# **NEUTRINO THEORY**

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# Plan of the talk

- Neutrino Masses and Mixings
- The Seesaw and Non-Seesaw Paradigms
- Matter-AntiMatter Asymmetry
- Lepton Flavour and Number Violation at the LHC
- Lepton Flavour and Number Violation in Low-Energy Experiments
- Conclusions

#### • Neutrino Masses and Mixings

[T. Schwetz, M. A. Tortola, J. W. F. Valle, arXiv:0808.2016v3, Feb 2010]



At the  $3\sigma$  CL:

$$\begin{split} \Delta m_{\odot}^2 \left[ 10^{-5} \text{eV}^2 \right] &= 7.03 - 8.27 \,, \\ \Delta m_{\text{atm}}^2 \left[ 10^{-3} \text{eV}^2 \right] &= 2.07 - 2.75 \,, \\ \sin^2 \theta_{12} &= 0.27 - 0.38 \,, \\ \sin^2 \theta_{23} &= 0.36 - 0.67 \,, \\ \sin^2 \theta_{13} &\leq 0.053 \,, \end{split}$$

with  $\Delta m_{\odot}^2 = m_{\nu_2}^2 - m_{\nu_1}^2$  and  $\Delta m_{\rm atm}^2 = m_{\nu_3}^2 - m_{\nu_1}^2$ .

# **Cosmological and astronomical limits (WMAP + SDSS)**:

[M. Tegmark et al., PRD69 (2004) 103501]

$$\sum_{i=1}^{3} m_{\nu_i} \lesssim 1.74 \text{ eV} \quad (95\% \text{ CL}) .$$

- The Seesaw Paradigm [P. Minkowski, PLB67 (1977) 421; T. Yanagida, (1979) . . . ]

# $SO(10) \rightarrow SU(4)_{PS} \otimes SU(2)_R \otimes SU(2)_L$ $\rightarrow SU(3)_c \otimes SU(2)_R \otimes SU(2)_L \otimes U(1)_{(B-L)}$ $\rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \equiv SM + 3\nu_R's$

$$\left(\begin{array}{c}\nu_L\\l_L\end{array}\right)_i,\quad l_{iR},\quad \nu_{iR};\qquad i=1,2,3=e,\mu,\tau$$

- The Seesaw Paradigm [P. Minkowski, PLB67 (1977) 421; T. Yanagida, (1979) . . . ]

 $\begin{aligned} & \mathsf{SO}(10) \to \mathsf{SU}(4)_{\mathrm{PS}} \otimes \mathsf{SU}(2)_R \otimes \mathsf{SU}(2)_L \\ & \to \mathsf{SU}(3)_c \otimes \mathsf{SU}(2)_R \otimes \mathsf{SU}(2)_L \otimes \mathsf{U}(1)_{(B-L)} \\ & \to \mathsf{SU}(3)_c \otimes \mathsf{SU}(2)_L \otimes \mathsf{U}(1)_Y \equiv \mathsf{SM} + 3\,\nu_R \mathsf{'s} \end{aligned} \\ & \left( \begin{array}{c} \nu_L \\ l_L \end{array} \right)_i, \quad l_{iR}, \quad \nu_{iR}; \qquad i = 1, 2, 3 = e, \mu, \tau \end{aligned} \\ & \mathcal{L}_{\mathrm{mass}} = -\frac{1}{2} (\bar{\nu}_L, \ \bar{\nu}_R^C) \underbrace{\left( \begin{array}{c} 0 & m_D \\ m_D^T & m_M \end{array} \right)}_{:6 \times 6 \text{ matrix}} \left( \begin{array}{c} \nu_L \\ \nu_R \end{array} \right) + \mathrm{H.c.} \end{aligned}$ 

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Seesaw approximation:  $(m_M \gg m_D \sim m_t)$ 

 $m_N^{\text{heavy}} \approx m_M \sim 10^{15} \text{ GeV} \leftarrow 3 \text{ heavy neutrinos}$  $m_{\nu}^{\text{light}} \approx -m_D \frac{1}{m_M} m_D^T \sim 4 \times 10^{-2} \text{ eV} \leftarrow 3 \text{ light neutrinos}$  $\theta_{\nu N} \approx m_D m_M^{-1} \approx \sqrt{m_{\nu}^{\text{light}}/m_N^{\text{heavy}}} \sim 10^{-12}$ 

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$$\begin{split} m_N^{\text{heavy}} &\approx m_M \sim 10^{15} \text{ GeV} \leftarrow 3 \text{ heavy neutrinos} \leftarrow \text{ unobservable !} \\ m_{\nu}^{\text{light}} &\approx -m_D \frac{1}{m_M} m_D^T \sim 4 \times 10^{-2} \text{ eV} \leftarrow 3 \text{ light neutrinos} \\ \theta_{\nu N} &\approx m_D m_M^{-1} \approx \sqrt{m_{\nu}^{\text{light}}/m_N^{\text{heavy}}} \sim 10^{-12} \leftarrow \text{ unobservable !} \end{split}$$

– The Non-Seesaw Paradigm

[A.P., PRL95 (2005) 081602 [hep-ph/0408103]; based on A.P., ZPC55 (1992) 275;
D. Wyler, L. Wolfenstein, NPB218 (1983) 205;
R.N. Mohapatra, J.W.F. Valle, PRD34 (1986) 1642.]

**Break SO(3)** and  $U(1)_l$  flavour symmetries:

$$\mathsf{SO}(3) \ \stackrel{\sim h_{ au}}{\longrightarrow} \ \mathsf{SO}(2) \simeq \mathsf{U}(1)_l \ \stackrel{\sim h_e}{\longrightarrow}$$

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 $U_l(1)$ -broken Yukawa sector:

$$m_D = \frac{v}{\sqrt{2}} \begin{pmatrix} \varepsilon_e & a e^{-i\pi/4} & a e^{i\pi/4} \\ \varepsilon_{\mu} & b e^{-i\pi/4} & b e^{i\pi/4} \\ \varepsilon_{\tau} & c e^{-i\pi/4} & c e^{i\pi/4} \end{pmatrix},$$

with  $a \sim b \sim 10^{-2} \sim h_{\tau}$ ,  $c \lesssim 10^{-4}$  &  $|\varepsilon_l| \sim 10^{-7} \sim h_e$ .

$$\implies m_{\nu}^{\text{light}} \sim \frac{m_e^2}{m_N} \sim 0.1 \text{ eV} \implies m_N \sim 100 - 500 \text{ GeV}$$

 $\Rightarrow$  3 nearly degenerate heavy Majorana neutrinos.

# Light neutrino-mass spectrum:

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$$m_{\nu}^{\text{light}} = \frac{v^2}{2m_N} \begin{pmatrix} \frac{\Delta m_N}{m_N} a^2 - \varepsilon_e^2 & \frac{\Delta m_N}{m_N} ab - \varepsilon_e \varepsilon_\mu & \frac{\Delta m_N}{m_N} ac - \varepsilon_e \varepsilon_\tau \\ \frac{\Delta m_N}{m_N} ab - \varepsilon_e \varepsilon_\mu & \frac{\Delta m_N}{m_N} b^2 - \varepsilon_\mu^2 & \frac{\Delta m_N}{m_N} bc - \varepsilon_\mu \varepsilon_\tau \\ \frac{\Delta m_N}{m_N} ac - \varepsilon_e \varepsilon_\tau & \frac{\Delta m_N}{m_N} bc - \varepsilon_\mu \varepsilon_\tau & \frac{\Delta m_N}{m_N} c^2 - \varepsilon_\tau^2 \end{pmatrix},$$

where

$$\Delta m_N = 2(\Delta m_M)_{23} + i[(\Delta m_M)_{33} - (\Delta m_M)_{22}], \quad \frac{b}{a} = \frac{19}{50},$$

and (in  $\sim 10^{-7}$  units)

$$\sqrt{\frac{\Delta m_N}{m_N}}a = 2, \quad \varepsilon_e = 2 + \frac{21}{250}, \quad \varepsilon_\mu = \frac{13}{50}, \quad \varepsilon_\tau = -\frac{49}{128}$$

<u>Prediction</u>: inverted mass hierarchy,  $m_{\nu_3} < m_{\nu_1} < m_{\nu_2}$ , with

$$m_{\nu_2}^2 - m_{\nu_1}^2 = 7.54 \times 10^{-5} \text{ eV}^2, \qquad m_{\nu_1}^2 - m_{\nu_3}^2 = 2.45 \times 10^{-3} \text{ eV}^2,$$
$$\sin^2 \theta_{12} = 0.36, \qquad \sin^2 \theta_{23} = 0.35, \qquad \sin^2 \theta_{13} = 0.047.$$

# • Matter-AntiMatter Asymmetry

$$\eta_B^{\text{CMB}} = \frac{n_B}{n_\gamma} = 6.1^{+0.3}_{-0.2} \times 10^{-10} \qquad (\eta_B^{\text{BBN}} = 3.4 - 6.9 \times 10^{-10}, \text{ at } 95\% \text{ CL})$$

Sakharov's conditions for generating the BAU:

[A.D. Sakharov, JETP Lett. 5 (1967) 24.]

- B-violating interactions
- C and CP violation
- Out-of-equilibrium dynamics

# → Baryogenesis through Leptogenesis

Out-of-equilibrium *L*-violating decays of heavy Majorana neutrinos produce a net lepton asymmetry, converted into the BAU through (B + L)-violating sphaleron interactions.

[M. Fukugita and T. Yanagida, PLB174 (1986) 45.]



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#### $\implies$ **Resonant** Leptogenesis

Resonant conditions for O(1) leptonic asymmetries:

[A.P., PRD56 (1997) 5431.]

$$\Rightarrow \qquad m_{N_2} - m_{N_1} \sim \frac{1}{2} \Gamma_{N_{1,2}} \\ \Rightarrow \qquad \frac{\operatorname{Im} (h^{\nu \dagger} h^{\nu})_{ij}^2}{(h^{\nu \dagger} h^{\nu})_{ii} (h^{\nu \dagger} h^{\nu})_{jj}} \sim 1$$

# **Resonant Leptogenesis**



#### **Resonant** Leptogenesis



#### **Resonant** $\tau$ -Leptogenesis

[A.P., PRL95 (2005) 081602; F. Deppisch, A.P., **PRELIMINARY**.]



# • Lepton Flavour and Number Violation at the LHC

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• Heavy Majorana Neutrino Production at the LHC

[ A.P., ZPC55 (1992) 275; A. Datta, M. Guchait, A.P., PRD50 (1994) 3195;
 T. Han, B. Zhang, PRL97 (2006) 171804;
 F. del Aguila, J. A. Aguilar-Saavedra, R. Pittau, JHEP0710 (2007) 047.]







Signal: 2 leptons + 2 jets + no  $\not p_T$ 

- LNV signatures:  $pp \rightarrow e^+e^+, \ e^+\mu^+, \ e^-e^-, \ e^-\mu^-, e^-\tau^- \dots$
- LFV signatures:  $pp \rightarrow e^+\mu^-, \ e^-\mu^+, \ e^-\tau^+\dots$

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- CP Asymmetries

[S. Bray, J.S. Lee, A.P., NPB786 (2007) 95.]

LHC sensitivity with 100 fb $^{-1}$ 



[T. Han, B. Zhang, PRL97 (2006) 171804; Manchester–ATLAS Group, work in progress.]



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# **CP** Asymmetries

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# **CP** Asymmetries

# • Lepton Number Violation:

$$A_{\rm CP}({\rm LNV1}) = \frac{\sigma(pp \to e^+e^+W^-X) - K\sigma(pp \to e^-e^-W^+X)}{\sigma(pp \to e^+e^+W^-X) + K\sigma(pp \to e^-e^-W^+X)},$$
  

$$A_{\rm CP}({\rm LNV2}) = \frac{\sigma(pp \to e^+\mu^+W^-X) - K\sigma(pp \to e^-\mu^-W^+X)}{\sigma(pp \to e^+\mu^+W^-X) + K\sigma(pp \to e^-\mu^-W^+X)},$$
  

$$R_{\rm CP}({\rm LNV}) = \frac{\frac{\sigma(pp \to e^+e^+e^+W^-X)}{\sigma(pp \to e^+\mu^+W^-X)} - \frac{\sigma(pp \to e^-e^-W^+X)}{\sigma(pp \to e^+\mu^+W^-X)}}{\frac{\sigma(pp \to e^+e^+W^-X)}{\sigma(pp \to e^+\mu^+W^-X)} + \frac{\sigma(pp \to e^-e^-W^+X)}{\sigma(pp \to e^-\mu^-W^+X)}}.$$

### **CP** Asymmetries

# • Lepton Number Violation:

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$$R_{\rm CP}({\rm LNV}) = \frac{\frac{\sigma(pp \to e^+e^+W^-X)}{\sigma(pp \to e^+\mu^+W^-X)} - \frac{\sigma(pp \to e^-e^-W^+X)}{\sigma(pp \to e^+\mu^+W^-X)}}{\frac{\sigma(pp \to e^+e^+W^-X)}{\sigma(pp \to e^+\mu^+W^-X)} + \frac{\sigma(pp \to e^-e^-W^+X)}{\sigma(pp \to e^-\mu^-W^+X)}}.$$

#### • Lepton Flavour Violation:

$$A_{\rm CP}({\rm LNC}) = \frac{\sigma(pp \to e^+\mu^-W^{\pm}X) - \sigma(pp \to e^-\mu^+W^{\pm}X)}{\sigma(pp \to e^+\mu^-W^{\pm}X) + \sigma(pp \to e^-\mu^+W^{\pm}X)},$$
  

$$R_{\rm CP}({\rm LNC}) = \frac{\frac{\sigma(pp \to e^+\mu^-W^{\pm}X)}{\sigma(pp \to e^-\mu^+W^{\pm}X)} - \frac{\sigma(pp \to e^-\mu^+W^{\pm}X)}{\sigma(pp \to e^+\mu^-W^{\pm}X)}}{\frac{\sigma(pp \to e^+\mu^-W^{\pm}X)}{\sigma(pp \to e^-\mu^+W^{\pm}X)} + \frac{\sigma(pp \to e^-\mu^+W^{\pm}X)}{\sigma(pp \to e^+\mu^-W^{\pm}X)}}.$$

#### **Resonant CP Violation through Mixing of Heavy Majorana Neutrinos**

[A.P., NPB504 (1997) 61; S. Bray, J.S. Lee, A.P., NPB786 (2007) 95.]



• Lepton Flavour and Number Violation in Low-Energy Experiments

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# • $0\nu\beta\beta$ Decay

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z+2}X + 2e^{-}$$

Half-life for  $0\nu\beta\beta$  decay:

$$[T_{1/2}^{0\nu\beta\beta}]^{-1} = \frac{|\langle m \rangle|^2}{m_e^2} |\mathcal{M}_{0\nu\beta\beta}|^2 G_{01} .$$

 $R\tau L$  realize inverted light-neutrino hierarchy, with:

$$|\langle m_{0\nu\beta\beta}\rangle| = |(\mathbf{m}^{\nu})_{ee}| = \frac{v^2}{2m_N} |\frac{\Delta m_N}{m_N} a^2 - \varepsilon_e^2| \approx 0.013 \text{ eV}.$$

Future  $0\nu\beta\beta$  experiments will be sensitive to  $|\langle m\rangle|\sim 0.01\text{--}0.05$  eV, such as SuperNEMO . . .

•  $\mu 
ightarrow e\gamma$ 



[T.P. Cheng, L.F. Li, PRL45 (1980) 1908;
 A. Ilakovac, A.P., NPB437 (1995) 491]

For 
$$h_{eN_{2,3}}^{\nu} = h_{\mu N_{2,3}}^{\nu} = 8 \times 10^{-3}$$
  
and  $m_N = 250$  GeV:

$$\begin{aligned} & \mathcal{B}(\mu \to e\gamma) \\ & \sim 7 \cdot 10^{-4} \times \frac{(h_{eN}^{\nu} h_{\mu N}^{\nu})^2 v^4}{m_N^4} \\ & \sim 10^{-12} \,. \end{aligned}$$

MEG sensitivity:  $B(\mu \rightarrow e\gamma) \sim 10^{-13} - 10^{-14}$ . •  $\mu 
ightarrow e \gamma$ 



• Coherent  $\mu \rightarrow e$  Conversion in Nuclei ( $^{48}_{22}$ Ti,  $^{197}_{79}$ Au)

[A.P., T. Underwood, PRD72 (2005) 113001]









 $m_N=250$  GeV:

$$B(\mu \to e) \approx 0.5 \times B(\mu \to e\gamma) \sim 5 \times 10^{-13}$$
.

COMET/PRISM will be sensitive to  $B(\mu \rightarrow e) \sim 10^{-13} - 10^{-18}$ .

•  $\mu \rightarrow eee$ 

[A. Ilakovac, A.P., NPB437 (1995) 491.]





 $m_N=250$  GeV:

 $\mathbf{B}(\mu \to eee) \approx 1.4 \cdot 10^{-2} \times \mathbf{B}(\mu \to e\gamma) \sim 1.4 \times 10^{-14} .$ 

•  $\mu \rightarrow eee$ 

[A. Ilakovac, A.P., NPB437 (1995) 491.]



(d)

 $m_N=250$  GeV:

 $B(\mu \to eee) \approx 1.4 \cdot 10^{-2} \times B(\mu \to e\gamma) \sim 1.4 \times 10^{-14}$ .

#### No new experiment proposed yet!

•  $\mu 
ightarrow e\gamma$ ,  $\mu 
ightarrow eee$  and  $\mu 
ightarrow e$  conversion







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- $B(\mu \rightarrow e\gamma) \sim 10^{-13}$  + successful leptogenesis  $\implies$  large  $\theta_{13}$  for models with inverted light neutrino hierarchy.
- Strong correlations among the predictions for LFV and LNV at the observable level ( $m_N = 250$  GeV):

$$egin{aligned} &m{B}(\mu 
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u eta eta} 
angle| \ pprox 0.01 \ {
m eV}. \end{aligned}$$