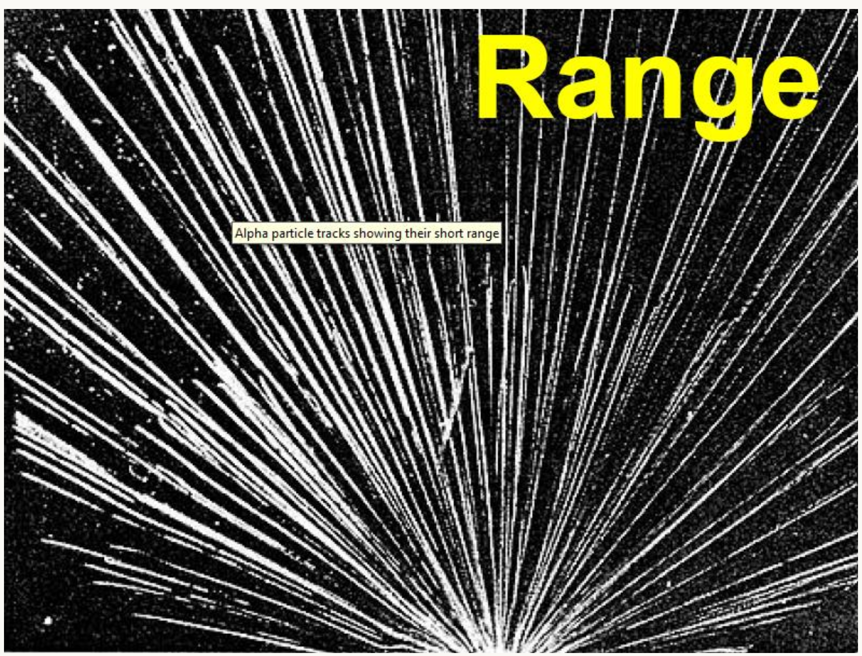
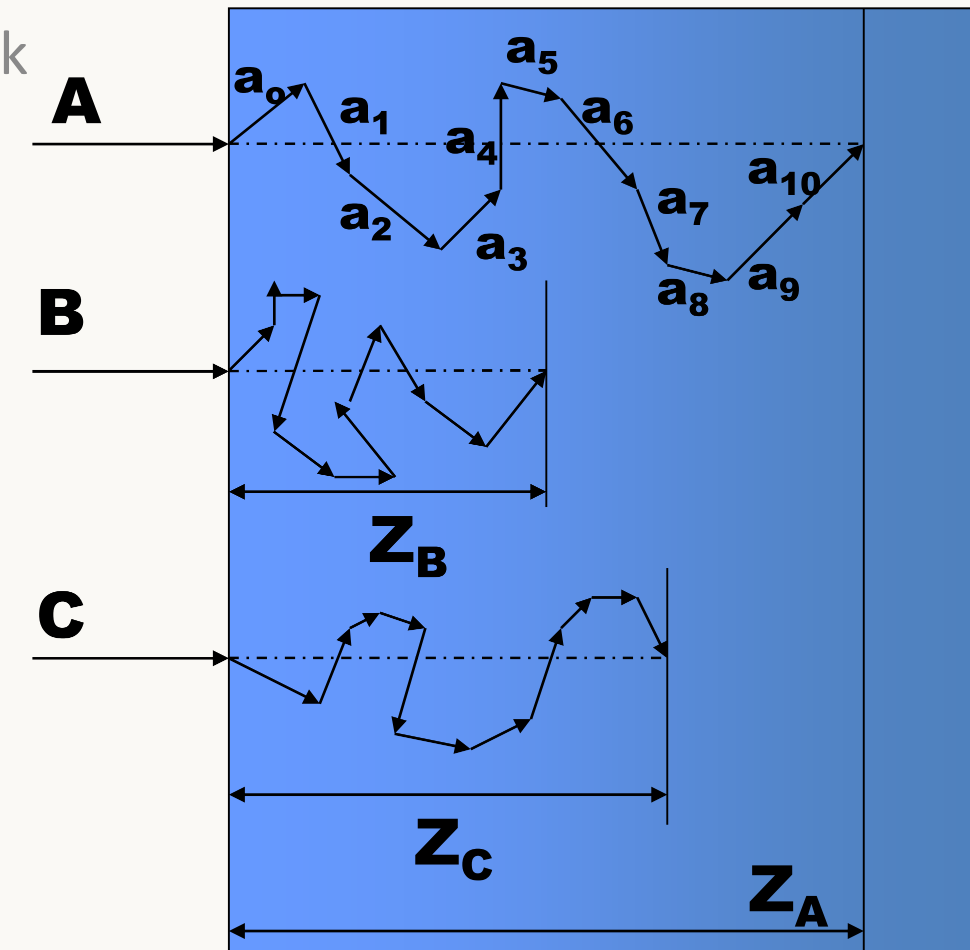
MPHY0032 – Ionising Radiation Physics

Lecture 3 – Charged Particle Interactions

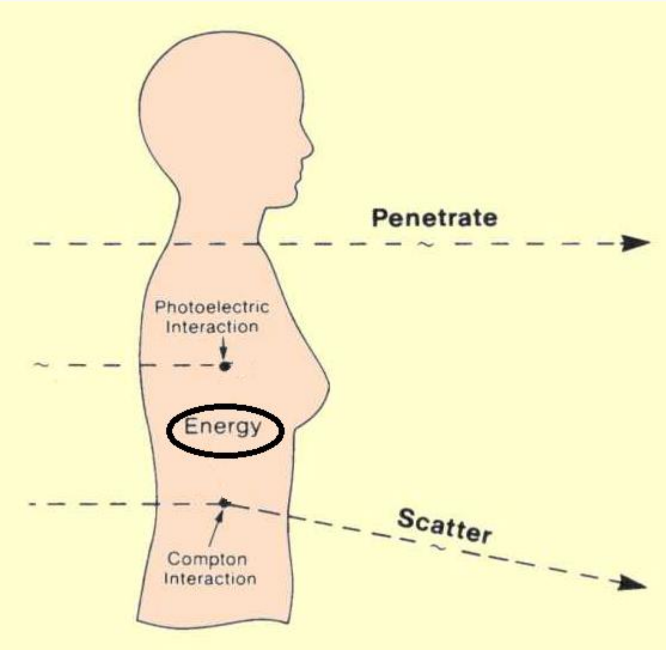
Billy Dennis – w.dennis@ucl.ac.uk





Review of last lecture

- Key photon interaction mechanisms
  - Absorption
    - Photoelectric effect; $\tau \propto \frac{Z^4}{E^3}$
      - Dominates at low photon energies, especially high atomic number materials
    - Pair production; $\kappa \propto Z^2 . E$
      - when hν >1.02 MeV
      - Dominates at very high photon energies, especially high atomic number materials
  - Scatter
    - Thompson, Raleigh; $\phi \propto \frac{Z^2}{E}$
      - Most significant at low energies
    - Compton; $\sigma \propto \frac{1}{E}$
      - Dominates at medium photon energies, especially low atomic number materials

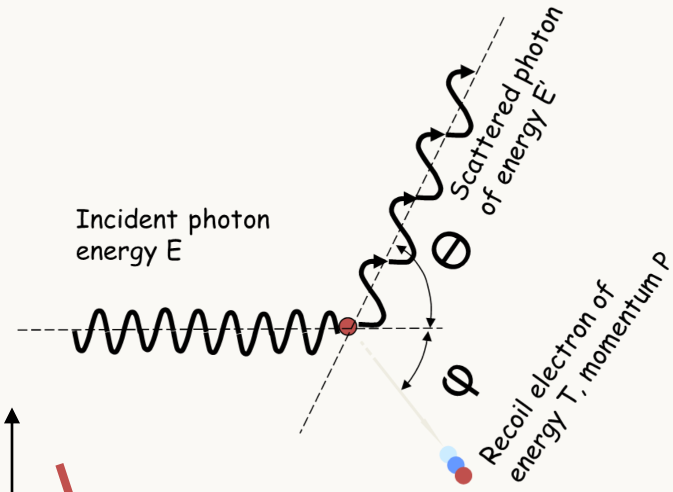


Review of last lecture

- Compton shift equation
  - Shift varies with scatter angle
  - Photon energy but also wavelength

- $E_f = \dfrac{E_i}{1+\frac{E_i}{m_e c^2}(1-\cos\theta)}$

- Combined interactions vary with photon energy



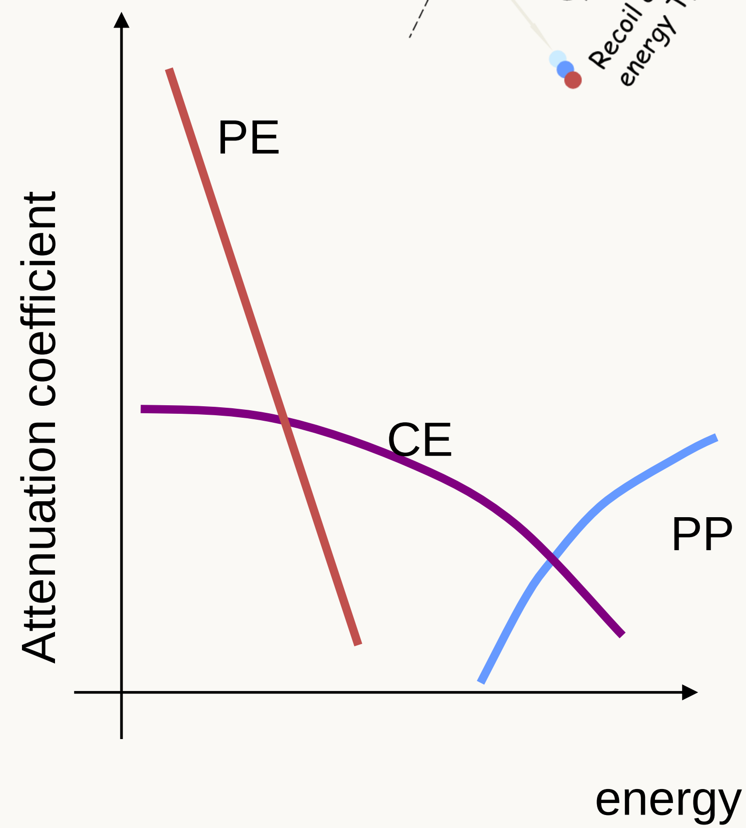


Image quality in photon imaging

Low transmission
(high skin dose)

Good contrast
(photoelectric effect
$\mu \propto z^4$)

Mammography

Acceptable signal and contrast

Compton effect

X-ray imaging

CT - imaging

High signal

Reduced contrast

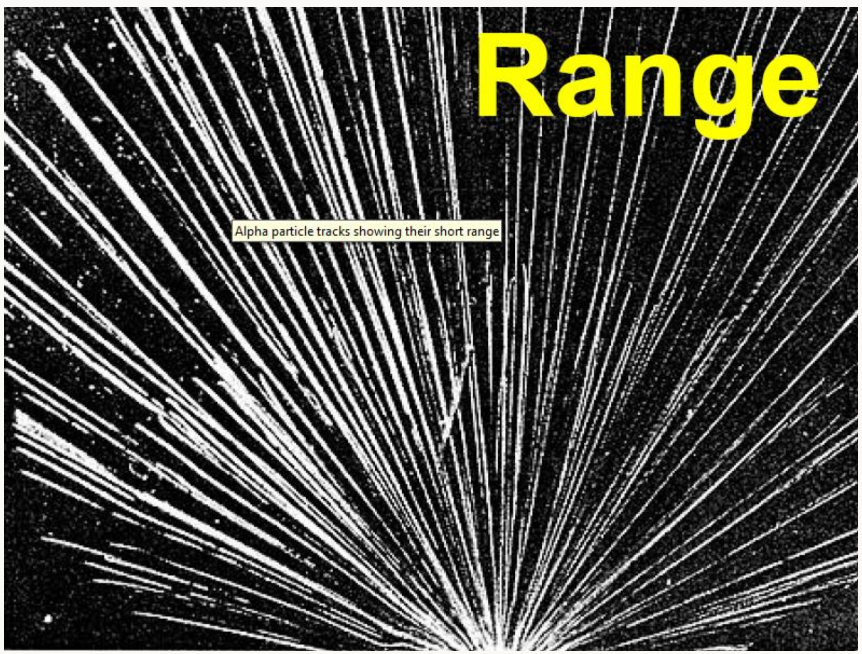
Reduced image quality

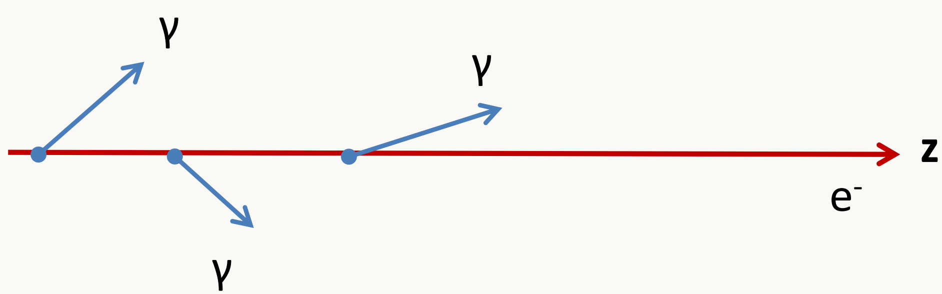
High doses

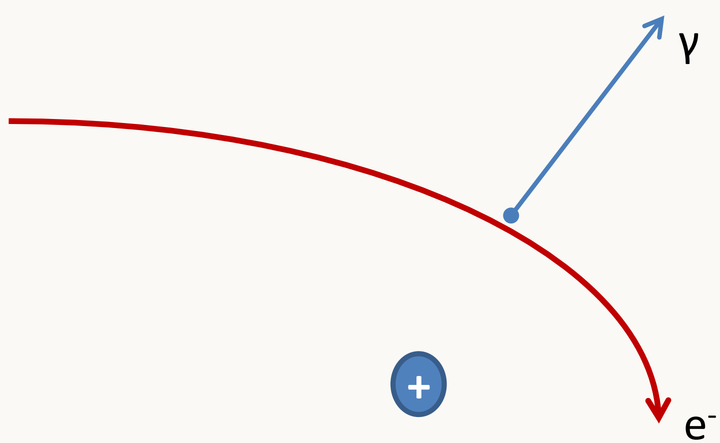
30kev **80kev** **120kev** **1Mev**

Photon interactions tougher example

2. Charged Particle Interactions



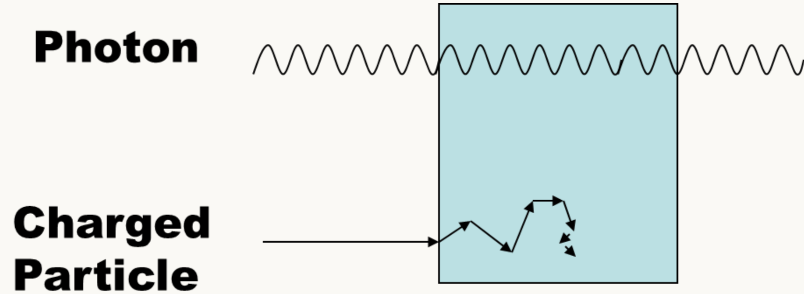




Charged particle interactions

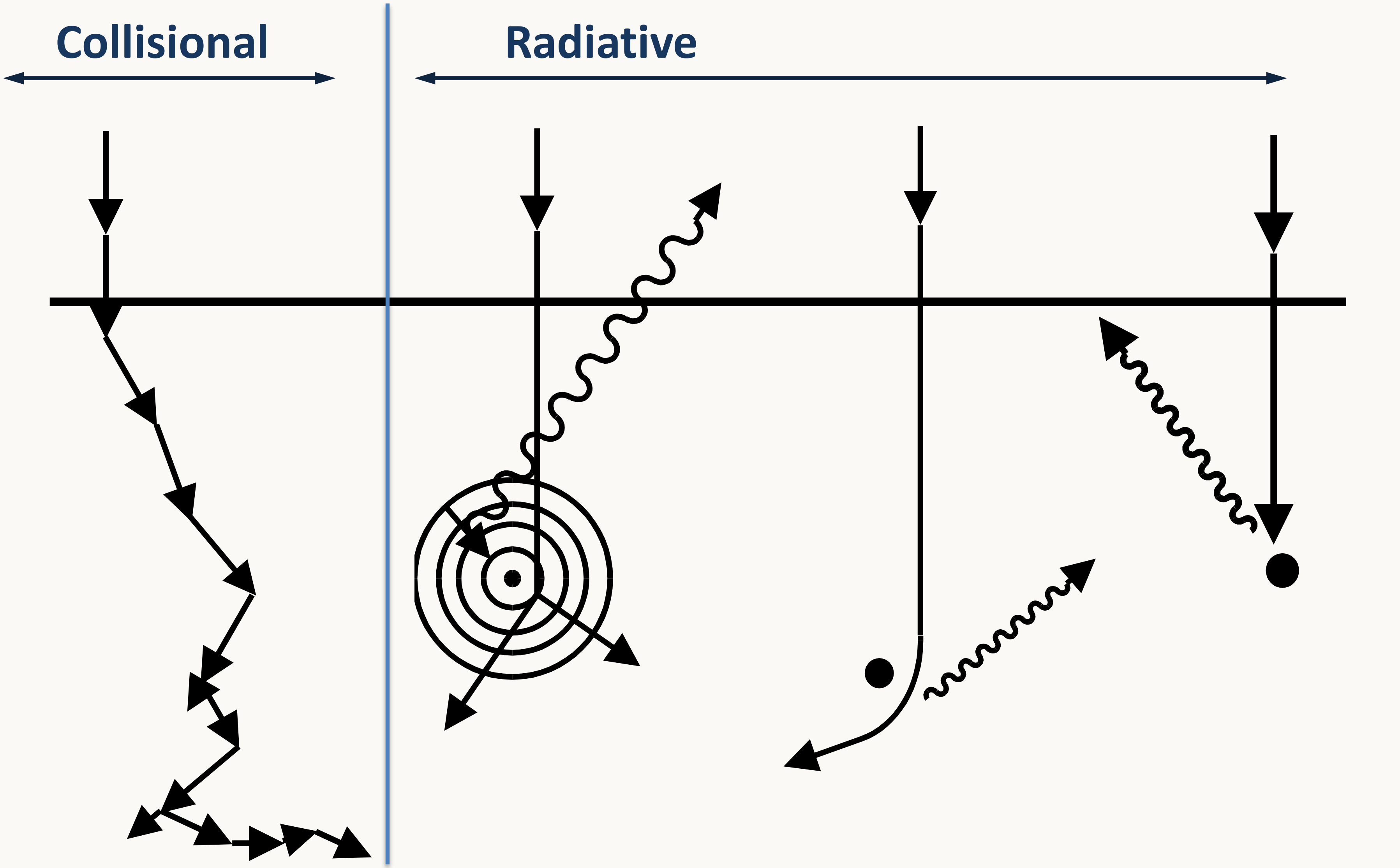
- Source of charged particles for medical physics?
- CPs behave differently from photons
  - Mass/charge
- Therefore need new equations to describe how they transmit in tissue

Comparing Photon and CP interactions



- Photons (x-rays, γ-rays) **Why?**
  - few interactions per unit length
  - Characteristic length ~10^{-1} m
  - large energy loss per interaction
  - Infinite penetration in theory (% loss / unit length)
- Charged particles (electrons, protons, alpha)
  - many interactions per unit length
  - characteristic length ~10^{-5} – 10^{-3} m
  - small energy loss per interaction
  - can calculate a fairly accurate penetration for given particle energy

Charged Particle Interactions



Charged particle interactions

- Elastic scatter
  - No loss of energy
  - Above a few hundred keV, sharply peaked in forward direction
- Inelastic scatter from nucleus/atomic electron
  - Photon emitted
  - Bremsstrahlung

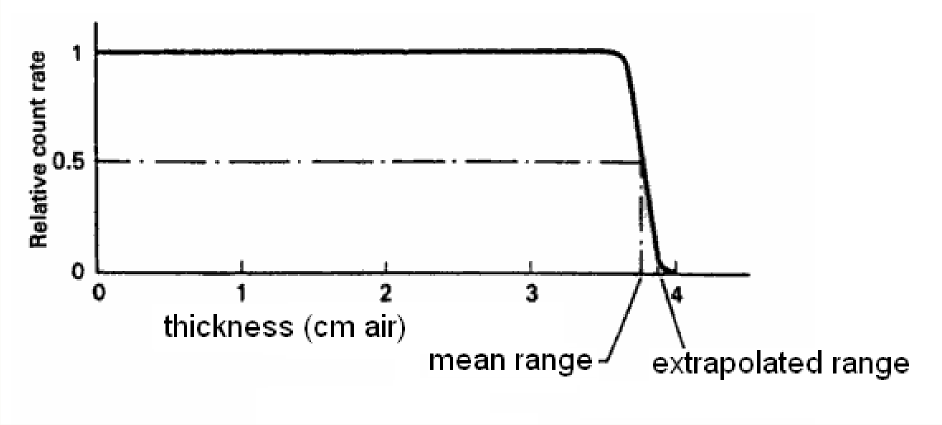
Radiative energy loss

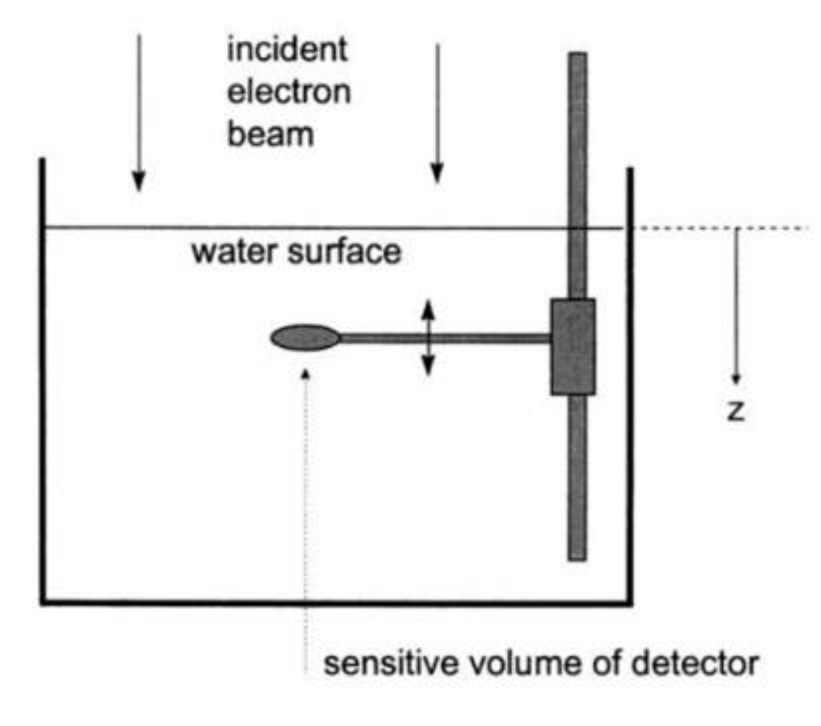
- Inelastic scatter causing excitation
  - Atomic electron excited
  - Delayed photon emitted
- Ionisation
  - 2 electrons exit the interaction
  - More energetic called the "primary"

Collisional energy loss

Cerenkov not considered significant

Experimental measurements of CP interactions





Stopping power - ${}^{dE}/_{dx}$

- Used to define energy loss of charged particles along path
- Equal to the average energy loss per unit length travelled in medium
- Made up of collisional and radiative parts

$$Linear\ stopping\ power, \qquad S = \frac{dE}{dx} = \left(\frac{dE}{dx}\right)_{col} + \left(\frac{dE}{dx}\right)_{rad}$$

$$mass\ stopping\ power = \frac{linear\ stopping\ power}{density}$$

Charged particle range - r_0

- Used to define how far a charged particle of a given energy will travel in a medium before being stopped (E=0)

$$r_0 = \int_0^{E_0} \frac{1}{dE/dx} . dE$$

- Two approximations:
  - CSDA – Continuous slowing down approximation
  - Straight ahead approximation

Simplifies to $$r_0 = \frac{E_0}{dE/dx}$$

CSDA approximation

- All energy lost at continuous rate along particle track given by stopping power
  - All energy loss fluctuations neglected
- Only refers to interactions that result in energy loss
  - Ignores elastic scatter and thermal diffusion
  - Therefore, r_0 always smaller than mean value of path lengths actually travelled

Straight ahead approximation

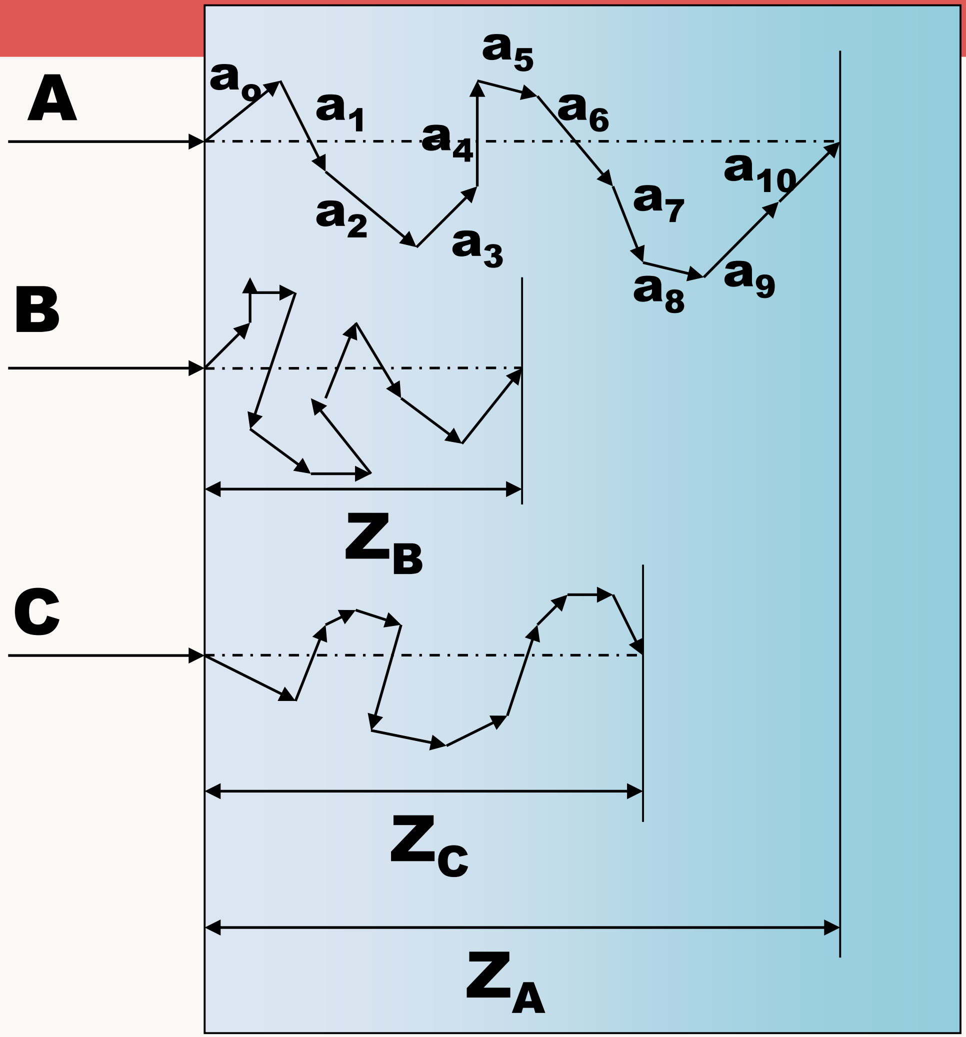
- Any changes in direction ignored
  - Track is assumed rectilinear
- Good approx. for heavy particles
  - Protons and alpha
- Departure from linearity described by detour factor, d

$$d = \frac{z_{av}}{r_0}$$

z_{av} = mean penetration depth

r_0 = CSDA range

Explained on next slide



$$d = \frac{Z_{av}}{r_o}$$

Mean penetration depth:

$$Z_{av} = \frac{Z_A + Z_B + Z_C}{3}$$

CSDA range:

$$r_o = \frac{\sum a_n + \sum b_n + \sum c_n}{3}$$

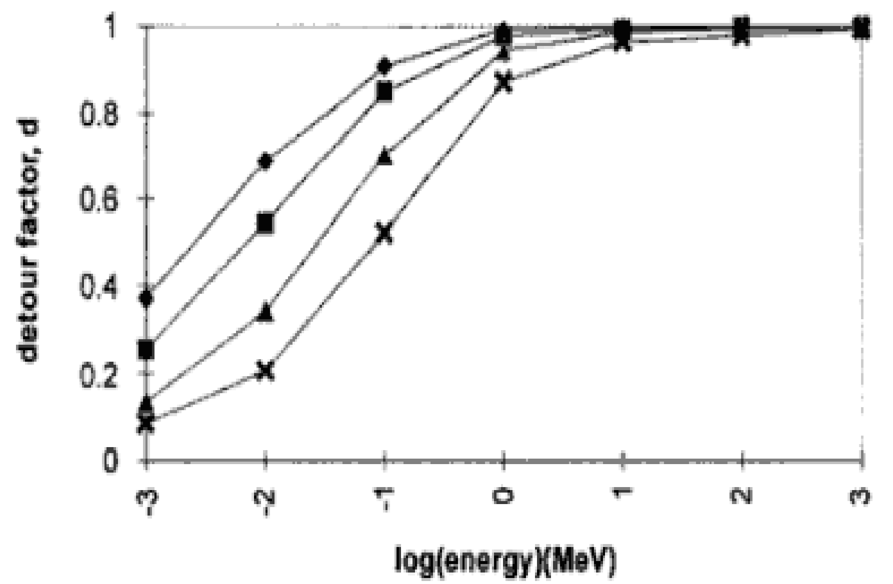
If using CSDA, assume all interactions have energy loss (average pathlength will be greater than r_o)

Detour factor - d

- Found experimentally by visual inspection of cloud chamber tracks

All values in cm x 10^{-5}

WATER	protons		alpha	
Energy (keV)	Csda range	Penetration depth	Csda range	Penetration depth
5	2.2	1.4	1.8	1.1
55	10.6	9.0	9.8	8.0
100	16.0	14.2	14.0	12.0

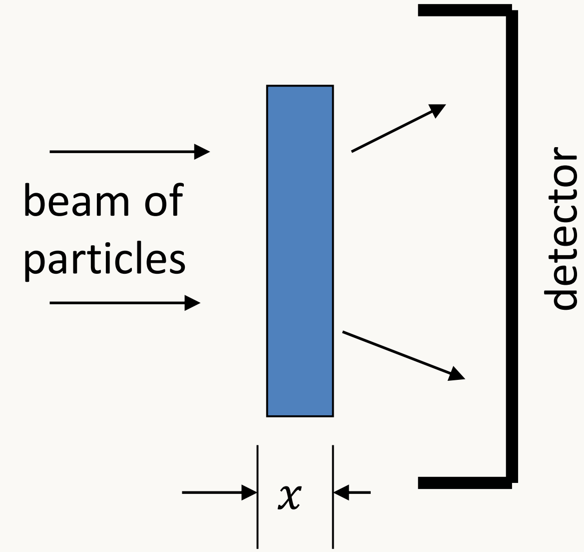


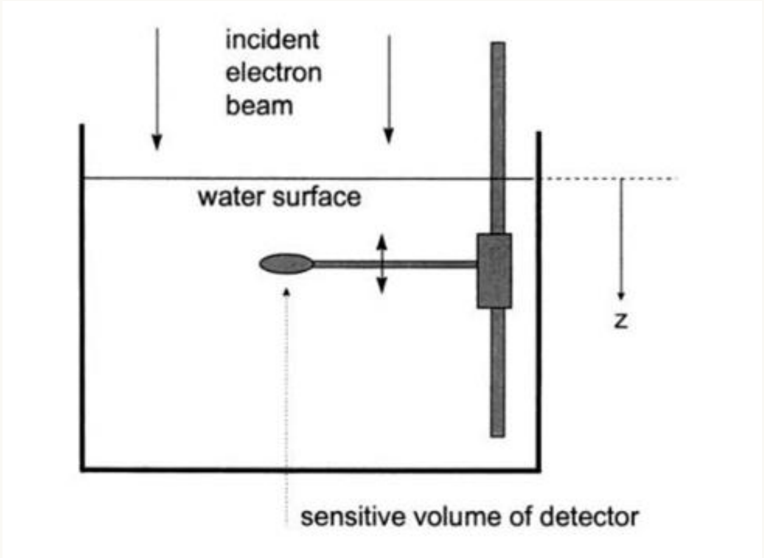
Detour factor, *d*, versus proton energy (MeV) in various materials.
◆ carbon ; ■ aluminium ; ▲ copper ; ✖ tungsten.

- Always less than 1
- Depends on
  - Energy
  - Z of medium

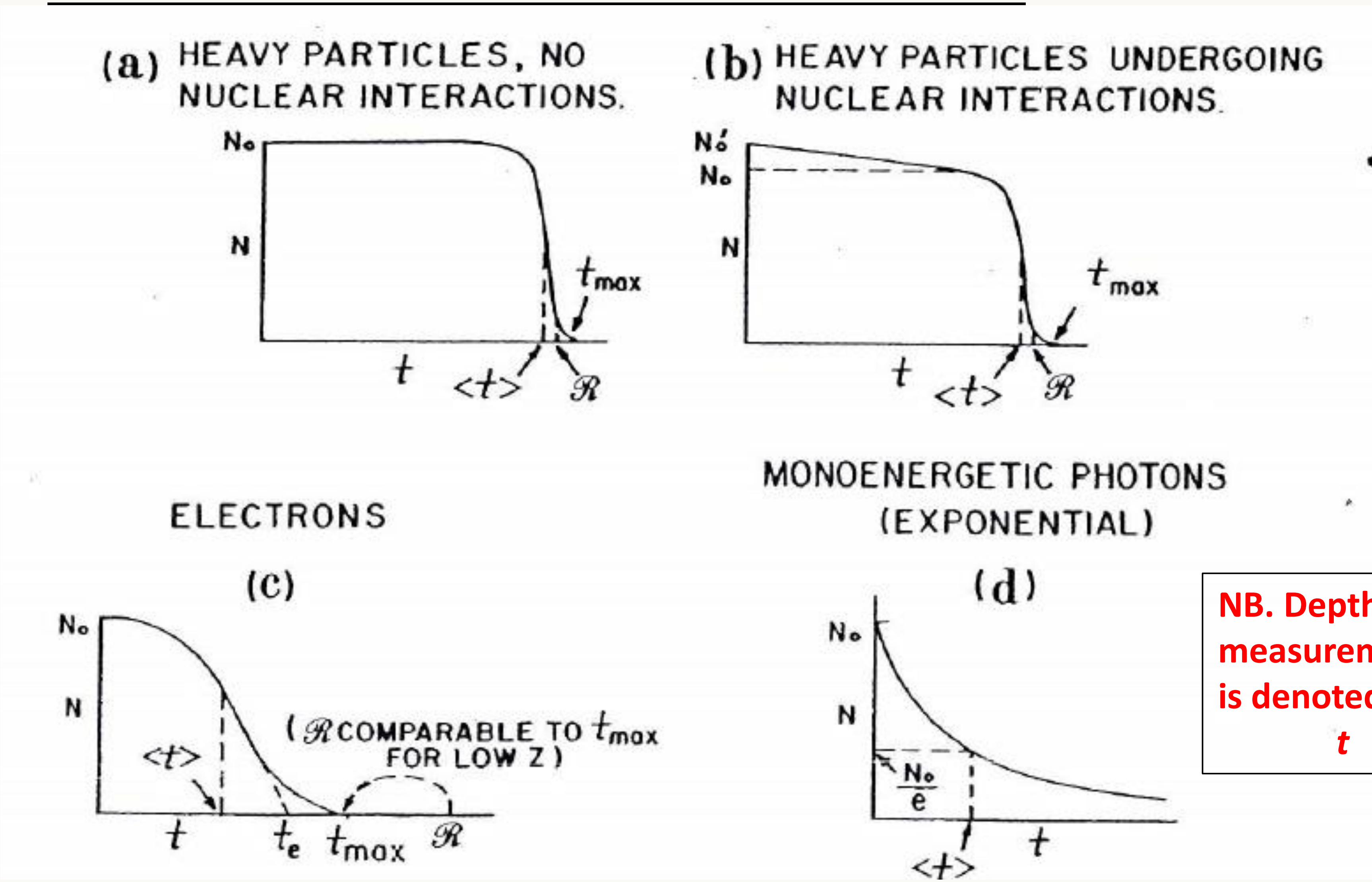
Measurement of CP range

- Two methods:
  1. Measure fraction transmitted through increasing thicknesses of medium
     - All particles must be collected so geometry important
     - Energy dependence of detector may affect result
  2. Measure depth dose in a medium
     - Routine in radiotherapy dept for electron beams
     - Different result to Method 1 (above)





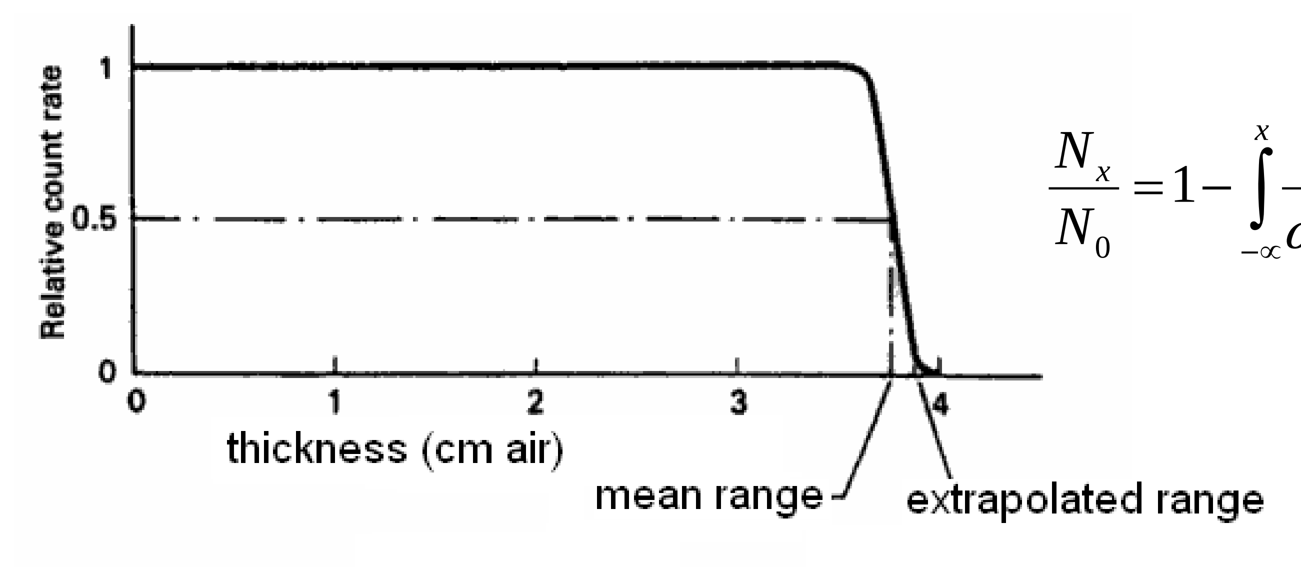
Method 1 – Transmission measurements



NB. Depth of measurement is denoted by: ***t***

Range straggling - α

- An explanation for why the range measurements have a "tail" rather than a sharp drop-off



$$\frac{N_x}{N_0} = 1 - \int_{-\infty}^{x} \frac{1}{\alpha\sqrt{\pi}} \exp\left(\frac{-(x-R)^2}{\alpha^2}\right) dx$$

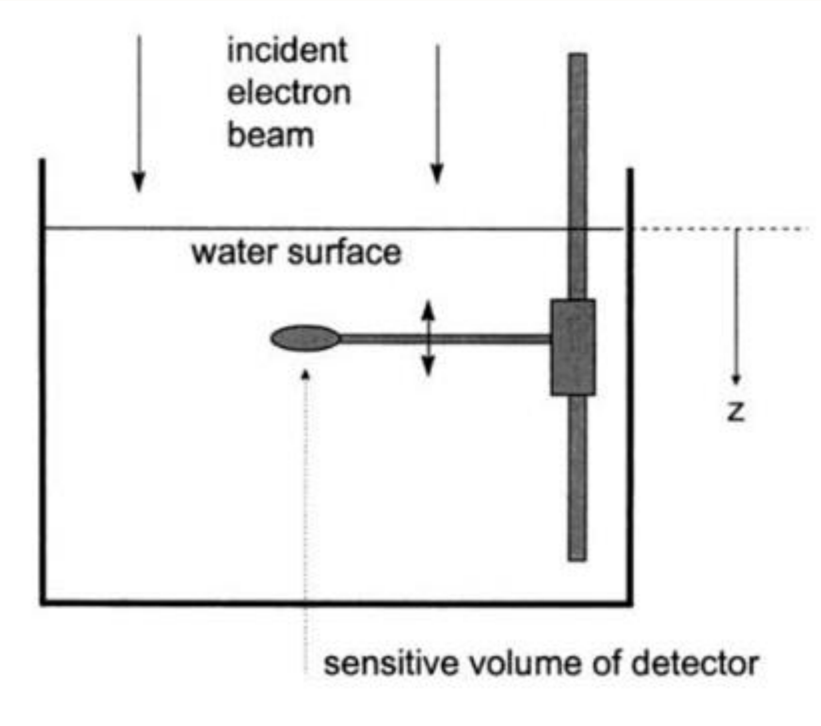
Not for exam

- Fluctuation in path length for particles of same initial energy
  - Value varies with energy/medium
  - See proton lectures with Amos for more detail

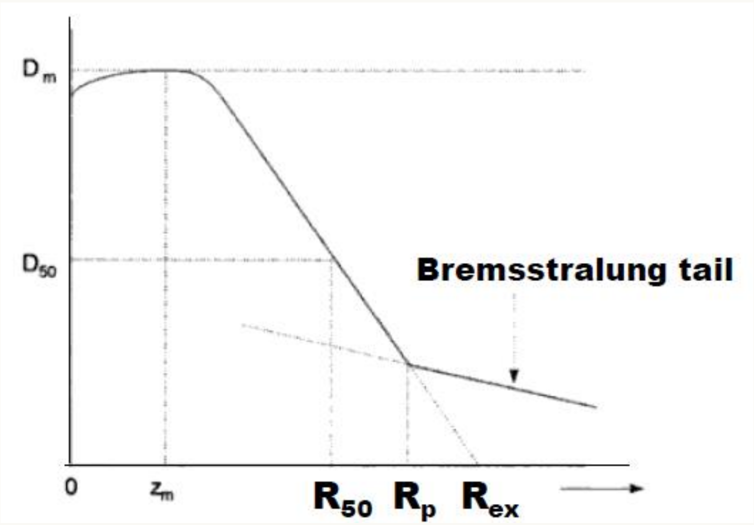
Method 2 – Depth dose measurement

- Routinely used in radiotherapy

- Includes effects of “build-up”
  - Learn more in dosimetry lectures
  - At large depths there is bremsstrahlung from electrons

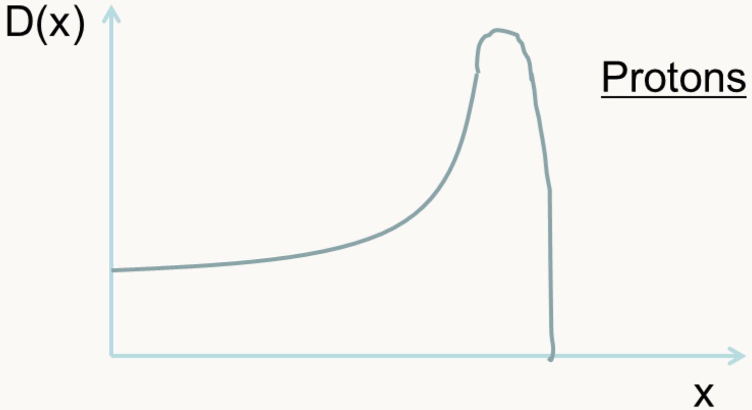


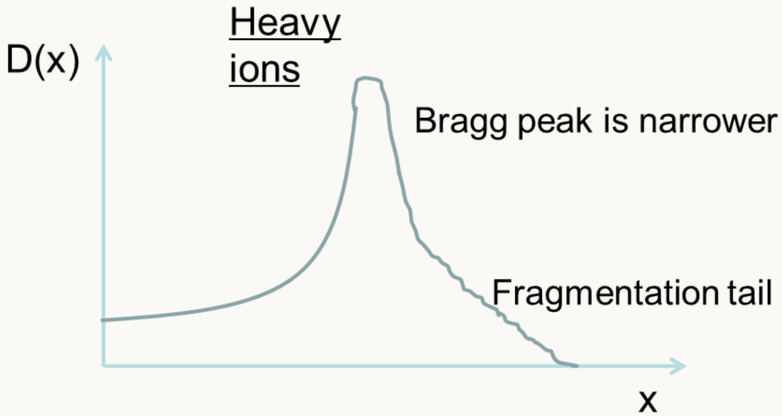
Depth dose curve



- Dose measured and plotted as function of depth

High energy electrons





Theoretical treatment of CP interactions

$$\sigma_{rad} = \frac{dT}{T + m_0 c^2} \frac{1}{N dx} = \frac{1}{137} r_0^2 Z^2 \int_0^1 B.d\left(\frac{hv}{T}\right) = \frac{r_0^2 Z^2 \overline{B}}{137}$$

$$\frac{1}{\rho}\left(\frac{dT}{dx}\right) = \frac{4\pi r_0^2 m_0 c^2}{\beta^2} \frac{1}{u} \frac{Z}{A} z^2 \left[\ln\left(\frac{2 m_0 c^2 \beta^2 \gamma^2}{I}\right) - \beta\right]$$

You won't have to learn these!

Coulomb's law

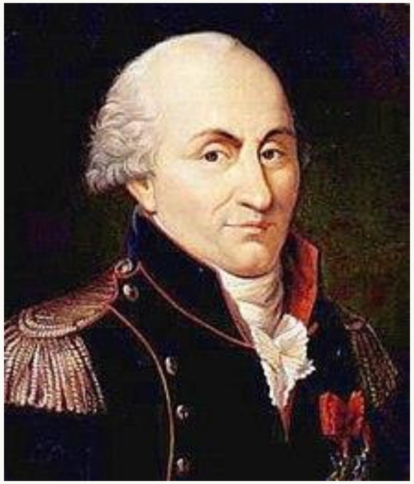
- Describes the force experienced by two electrically charged particles

$$F = k_e \frac{q_1 q_2}{r^2}$$

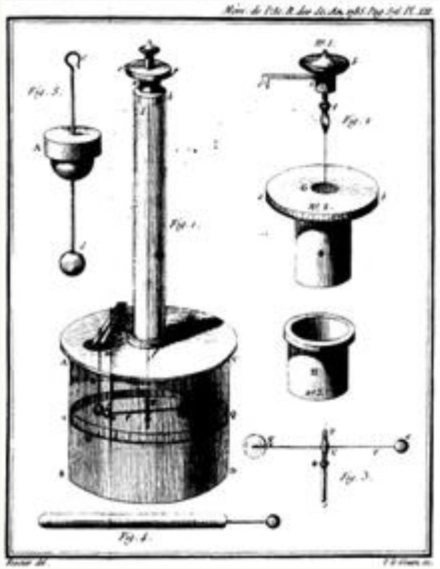
$$k_e = 8.99 \times 10^9 \ Nm^2C^{-2}$$

- Potentially large forces reduced by the inverse square law

Charles-Augustin de Coulomb



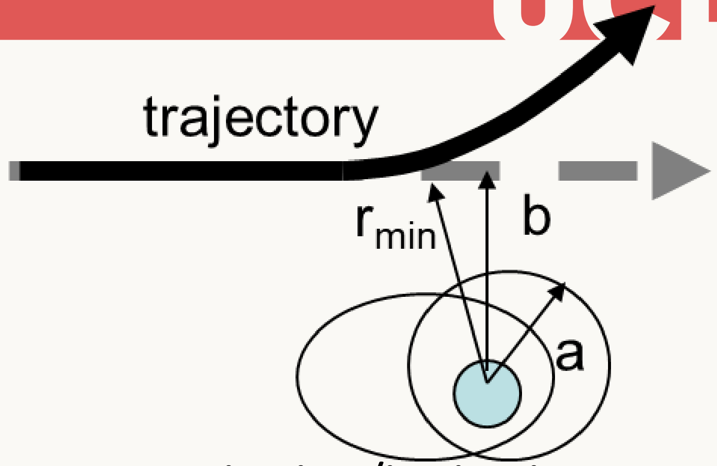
Published 1785



Types of interaction

- Incoming charged particles can interact with:
  - Individual electrons
    - Most frequent
    - Inelastic if energy is transferred (excite or ionise)
    - Elastic if sub-excitation collision
  - Nucleus **Why?**
    - Nuclear reactions possible (requires heavy charged particle)
    - Nuclear or Coulomb scattering
    - If incoming particle is electron – Bremsstrahlung inelastic scatter
  - Coulomb field of whole atom
    - Applies to heavy particles at low velocity

Which type?



- Depends on distance of approach
  - b>>a – soft collision
    - Coulomb force disrupts atomic structure – excitation/ionisation
    - Small energy transfer
  - b~a – hard collision
    - Individual electron ejected – delta ray
  - b<<a – nuclear interaction
    - Most relevant for incoming electrons

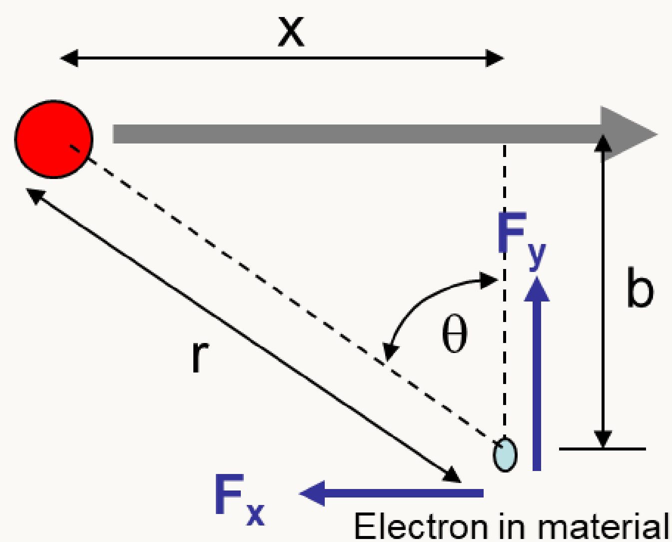
- Depends on velocity of particle and duration of collision

Heavy CP interactions – classical – part a)

- Consider a heavy charged particle (not electron) interacting with a single electron
  - Assume $M \gg m_0$
  - Force experienced by particle:

$$F = k\frac{ze^2}{r^2}$$

  - x-component will cancel during collision, thus only net force imparting momentum will be F_y



$$\Delta p = \int_{-\infty}^{\infty} F_y dt = \int_{-\infty}^{\infty} zke^2 \frac{\cos\theta}{r^2}.dt$$

Change in momentum -

$$\Delta p = \frac{zke^2}{bv}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos\theta.d\theta = \frac{2zke^2}{vb} = \frac{2zr_0 m_0 c^2}{vb}$$

Heavy CP interactions – classical – part b)

- Energy lost by particle M for a given value of b:

$$\Delta E(b) = \frac{(\Delta p)^2}{2m_0}$$

- In a real material all values of b will exist

- Energy loss given by:

$$\beta = {}^{v}/_{c}$$

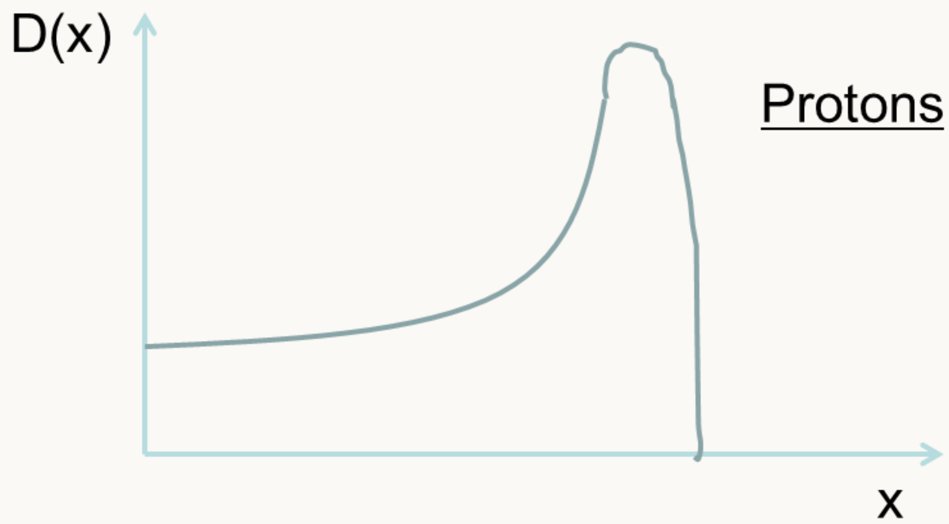
$$\frac{dE}{dx} = 4\pi N_e \rho \frac{z^2 {r_0}^2 \, m_0 c^2}{\beta^2}\left[\ln\left({}^{b_{max}}/_{b_{min}}\right)\right]$$

- Important factors $\frac{dE}{dx} \propto \frac{z^2}{v^2}$

Log factor, so requires a large change to make much difference

Bragg curve

- Why do protons have such potential for treatment?



$$\frac{dE}{dx} \propto \frac{z^2}{v^2}$$

- Particle energy changes dramatically as velocity is reduced

Heavy CP interactions – QM – part a)

The Bethe formula

- Assumes
  - E transferred is small compared to particle E
  - zZ << β
- Orbital velocity of atomic electrons << v
- Both soft and hard collisions are important
- Introduces
  - Mean excitation energy, I
  - E_c (cut-off energy) between hard and soft collisions

Hans Bethe

Published 1932

$$\frac{1}{\rho}\left(\frac{dT}{dx}\right)=\frac{4\pi r_0^2 m_0 c^2}{\beta^2}\frac{1}{u}\frac{Z}{A}z^2\left[\ln\left(\frac{2m_0c^2\beta^2\gamma^2}{I}\right)-\beta\right]$$

u = atomic mass unit

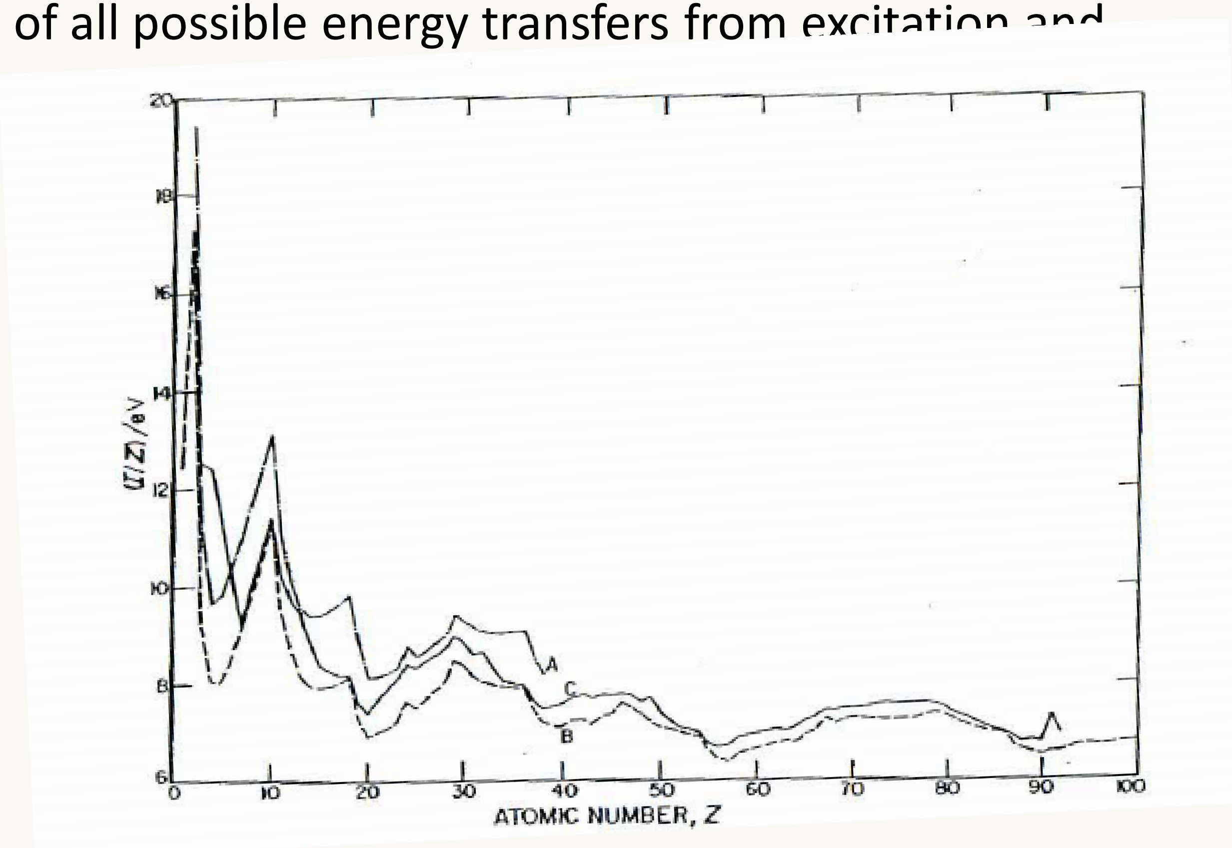
γ = Lorentz factor

Heavy CP interactions – QM – part b)

- Mean excitation energy, I
  - ln (I) dependence – thus slow to change
  - Weighted average of all possible energy transfers from excitation and ionisation
  - Depends on Z
  - Note cyclic nature

    I/Z vs Z

Why?



Heavy CP interactions – QM – part c)

Restricted stopping power

- Limits the stopping power value to energy changes with a certain “Cut-off energy”, E_c
- Interactions with large energy transfers are eliminated

- Useful for assessing dosimetry close to particle track

Heavy CP interactions – QM – part d)

Bethe-Bloch equation

- Includes extra correction factors:
  - Shell term
    - Corrects for shells whose orbital electron velocity > particle velocity
  - Barkas term
    - Accounts for the sign of incoming charged particle
  - Bloch term
    - Accounts for departures from Born approximation

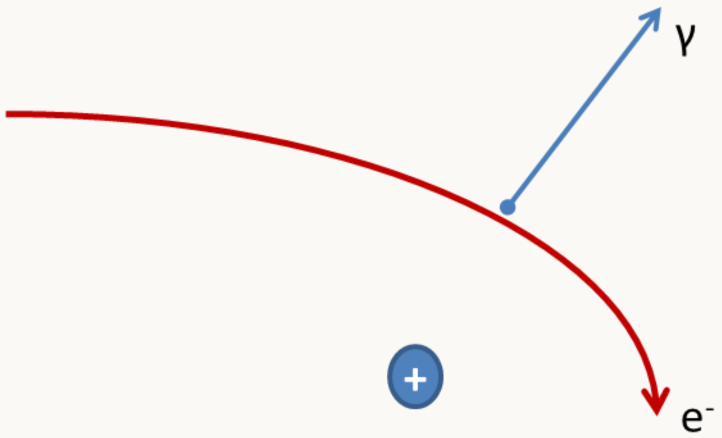
Light CP interactions – QM – part a)

- Mass of incoming particle = mass of struck electron
  - Includes electrons and positrons
  - Requires additional relativistic terms
- Interaction ejects atomic electron
  - Stopping power applies to the faster of the two emerging particles
  - Max energy transfer
    - 50% of original electron energy

$$\frac{1}{\rho}\left(\frac{dT}{dx}\right)=\frac{2\pi r_0^2 m_0 c^2}{\beta^2}\frac{Z}{uA}\left[2\ln\left(\frac{T}{I}\right)+\ln\left(\frac{2+\tau}{2}\right)+F^{\pm}(\tau)-\delta\right]$$

- Breaks down at low electron energies
  - Fractional loss per interaction becomes too large for CSDA to apply

Bremsstrahlung radiation loss



- "Breaking radiation" due to interaction with nuclear Coulomb field
- Outweighs collisional loss for electrons
- Release of photons due to acceleration of charged particle
- Total cross section

$$\sigma_{rad} = \frac{dT}{T + m_0c^2}\frac{1}{Ndx} = \frac{1}{137}r_0^2Z^2\int_0^1 B.d\left(\frac{h\nu}{T}\right) = \frac{r_0^2Z^2\overline{B}}{137}$$

You don't have to learn this!

Bremsstrahlung

- When a charged particle is accelerated it emits photons with an amplitude proportional to the acceleration

- Acceleration

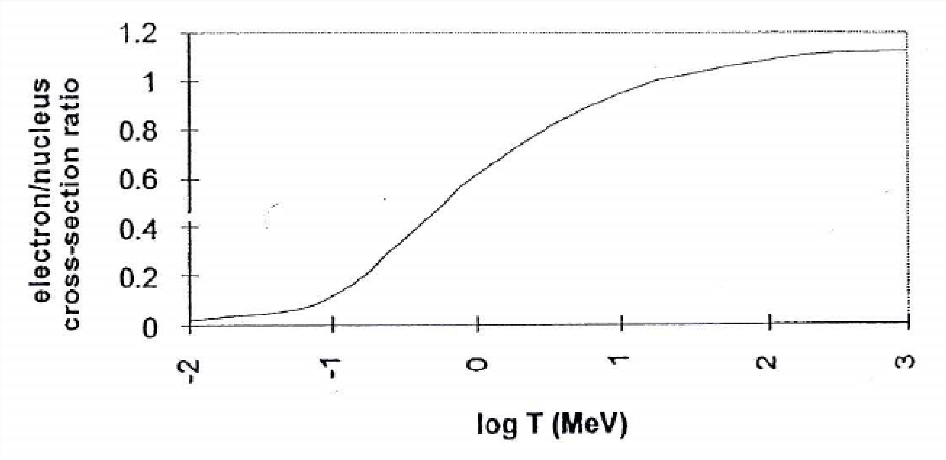
$$a \propto \frac{zZe^2}{m}$$

- Intensity of photons

$$I \propto \frac{z^2 Z^2 e^4}{m^2}$$

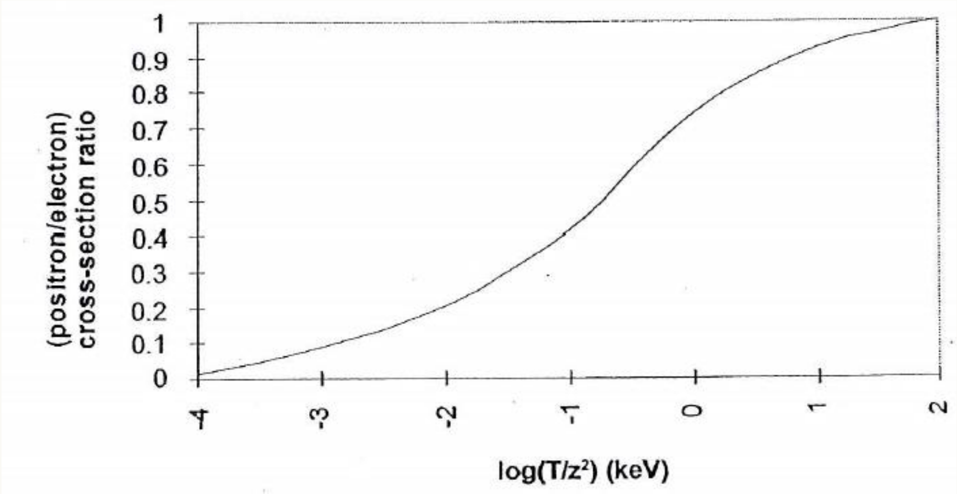
Why with electrons and not protons?

Bremsstrahlung from electrons vs nucleus



Electron-electron vs electron-nucleus

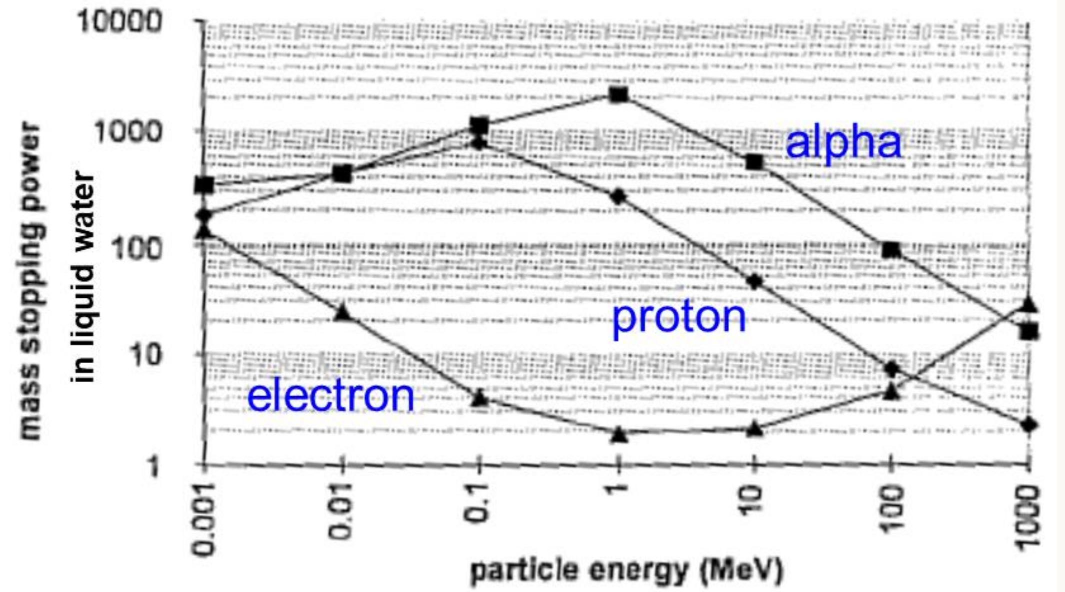
Positron-nucleus vs electron-nucleus



Electron induced Bremsstrahlung becomes significant at energies beyond medical range

Summary of stopping power for charged particles

- Stopping power varies with particle kinetic energy
- Shape of curves are similar for a given particle
- Electrons have high radiative losses at high energies and high collisional losses at low energies



Any Questions?

w.dennis@ucl.ac.uk

Next time

X-ray Production

Using all we've learnt so far