# Renormalons and $N^3$ LO CORGI approach for $\hat{R}(s)_{\tau}$

#### Pascalius Lai Ho Shie / Dr Chris J Maxwell

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#### Renormalons, CIPT & CORGI in Brief

Chains & Bubbles Large Nf and Leading b Approximation Borel Transform and Renormalons CIPT + CORGI

#### Theoretical Results of $N^3$ LO CORGI $\hat{R}(s)_{\tau}$ $\hat{R}(s)_{\tau}$ ALEPH Comparison Conclusions

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Outline	Renormalons, CIPT & CORGI in Brief • • • •	Theoretical Results of $N^3$ LO CORGI $\hat{R}(s)_{\tau}$ 0000 00 00
Chains & Bubbles		

#### Chains & Bubbles

- Consider a QED 1-loop vacuum diagram
- Connects n of them....
- ...then insert back into the bubble



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$$\frac{1}{t}$$

$$B^{\mu\nu}(k^2) = \left(\frac{-i}{k^2}\right)\left[g^{\mu\nu} - \frac{k^{\mu}k^{\nu}}{k^2}\right]\frac{1}{1+\Pi_0} + \left(\frac{-i}{k^2}\right)\frac{k^{\mu}k^{\nu}}{k^2}\xi$$

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# Large Nf and Leading b Approximation

The summation of vacuum diagrams with 1 to n-bubbles is...

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# Large Nf and Leading b Approximation

- ▶ The summation of vacuum diagrams with 1 to n-bubbles is...
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$$\beta(\mathbf{a}) = -ba^2(1+ca+c2a^2+c3a^3+\ldots)$$

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## Borel Transform and Renormalons

• with a resummation of  $d_n$  all orders results in

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• with a resummation of  $d_n$  all orders results in

$$D^{(L)}(Q^2) = \int_0^\infty dz e^{-z/a} B[D^{(L)}](z)$$

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$$D^{(L)}(Q^2) = \int_0^\infty dz e^{-z/a} B[D^{(L)}](z)$$

which B[D<sup>(L)</sup>](z) is the Borel transform given by...

$$B[D_{PT}^{(L)}](z) = \sum_{n=1}^{\infty} \frac{A_0(n) - A_1(n)z_n}{(1 + \frac{z}{z_n})^2} + \frac{A_1(n)z_n}{(1 + \frac{z}{z_n})} + \sum_{n=1}^{\infty} \frac{B_0(n) + B_1(n)z_n}{(1 - \frac{z}{z_n})^2} - \frac{B_1(n)z_n}{(1 - \frac{z}{z_n})}$$

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▶ with z=±z<sub>n</sub> at where the singularities lie are called RENORMALONS.

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• Consider the ratio total  $\tau$  hadronic to leptonic decay width



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$$R_{\tau} = \frac{\Gamma(\tau \to \nu_{\tau} + hadrons)}{\Gamma(\tau \to \nu_{\tau} e^{-}\overline{\nu}_{e})} = N(V_{ud}^{2} + V_{us}^{2})S_{EW}[1 + \frac{5\alpha(m_{\tau}^{2})}{12\pi} + \hat{R}(s)_{\tau} + \delta_{pc}]$$

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CIPT + CORGI		

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RS-Invariance, hence setting c2...cn = 0

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- Thus the perturbative corrections  $\hat{R}(s)_{\tau}$  is...

$$\hat{R}(s)_{ au} = rac{1}{2\pi}\int_{-\pi}^{\pi}W( heta)D(s_{0}e^{i heta})d heta$$

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• with  $D(s_0e^{i\theta})$  integrated along the complex s-plane

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$\hat{R}(s)$		

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- ...1st one is pure leading CORGI with X<sup>[L]</sup><sub>n</sub> terms constructed in V-scheme given by (C<sub>n+1</sub> denotes the coefficient)...

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▶ ...2nd one is the substitution of  $X_n^{[L]}$  with  $X_n$  for n = 1, 2, 3 constructed from c1, c2 and latest c3 aka NLO,  $N^2LO$ ,  $N^3LO$  thus  $X_1 = 0, X_2 = d_2 - d_1^2 - cd_1 - c2...$ 

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 ▶ and X<sub>3</sub> = d<sub>3</sub> - d<sub>1</sub><sup>3</sup> - <sup>5c</sup>/<sub>2</sub>d<sub>1</sub><sup>2</sup> - (3X<sub>2</sub> - 2c2)d<sub>1</sub> - <sup>c3</sup>/<sub>2</sub>

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 $\hat{R}(s)_{\tau}$ 

# Numerical Procedure of $\hat{R}(s)_{\tau}$

*R̂(s)<sub>τ</sub>* can be split into K steps of size Δθ = π/K ranging from θ=0,π

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*R̂(s)<sub>τ</sub>* can be split into K steps of size Δθ = π/K ranging from θ=0,π

$$\frac{R(s_0)}{2\pi} \simeq \frac{\Delta\theta}{2\pi} [W(0)D(s_0) + 2Re\sum_{n=1}^{K} W(\theta_n)D(s_n)]$$

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•  $W(\theta_n)$  is the Weight function and  $D(s_n) = \bar{a}_n + \sum_{n>2}^{\infty} X_n \bar{a}_n^{n+1}$ 

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- $W(\theta_n)$  is the Weight function and  $D(s_n) = \bar{a}_n + \sum_{n>2}^{\infty} X_n \bar{a}_n^{n+1}$ • Using Taylor's Theorem
- Using Taylor's Theorem

$$\bar{\mathbf{a}}_{n+1} = \bar{\mathbf{a}}_n - i \frac{\triangle \theta}{2} b B(\bar{\mathbf{a}}_n) - \frac{\triangle \theta^2}{8} B(\bar{\mathbf{a}}_n) B'(\bar{\mathbf{a}}_n)$$
  
+  $i \frac{\triangle \theta^3}{48} b^3 [B(\bar{\mathbf{a}}_n) B'(\bar{\mathbf{a}}_n)^2 + B''(\bar{\mathbf{a}}_n) B(\bar{\mathbf{a}}_n)^2] + O(\triangle \theta^4)$ 

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 $\hat{R}(s)_{\tau}$ 

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W(θ<sub>n</sub>)is the Weight function and D(s<sub>n</sub>) = ā<sub>n</sub> + Σ<sub>n>2</sub><sup>∞</sup> X<sub>n</sub>ā<sub>n</sub><sup>n+1</sup>
 Using Taylor's Theorem

$$\bar{\mathbf{a}}_{n+1} = \bar{\mathbf{a}}_n - i \frac{\bigtriangleup \theta}{2} bB(\bar{\mathbf{a}}_n) - \frac{\bigtriangleup \theta^2}{8} B(\bar{\mathbf{a}}_n) B'(\bar{\mathbf{a}}_n)$$
  
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• ...which  $B(x) = x^2 + cx^3 + c2x^4 + ...$  is the truncated beta function

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$\hat{R}(s)_{\tau}$		

$$\hat{R}(s)_{ au}$$
 vs  $lpha_{s}(m_{ au}^{2})$ 



We then convert  $\alpha_s(m_{\tau}^2) \rightarrow$ 

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Theoretical Results of  $N^3$ LO CORGI  $\hat{R}(s)_{\tau}$ 0000

 $\hat{R}(s)_{\tau}$  vs  $\alpha_s(m_Z)$  $\rightarrow \alpha_s(m_Z)$  through flavour threshold...



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 $\hat{R}(s)_{\tau}$ 

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Outline	Renormalons, CIPT & CORGI in Brief 0 0 0 0	Theoretical Results of $N^3$ LO CORGI $\hat{R}(s)_{\tau}$
ALEPH Comparison		

#### ALEPH Comparison

We plot ratio total τ hadronic to leptonic decay width to energy of the particle (s)...

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and made direct comparison with data from ALEPH....

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Conclusions		



 N<sub>3</sub>LO shows the reliability of CIPT + CORGI prediction in comparison to FOPT<sub>Fix</sub> Order Perturbation Theory

Pascalius Lai Ho Shie / Dr Chris J Maxwell Renormalons and  $N^3$ LO CORGI approach for  $\hat{R}(s)_{\tau}$  ▲□ > ▲圖 > ▲ 圖 > ▲ 圖 > ◎ ④ (

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Outline	Renormalons, CIPT & CORGI in Brief 0 0 0 0	Theoretical Results of $N^3$ LO CORGI $\hat{R}(s)_{\tau}$
Conclusions		



- N<sub>3</sub>LO shows the reliability of CIPT + CORGI prediction in comparison to FOPT<sub>Fix</sub> Order Perturbation Theory
- Prediction matches with ALEPH data for energy s > 0.525.

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THANK YOU

#### THANK YOU ▶ and any Questions?

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