Brunel University
Queen Mary, University of London
Royal Holloway, University of London
University College London

Intercollegiate post-graduate course in High Energy Physics

Paper 1: The Standard Model and Beyond part 1

Monday, 10 January 2014

Time allowed for Examination: 3 hours

Answer **ALL** questions

Books and notes may be consulted

The Standard Model

Question 1 (20 marks)

At a collider, two high energy particles A and B with momenta p_A and p_B and masses m_A and m_B collide head on. Derive the expression for the centre-of-mass energy squared when the particle masses are neglected **[2]**

and when the masses are not neglected **[2]**

Using this latter expression, calculate the centre-of-mass energy of a possible collider where collisions are made between protons with momentum of 20 GeV and electrons with momentum of 5 GeV, and compare it to the centre-of-mass energy of an asymmetric proton-proton collider with the same beam momenta. **[2]**

Now consider one proton to be at rest. Derive the formula for the centre-of-mass energy of electron-proton and proton-proton fixed-target experiments. **[2]**

Write the Klein-Gordon equation, and explain why it is possible to define a conserved 4-dimesional current. **[4]**

Write the expression of this current for a plane wave. **[2]**

Show how an electromagnetic potential A^μ can modify the momentum operator, and indicate the resulting electromagnetic potential.

[4]

Write a transition amplitude between two generic waves.

[2]

CONTINUED

Question 2 (20 marks)
Write the transition amplitude between two spinless states interacting through electromagnetic interactions.

[4]

Show that the electromagnetic potential

$$A^{\mu} = \frac{-g^{\mu\nu} j_{\nu}^{DB}}{q^2}$$

satisfies Maxwell's equations, in the form

$$\Box^2 A^{\mu}(x) = j^{\mu}(x)$$

with $j^{\mu}(x)$ being the current between two states B and D.

[6]

Discuss the relation between the transmission amplitude T_{fi} and the number of transitions for unit time and volume, and write an example of a transition amplitude to be used in the case of scattering between spinless particles.

[6]

Discuss the differences in the phase-space definition (flux factor) between a 2$\longrightarrow$ 2 interaction and a two-body decay of a heavy particle.

[4]

CONTINUED

Question 3 (20 marks)

Write down Pauli's matrices, and their relation with the operators for spin and spin angular momentum. Write commutation and anti-commutation relations between the matrices.

[5]

Write the Dirac equation in standard and adjoint form, and the relations between the matrices α_i, β and the Pauli matrices.

[5]

Derive the Hermitian conjugate of the γ matrices. Show that the combination of the standard and adjoint form of the Dirac equation leads to a conserved four-current.

[5]

Write the free-particle solution of the Dirac equation, and derive the eigenvalues of the spin states.

[5]

CONTINUED

Question 4 (20 marks)

Define the Mandelstem variables, and draw Feynman diagrams for Compton scattering as well as those for $e^+e^- \to \gamma\gamma$.

[5]

Write the generic amplitude T_{fi} for Compton scattering, in the limit $m_e \to 0$

[5]

Using your expression for T_{fi}, derive the cross-section for the first diagram of Compton scattering in terms of the Mandelstem variables.

[8]

Describe the relationship between the cross-section for Compton scattering and that for e^+e^- annihilation. **[2]**

Question 5 (20 marks)

Write the convolution of a wavefunction at initial coordinates $\Psi(\underline{r}, t)$ with a Green's function to produce a wave function after scattering. **[4]**

Write Schroedinger's equation for a plane wave incident on a potential $V(\underline{r}, t)$ on top of a constant energy level H_0. **[5]**

Show that in the approximation that the potential acts for a small time interval Δt_1, where it is much larger than the Hamiltonian H_0, Schroedinger's equation can be written as $\Delta\psi(\underline{r}_1, t_1) = -iV(\underline{r}_1, t_1)\psi(\underline{r}_1, t_1)\Delta t_1$ **[5]**

If a final-state planar wave is written as $\psi(\underline{r}_1, t_1) = \phi(\underline{r}_1, t_1) + \Delta\psi(\underline{r}_1, t_1)$, and applying the Green's function in point $(\underline{r}_1, t_1)$, show that after scattering in a position $(\underline{r}', t')$

$$\psi(\underline{r}', t') = \phi(\underline{r}', t') + \int d^4x_1 G_0((\underline{r}', t'; \underline{r}_1, t_1)V(\underline{r}_1, t_1)\phi(\underline{r}_1, t_1)$$

[6]

CONTINUED

Question 6 (20 marks)
Draw the Feynman diagram for electron-muon scattering at tree level, indicating the incoming and outgoing currents. **[4]**

Given an electron current of the form $j^{\mu}_{fi} = -e(p_f + p_i)^{\mu} e^{i(p_f - p_i)x}$, write the transition amplitude for a Coulomb potential, calling k and k' the initial and final momenta for the electron, and p and p' the same quantities for the muon. **[6]**

Denoting the electron mass by m and the muon mass by M, the squared scattering amplitude is:

$$|T_{fi}|^2 = \frac{e^4}{4q^4} Tr\Big[\left(\not{k}' + m\right)\gamma_\mu \left(\not{k} + m\right)\gamma_\nu\Big] Tr\Big[\left(\not{p}' + M\right)\gamma^\mu \left(\not{p} + M\right)\gamma^\nu\Big]$$

Show how this expression can be simplified in the relativistic limit, considering that products involving an odd number of γ matrices are zero. **[6]**

Rewrite the simplified expression for $|T_{fi}|^2$ using Mandelstam variables. **[4]**

END OF PAPER