Maximum Likelihood Fits

$$y(\theta; p) = \frac{dn}{d\cos(\theta)} = Np\mathcal{F}(\theta)$$

p is the parameter we wish to determine

N is the normalisation factor such that

$$\int_{-1}^{+1} \mathrm{d}\cos(\theta) y(\theta; p) = 1$$

Thus y is normalised so that it behaves as a probability distribution

For the ith event

$$y_i = Np\mathcal{F}(\theta_i)$$

Likelihood

$$\mathcal{L}(p) = \prod_{i=1}^{n} y_i$$

Plots of $\mathcal{L}$ tend to be Gaussian (at least in the vecinity of the max. of the distribution)

Log Likelihood Fits

$$\ell = \log \mathcal{L}$$
$$= \sum_{i=1}^{n} \log y_i$$

Errors on Likelihood Fits

When $\mathcal{L}$ is Gaussian, the following quantities are all identical, and any of them can be used as the definition of the error on p

(i) the root mean square deviation of the $\mathcal{L}$ distribution about its mean

(ii) $[\frac{(-\partial^2 \ell)}{(\partial p^2)}]^{-\frac{1}{2}}$.

(iii) the change in p required to reduce ℓ from its maximum value by 0.5, i.e.

$$\ell(p_0 \pm \delta p) = \ell(p_o) - \frac{1}{2}$$

However, when $\mathcal{L}$ is not Gaussian, estimating the error on p is much harder. There are a number of methods

(i) Find a range of p values $p_1 \leq p \leq p_2$ where

$$\ell(p_i) = \ell(p_o) - \frac{1}{2}, i = 1 \mathrm{or} 2$$

probability that parameter lies in this range $= 66\%$

(ii)

$$\langle [\frac{(-\partial^2 \ell)}{(\partial p^2)}]^{-\frac{1}{2}} \rangle$$

Where the average extends over a suitable range of p, e.g. over the range of p for which L is large.

It is clear that non-Gaussian distributions are unpleasant, it is therefore desirable to choose the varying parameters so that the error distributions are more gaussian, when ever possible!

Comments

- MLM uses events one at a time - no need to construct histograms
- Can use MLM when the density of events in the physical region is low
- Can transform from one variable to another and the answer remains unique
- In MLM constraints on parameters are easily imposed
- Background subtraction techniques can be a nuisance
- MLM requires an assumed functional form before anything can be deduced
- Large amounts of computation is often involved
- Normalisation factor cannot normally be obtained analytically