.

Definition of the Primary Measurement - angle version



Definition of the primary measurement – angle version

Ignore for the moment the variations of incident beam positions and angle from beam jitter, and consider a system with three magnets having lengths and strengths such that BL2 = 2*BL1 = 2*BL3, BPM's with gains calibrated and encoder values known for beam in their electrical centers with straight undeflected beam (B = 0), and beam on the axis of BPM1 and BPM8. The primary energy measurement is obtained from absolute measurements of the quantities:

1) BL2(+) – total integral BdL in B2 along the beam path with magnets in the + polarity.
2) Z1 – separation (m) of BPM's on the incident leg
3) Z2 – separation (m) of BPM's on the deflected leg
4) x – encoder positions(m) of BPM's 2-3-4 and BPM's 5-6-7

From these measurements the energy would be obtained from the deflection angle θ_2 between the trajectories before and after the bend B2, determined from the difference of the trajectories measured at $\theta_1(+)$ and $\theta_3(+)$. The contribution to the energy error from the uncertainties in beam position measured in the BPM's would be the sum in quadrature of the errors on the angles:

$$E \pm \Delta E \approx \theta_1 - \theta_3 \pm ((\Delta\theta_1)^2 + (\Delta\theta_3)^2)^{1/2}$$

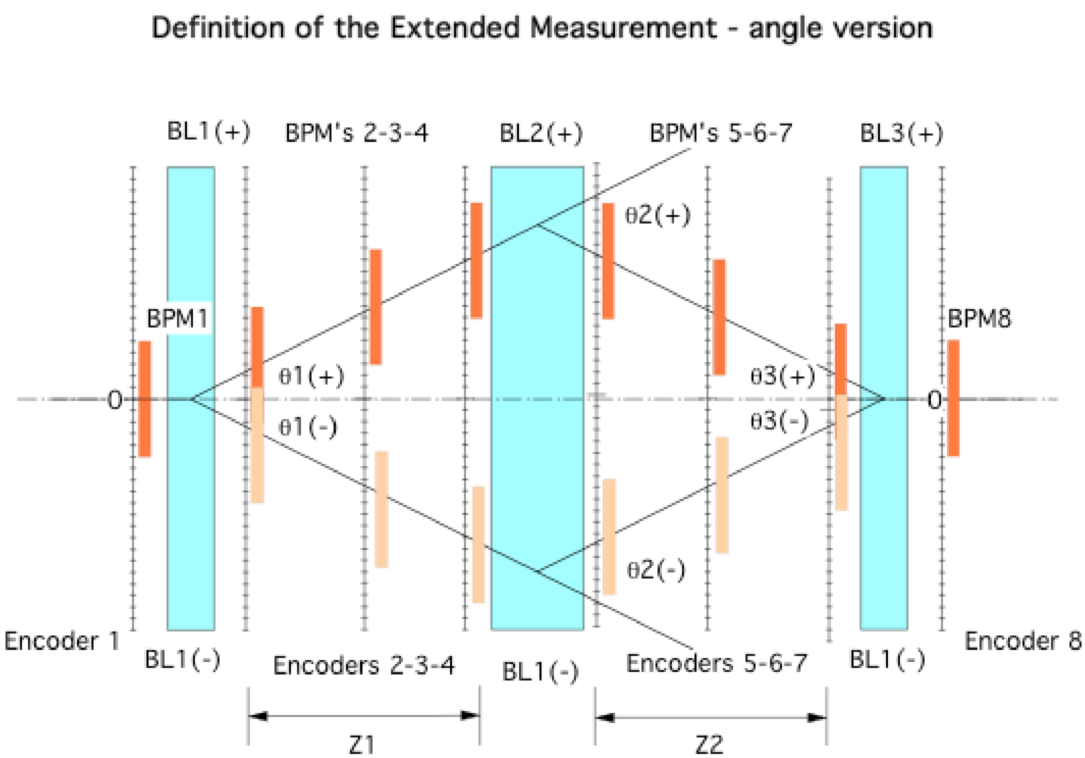
From this definition of the primary measurement we can draw some conclusions about the measurement strategies and contributions to the total error:

1) B2 is the only magnet in the energy spectrometer. The values of BL1 and BL3 do not enter the energy measurement. They are only used to deflect the beam from and to the main beam line.
2) For given errors $\Delta\theta_1$ and $\Delta\theta_3$, the fractional error on the energy is reduced when θ_2 large. The angle θ_2 large corresponds to large dispersion or offset at B2. Large dispersion at B2 reduces the contribution of error from measured angles similarly to the way in which large dispersion reduces the contribution to error from measured position at BPM2 in the Δx version of the chicane. The desire for large θ_2 to reduce the error is in conflict with the need to keep the emittance growth small. Accumulation of dispersion at B2 by increasing the separation between B1-B2 and between B2-B3 does not improve the total error unless the distance is used to reduce the error in trajectories at θ_1 and θ_3 by lengthening the BPM separations Z1 and Z2. Accumulation of dispersion at B2 with no commensurate gain in angle precision is bad, as would occur if B2 is lengthened to reduce emittance growth while keeping Z1 and Z2 the same.
3) The absolute distance between BPM's 2-3-4 with respect to BPM's 5-6-7 and the distances between the BPM groups and any of the magnets do not enter the measurement. The magnet B2 need not be equal distance from the BPM groups. The only dimensions in the z direction that count are the separations Z1 and Z2 of the BPM's within each group.
4) The precision of the beam trajectory measurements $\theta_1(+)$ and $\theta_3(+)$ is determined mostly by the separations Z1 and Z2 and by the BPM resolutions. This assumes that the relative calibration of encoder values for BPM's centered on beam at known positions in space are known accurately (centered on undeflected beam, for example). The use of BPM3 and BPM6 in the middle of each group will give some reduction in the error, but they are

not crucial. Long distances Z1 and Z2 between BPM's would reduce the angle error, but this is in conflict with keeping the overall system short. The system redundancy could be increased with a small decrease in angle precision if we are willing to use two additional BPM's and deploy pairs at BPM2, BPM4, BPM5, and BPM7 and eliminate BPM3 and BPM6.

5) The constraint from keeping the emittance growth small would drive the lengths and separations of the magnets to similar values as for the Δx version of the chicane. Therefore we can take Case 4 as a preliminary magnet configuration. Note, in this case, unless we consider magnets much longer than 6m, the B2 magnet would actually be two magnets. This doubles the effort required to construct and characterize the primary magnets critical to the energy measurement. The need to keep the bends at the B2 position long and soft to suppress emittance growth means that the angle version of the chicane is not simpler to build because B2 is a single magnet.
6) BPM resolution is a critical parameter. Take the parameters of Case 4 with Z = Z1 = Z2 = 16 m (leaving some room on each end for fittings, etc), and $\theta_1(+) = 230 \times 10^{-6}$ rad, and angle resolution from extremes of σ_x resolution of just the two end BPM' in each group that would be required to achieve $\Delta\theta_1/\theta_1 < 10^{-4}$ to be $\Delta\theta_1 = 2\sigma_x/Z$. This gives $\sigma_x = 1.8 \times 10^{-7}$ m, or 0.18 μm, for the required BPM resolution. This is a factor of 5 smaller than the 1 μm we assumed (so far) for the Δx version of the chicane.
6) In principle a measurement of the energy could be made for each beam bunch from measurement of θ_2 by comparing the $\theta_1(+)$ and $\theta_3(+)$ for the bunch measured in all the BPM's 2-3-4 and BPM's 4-5-6. This assumes that all the "constant" factors that enter parameters 1)-4) above are known. The time scale on which they have to be kept constant is the duration of one bunch. In principle if "constant" factors drift between beam bunches and can be measured fast enough, the drifts could be corrected so that each bunch would be an independent measurement of the energy.
7) Consider bunch-by-bunch measurement of θ_2. A separate measure of beam jitter outside the chicane from BPM1 and BPM8 is, in principle, not required. Since the beam trajectory for each bunch is measured before and after the bend in B2, the betatron jitter in x position and x angle and the energy jitter are all measured. If BPM1 and BPM8 are used to help constrain the position and angle jitter, then it would also be necessary to know magnet fields BL1 and BL3 and have cross calibrations of the BPM1 and BPM8 encoders with those in the middle of the chicane. Using BPM1 and BPM8 could potentially reduce errors if the corresponding increase from errors in the additional parameters that would introduce were not large. Ignoring BPM1 and BPM8 and the "constant" factors, the precision of a single bunch energy would be determined mostly by BPM resolution and Z1 and Z2.
8) Measurement of the beam trajectories to extract θ_2 relies on knowing accurately the relative positions of the BPM's. The relative x positions are more critical than the relative z positions that form Z1and Z2. Accurate knowledge of the relative x positions requires accurate cross calibration of the relative positions of the electromagnetic centers of the BPM's and their encoder readings. The best way to achieve this is to record the encoder readings when all the BPM's are "centered" on a straight beam. This requires either assuring that B2 = 0, or if B2 in the off state has some residual field, it is required to know the BL at "zero" accurately to get the cross calibration for encoders on opposite sides of the magnet. Any ambient field from Earth field or equipment sources would not

contaminate the energy measurement if it remains constant for the measurements with B2 off and on. Constant ambient field would simply give an additional constant angle kick that would be included in the encoder calibrations and would cancel in the calculation of θ_2 by comparing the $\theta_1(+)$ and $\theta_3(+)$, with B2 on. Any miss calibration of encoder cross registration from one side of B2 to the other from residual field would result directly in a bias in the measured energy. The encoder cross calibration should not be worse than the BPM resolution. For chicane Case 4 above, this would require cross calibration to better than 0.18 μm. If we take the parameters of Case 4, with $\theta_2(+) = 460 \times 10^{-6}$ rad and estimate the permitted variation in the deflection in B2 from residual field in the "off" state to be $\Delta\theta_2/\theta_2 < 10^{-4}$, we get the BL2< 8×10^{-4} kG-m at 500 GeV. For two magnets of 6 m each as in Case 4, that means the residual field must be kept below 6.4×10^{-5} kG. This is very small, about 2% of the Earth field. Typical residual fields from hysteresis in iron magnets is in the range of 10's of Gauss. This sensitivity of the primary method to the encoder calibration at B = "zero" is similar to that in the Δx measuring chicane, and is the principle reason to use the extended method with +B and –B.



Definition of the extended measurement - angle version

We consider extensions of the primary measurement to include measurements with B fields set to +B and to –B. The extended measurement is illustrated in the figure above.

Again as for the primary measurement ignore for the moment the variations of incident beam positions and angle from beam jitter, and consider a system with three magnets having lengths and strengths such that BL2 = 2*BL1 = 2*BL3, BPM's with gains calibrated and encoder values known for beam in their electrical centers with straight undeflected beam (B2 = 0), and beam on the axis of BPM1 and BPM8. The primary energy measurement is obtained from absolute measurements of the quantities in the +B state:

1) BL2(+) – total integral BdL in B2 along the beam path with magnets in the + polarity.
2) Z1(+) – separation of BPM's on the incident leg (m)
3) Z2(+) – separation of BPM's on the deflected leg (m)
4) X(+) – encoder positions(m) of BPM's 2-3-4 and BPM's 5-6-7

and the corresponding quantities in the –B state.

Measurements are made with field in the +B configuration for some number of bunches extending over some number of beam pulses (at 5 Hz). Then the magnet polarities are reversed to the –B state, and BPM's 2-3-4 and BPM's 5-5-6 are moved to the Δx(-) position and a new set of measurements of 1)-4) are made at some later time. The beam energy is then obtained from the total bend angle $\theta_2(+) + \theta_2(-)$:

$$E \pm \Delta E \approx \theta^{+}{}_{1} - \theta^{+}{}_{3} + \theta^{-}{}_{1} - \theta^{-}{}_{3} \pm ((\Delta\theta^{+}{}_{1})^2 + (\Delta\theta^{+}{}_{3})^2 + (\Delta\theta^{+}{}_{1})^2 + (\Delta\theta^{+}{}_{3})^2)^{1/2}$$

and knowledge of all the "constant" factors in 1)-4).

The extended measurements cannot be made for single bunches. The extended result is extracted from average quantities measured over some number of bunches in the +B and –B state. This introduces additional sources of errors from the comparison of system parameters in two states at different times. Extraction of the energy from the sum of angles $\theta_2(+)$ and $\theta_2(-)$ makes result more insensitive to some parameters, in particular to the encoder calibration obtained with B = "zero".

From this definition of the primary measurement we can draw some conclusions about the measurement strategies and contributions to the total error:

1) As in the primary measurement, the following general statements still apply: a) B2 magnet is the only one that enters; b) For given errors $\Delta\theta_1$ and $\Delta\theta_2$, the fractional error on the energy is smaller when θ_2 is large; c) The only z distances that matter are the separations of the BPM's within each group; d) Measurements using BPM1 and BPM8 might reduce errors if errors from fields BL1 and BL3 and BPM1 and BPM8 encoder cross calibrations were not large; e) the optimum chicane length and magnet lengths and strengths to control emittance growth would not be affected.
2) The BPM resolution is still a critical parameter and must be less than 0.18 μm to keep the angle error below 10^{-4}. This follows from the fact that the ultimate error on each $\theta(+)$ and $\theta(-)$ is derived from the BPM resolution and the Z1 and Z2 distances.
3) The measurement still requires accurate cross calibration of the encoder x positions. The encoder calibration should still be less than the BPM resolution of 0.18 μm.
4) The error from miss calibration of encoders if B2 has residual field at B = "zero" can be eliminated with careful operating procedure. This requires that the encoder calibration used for the +B state and –B state be the same. In that case any kink from residual field in B2 will be included in the encoder calibrations, and is removed from the total bend

angle $\theta_2(+) + \theta_2(-)$ when signs of the angles are taken into account. This is a major advantage of the extended method.

Advantages of the Δx chicane

The primary advantages of the Δx chicane over the angle measuring chicane are:

1) The use of two magnets B1 and B2 in the primary instrument to map beam energy into Δx offset significantly reduces the requirements for the BPM resolutions and encoder calibrations compared to that needed to measure angles accurately.
2) The same number of magnets (two) is required to be accurately built and characterized for each chicane. This follows from the requirement for making B2 long, and probably optimized as two magnets instead of one, to keep the emittance growth down.
3) The Δx chicane could be optimized with fewer BPM's, even including pairs of BPM's at each BPM station for redundancy. In addition to reducing the system complexity and cost, this has the practical advantages that it requires fewer BPM's that need to be calibrated.
4) In the Δx chicane, the Δx offset from energy is decoupled in the measurement from the offsets caused by betatron position and angle jitter. The offsets at BPM2 from position and angle jitter can be measured external to the chicane in BPM1 and BPM3, or potentially in other BPM's in the BDS system, and applied as a jitter correction to BPM2. The energy jitter only appears in BPM2 data. In the angle chicane the betatron jitter and energy jitter are totally entangled in the trajectory measurements and cannot be separated. This may have some practical advantages in understanding the data under various beam conditions. Since the betatron jitter appears only on BPM1 and BPM3 while the energy jitter also appears on BPM2, it would be easier to study them separately. It may also make it easier to incorporate information from other BPM's in the BDS. Betatron jitter information from other BPM's applies only to the BPM1 and BPM3 data, whereas energy jitter measured elsewhere applies only to the BPM2 data.

The primary reason the Δx chicane is superior is that we would put our money into building and calibrating the BL1 and BL2 once and for all so we could extract the energy measurement from position offset Δx in only one pair of BPM's, modulo corrections for betatron jitter from BPM's outside the chicane. The reverse bend in BL2 eliminates the need to measure angle for each bunch. The beam angle has already been measured in a sense by the precise knowledge of the BL2 and BL1. Measurement of beam position to extract Δx on each bunch puts easier requirements on the BPM resolution and the encoder calibrations than does the requirement to measure angle for each bunch. You pay your money up front for a precisely calibrated BL1 and BL2 and don't have to keep measuring angles with BPM's and encoders for the life of the system. This is the better strategy.

Conclusions

Summarize the results so far. (to be written)