Spin density matrices

For the interaction $e^-(\lambda) + e^+(\lambda') \rightarrow W^-(\tau_-) + W^+(\tau_+)$ where $\lambda, \lambda', \tau_\pm$ are the respective helicity states, there are nine different helicity amplitudes, $F^{(\lambda)}_{\tau_-\tau_+}$ for each of the two possible helicity states of the e^+e^-system ($\lambda = -\lambda' = \pm\frac{1}{2}$) in the limit where electron and positron mass are neglected.
An observable which describes the polarisation of the W boson pairs is the two-particle joint density matrix, whose elements are given by:

$$\rho_{\tau_-\tau'_-\tau_+\tau'_+}(\cos\theta) = \frac{\Sigma_\lambda F^{(\lambda)}_{\tau_-\tau_+}\left(F^{(\lambda)}_{\tau'_-\tau'_+}\right)^*}{\Sigma_{\lambda\tau_+\tau_-}\left|F^{(\lambda)}_{\tau_-\tau_+}\right|^2} \qquad (1)$$

Altogether, the W^+W^- system has 81 density matrix elements.
With the normalisation

$$\sum_{\tau_+\tau_-} \rho_{\tau_-\tau'_-\tau_+\tau'_+} = 1 \qquad (2)$$

80 of these are independent. If CP is conserved in the iteraction, then this number reduces to 35.

Cross Sections and Projection Operators

The projection operators reflect the standard V-A couplings to the W boson in the W decay. The spin density matrices can be extracted using the threefold differential cross section

$$\frac{\mathrm{d}\sigma(e^+e^- \to W^+W^-)}{\mathrm{d}\cos\theta}\mathrm{Im}\rho^{W^-(\theta)}_{+0} = \frac{1}{\mathrm{Br}(W^- \to \ell^-\bar{\nu})}\int \frac{\mathrm{d}\sigma(e^+e^- \to W^+\ell^-\bar{\nu})}{\mathrm{d}\cos\theta\mathrm{d}\cos\theta^*\mathrm{d}\phi^*}\Lambda_{\tau\tau'}(^*\phi^*\mathrm{d}\cos\theta^*\mathrm{d}\phi^*) \tag{3}$$

Experimetally this corresponds to:

$$\rho_{\tau\tau'} = \frac{1}{N}\sum_{i=1}^{N} Lambda_{\tau\tau'}(\cos\theta_i^*, \phi_i^*)$$

Where N is the number of events.

CP-Violation

If the interaction is CP-invariant the following must be valid

$$\mathrm{Re}\rho^{W^-}_{\tau_1\tau_2} - \mathrm{Re}\rho^{W^+}{}_{-\tau_1-\tau_2} = 0 \tag{5}$$

and

$$\mathrm{Im}\rho^{W^-}_{\lambda_1\lambda_2} - \mathrm{Im}\rho^{W^+}{}_{-\lambda_1-\lambda_2} = 0 \tag{6}$$

Relevent Cross Sections for CP Studies

the CP invariance of $Z^0, W^\pm$ interactions can be tested by measuring linear combinations of appropriately defined weighted cross sections

$$\begin{aligned} \mathrm{d}\sigma^{\theta}_{+-} &\equiv \frac{\mathrm{d}\sigma(e^+e^- \to W^+W^-)}{\mathrm{d}\cos\theta}\mathrm{Im}\rho^{W^-(\theta)}_{+-} \\ &= \frac{1}{\mathrm{Br}(W^- \to \ell^-\bar{\nu})}\int \frac{\mathrm{d}\sigma(e^+e^- \to W^+\ell^-\bar{\nu})}{\mathrm{d}\cos\theta\mathrm{d}\cos\theta^*\mathrm{d}\phi^*}(-4\cos\phi^*\sin\phi^*)\mathrm{d}\cos\theta^*\mathrm{d}\phi^* \end{aligned} \tag{7}$$

$$\begin{aligned} \mathrm{d}\sigma^{\theta}_{+0} &\equiv \frac{\mathrm{d}\sigma(e^+e^- \to W^+W^-)}{\mathrm{d}\cos\theta}\mathrm{Im}\rho^{W^-(\theta)}_{+0} \\ &= \frac{1}{\mathrm{Br}(W^- \to \ell^-\bar{\nu})}\int \frac{\mathrm{d}\sigma(e^+e^- \to W^+\ell^-\bar{\nu})}{\mathrm{d}\cos\theta\mathrm{d}\cos\theta^*\mathrm{d}\phi^*}\left(\frac{8}{3\sqrt{2\pi}}(1-4\cos\theta^*)\sin\phi^*\right)\mathrm{d}\cos\theta^*\mathrm{d}\phi^* \end{aligned} \tag{8}$$

$$\begin{aligned} \mathrm{d}\sigma^{\theta}_{-0} &\equiv \frac{\mathrm{d}\sigma(e^+e^- \to W^+W^-)}{\mathrm{d}\cos\theta}\mathrm{Im}\rho^{W^-(\theta)}_{-0} \\ &= \frac{1}{\mathrm{Br}(W^- \to \ell^-\bar{\nu})}\int \frac{\mathrm{d}\sigma(e^+e^- \to W^+\ell^-\bar{\nu})}{\mathrm{d}\cos\theta\mathrm{d}\cos\theta^*\mathrm{d}\phi^*}\left(\frac{-8}{3\sqrt{2\pi}}(1+4\cos\theta^*)\sin\phi^*\right)\mathrm{d}\cos\theta^*\mathrm{d}\phi^* \end{aligned} \tag{9}$$