Strong coupling, discrete symmetry and flavour

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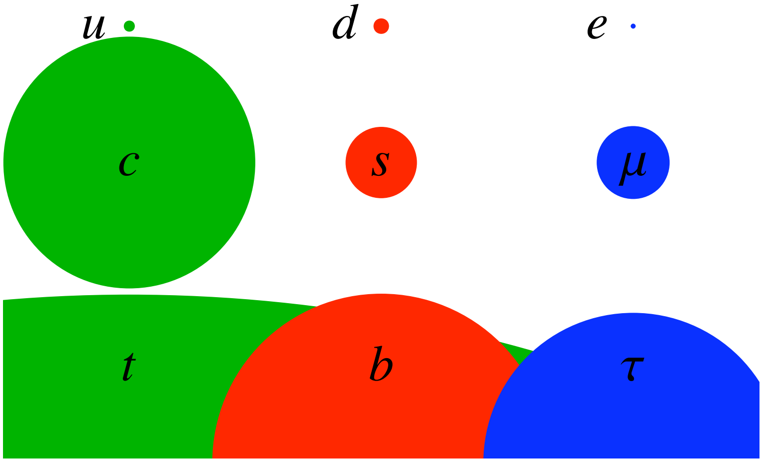
Problem 1: mass hierarchies



Problem 2: mixing matrices

Experiments determine quark and neutrino mixing matrices

$$|V_{\rm CKM}| = \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.226 & 0.973 & 0.042 \\ 0.009 & 0.041 & 0.999 \end{pmatrix} \sim \begin{pmatrix} 1 & \eta & \varepsilon\eta \\ \eta & 1 & \varepsilon \\ \varepsilon\eta & \varepsilon & 1 \end{pmatrix}$$

$$|V_{\rm PMNS}| \approx \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

- Why are there small but non-zero off diagonal terms in $V_{\rm CKM}$?
- Why are $V_{\rm CKM}$ and $V_{\rm PMNS}$ so different?

Some observations

Attempt to address these problems using two observations:

Observation

Neutrino mixing matrix suggests a **discrete flavour symmetry**.

Observation

Large hierarchies arise naturally in theories with **strong coupling**.

Mixing patterns from discrete symmetry

Observation

Neutrino mixing matrix suggests a **discrete flavour symmetry**.

- Highly symmetrical – **tribimaximal mixing**.
- Many successful models[1] apply a discrete flavour symmetry.
- Examples of groups: A_4, S_4, $PSL_2(7)$, $\Delta(27)\ldots$

Some general features:

- 3 generations $\implies$ discrete group with a **3** representation.
- Extended Higgs sector.
- Favour symmetry broken with particular **vacuum alignment**.
- Flavour symmetries **do not** reproduce quark mixing matrix.

[1] G.Altarelli, F.Feruglio - arXiv:1002.0211 [hep-ph]

Hierarchies from dynamical scales

Observation

Large hierarchies arise naturally in theories with **strong coupling**.

- Dynamical scales generated by renormalisation group flow:

$$\Lambda \sim E e^{-8b\pi^2/g^2(E)}$$

- Λ is dynamical **strong coupling** scale where $g \to \infty$.
- Exponential dependence $\implies$ naturally large hierarchies.

Addressing the flavour problem

Postulate

The flavour problem can be solved using two principles:

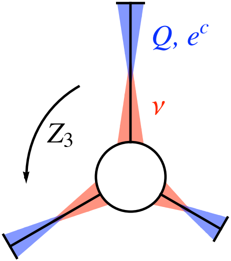
1. **Discrete flavour symmetry** for mixing.
2. **Strong coupling** for **all hierarchies** and flavour symmetry breaking.

Requirements

- Discrete flavour symmetry should contain a **3** representation.
- Flavour symmetry must be broken in the quark sector but **not** in the neutrino sector.
- **Multiple** strongly coupled sectors are needed to build hierarchies – one per generation.
- Should be a generic feature of strong coupling.
- Note – here strongly coupled $\equiv$ asymptotically free.

Pictorial interpretation

- Strongly coupled sectors on spokes.
- **Discrete flavour symmetry** on hub.
- Spoke lengths **generate hierarchies** and **break flavour symmetry**.
- Quarks (on spokes) feel hierarchies and symmetry breaking.
- Neutrinos (near hub) do not.
- **All mixing** occurs on hub.



Yukawa couplings

How does it work?

- Suppose Q_a is actually **bound state**[2] of strongly coupled gauge group G_a.

$$\Lambda_a Q_a \sim Y_a Y_a$$

- **Elementary** 'Yukawa' couplings are

$$V_{\mathrm{UV}} \supset \frac{1}{M_X} \xi_{ia} \bar{q}_i (Y_a Y_a) \phi$$

- M_X is **UV cutoff scale**.

[2] M.J.Strassler - arXiv:hep-ph/9510342

Yukawa couplings

Match to low energy couplings

$$\frac{1}{M_X}\xi_{ia}\bar{q}_i(Y_aY_a)\phi \sim \lambda_{ia}\bar{q}_iQ_a\phi \quad \Longrightarrow \quad \lambda_{ia} \sim \frac{\Lambda_a}{M_X}\xi_{ia}$$

- ξ_{ia} are parameters in elementary theory.
- Expect $\xi_{ia} \sim 1$ and flavour symmetry preservation.
- Λ_a are dynamical scales.
- $\Lambda_1 \ll \Lambda_2 \ll \Lambda_3$ natural $\Longrightarrow$ **hierarchy in quark Yukawas**.
- Differences typically **break flavour symmetry**.

Mass hierarchies

Strong coupling with composite left handed quark doublets and $\Lambda_1 \ll \Lambda_2 \ll \Lambda_3$ results in quark Yukawas

$$\lambda \sim \begin{pmatrix} \varepsilon\eta & \varepsilon & 1 \\ \varepsilon\eta & \varepsilon & 1 \\ \varepsilon\eta & \varepsilon & 1 \end{pmatrix}$$

- $\varepsilon\,,\eta \ll 1$ parameterise hierarchy.
- Expect quark masses $\sim \varepsilon\eta$, ε, 1.

Result – large quark mass hierarchies

$$\frac{m_d}{m_s} \sim \frac{m_u}{m_c} \sim \eta \quad \text{and} \quad \frac{m_s}{m_b} \sim \frac{m_c}{m_t} \sim \varepsilon$$

- Account for discrepancies with **confinement scheme**.
- Set $\Lambda_a \bar{e}_a \sim Y_a Y_a$ for charged lepton mass hierarchy.

Mixing hierarchies

Quark mixing matrix attained by diagonalising quark Yukawas.

- Assume λ_u already diagonal for simplicity.
- Diagonalise λ_d with **biunitary rotation**

$$\hat{\lambda}_d \sim U_d^\dagger \begin{pmatrix} \varepsilon\eta & \varepsilon & 1 \\ \varepsilon\eta & \varepsilon & 1 \\ \varepsilon\eta & \varepsilon & 1 \end{pmatrix} V_d$$

- U_d sees no hierarchy in rows of $\lambda_d \implies U_d \sim 1$.
- $V_d = V_{\mathrm{CKM}}^\dagger$ sees hierarchy in columns of λ_d.

Result – large quark mixing hierarchies

$$V_d \sim \begin{pmatrix} 1 & \eta & \varepsilon\eta \\ \eta & 1 & \varepsilon \\ \varepsilon\eta & \varepsilon & 1 \end{pmatrix}$$

Mixing hierarchies

Different story in neutrino[3] sector:

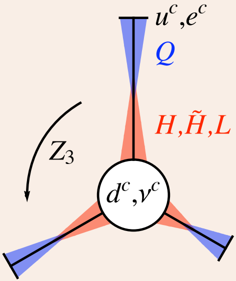
- Assume **elementary** neutrinos $\implies$ no hierarchies in Yukawa.
- Neutrinos **do not** see strong coupling.
- Mixing determined **only** by elementary coupling constants.
- Preserves flavour symmetry $\implies$ **tribimaximal mixing**.

[3] N.Haba - arXiv:hep-ph/9807552

Summary

The flavour problem can be addressed using two principles:

- Discrete flavour symmetry for mixing.
- Strong coupling for hierarchies and flavour symmetry breaking.
- Quarks see strong coupling effects, neutrinos do not.
- Mixing only on hub.



Simple examples use Z_3 permutation symmetry with strong coupling described by:

- S-confinement
- AdS/CFT correspondence

with same results.