Brunel University
Queen Mary, University of London
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Intercollegiate post-graduate course in High Energy Physics

Paper 2: The Standard Model and Beyond part 2

Monday, 4 February 2015

Time allowed for Examination: 3 hours

Answer four questions out of six (80 marks)

Books and notes may be consulted

1. Higgs sector of the Glashow Salam Weinberg model.

The kinetic part of the Higgs Lagrangian in the Standard Model is given by

$$\mathcal{L}_{\Phi-\text{kinetic}} = \left(\boldsymbol{D}_\mu \Phi\right)^\dagger \left(\boldsymbol{D}^\mu \Phi\right) ,$$

with

$$\boldsymbol{D}_\mu = \partial_\mu + ig'\frac{Y_\phi}{2}B_\mu + ig_W \boldsymbol{W}_\mu \, , \qquad \text{and} \qquad \Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v' + h \end{pmatrix} .$$

In the expression for $\boldsymbol{D}_\mu$, B_μ is the hypercharge gauge field, Y_ϕ the hypercharge of the Higgs doublet Φ, and g' is the hypercharge gauge field coupling constant. Furthermore, $\boldsymbol{W}_\mu = \tau^a W_\mu^a$, where W_μ^a $(a = 1, 2, 3)$, are the three W gauge boson fields, and τ^a are the Pauli spin matrices divided by two. Lastly, g_W is the weak coupling constant.
Inside Φ, h is the Higgs field and $v'/\sqrt{2}$ is its vacuum expectation value.

(a) Show that $\mathcal{L}_{\Phi-\text{kinetic}}$ can equivalently be written

$$\begin{aligned} \mathcal{L}_{\Phi-\text{kinetic}} &= \frac{1}{8}g_W^2 \left(\left(\frac{W_\mu^1 + iW_\mu^2}{\sqrt{2}}\right)\left(\frac{W^{1\mu} - iW^{2\mu}}{\sqrt{2}}\right) + \text{c.c.}\right)(v' + h)^2 \\ &+ \frac{1}{2}\left(\partial_\mu h\right)^2 + \frac{1}{8}\left(g_W W^3 - g' Y_\phi B\right)^2 (v' + h)^2 , \end{aligned}$$

where 'c.c.' represents the complex conjugate of the first term in the first set of round brackets. **[10]**

(b) What are the following *four* combinations of fields physically identified as:

$$\frac{1}{\sqrt{g_W^2 + g'^2}}\left(g_W W_\mu^3 - g' Y_\phi B_\mu\right) , \qquad \frac{1}{\sqrt{g_W^2 + g'^2}}\left(g' W_\mu^3 + g_W Y_\phi B_\mu\right) , \qquad \frac{W_\mu^1 \pm iW_\mu^2}{\sqrt{2}} \; ?$$

[3]

(c) Determine the masses of the physical W and Z boson fields in terms of v', g_W and $\cos\theta_W$, where

$$\cos\theta_W = \frac{g_W}{\sqrt{g_W^2 + g'^2}} .$$

[7]

2. Computation of the width for heavy Higgs boson decay to W^+W^-.

In this question we assume a scenario in which the Higgs boson mass (M_H) *is greater than twice the* W*-boson mass* (M_W)*:* $M_H > 2M_W$.
The amplitude for the decay $H \to W^+W^-$ is given by

$$-i\mathcal{M} = ig_W\, M_W\, g_{\mu\nu}\, \epsilon^\mu\,(p_+)\, \epsilon^\nu\,(p_-)\ ,$$

where g_W is the weak coupling constant, and $p_\pm$ are the $W^\pm$ boson momenta respectively.

(a) Compute the amplitude squared for the Higgs boson decaying into a W^+W^- pair, summed over polarizations, $\sum_{W^\pm\text{pols}} |\mathcal{M}|^2$, in terms of g_W, M_H and M_W.

Hint: you may find useful the kinematic identity $p_+ \cdot p_- = \frac{1}{2}M_H^2 - M_W^2$.

[10]

(b) The *longitudinal* polarization vectors $\epsilon_L^\mu\,(p_\pm)$ of the $W^\pm$ bosons, can be written respectively, as

$$\epsilon_L^\mu\,(p_\pm) \quad = \quad \frac{1}{4M_W}\left[\left(\frac{M_H^2 - 4M_W^2}{M_H^2}\right)^{+\frac{1}{2}}(2p_+^\mu + 2p_-^\mu) \pm \left(\frac{M_H^2 - 4M_W^2}{M_H^2}\right)^{-\frac{1}{2}}(2p_+^\mu - 2p_-^\mu)\right]\ ,$$

satisfying the usual Lorentz condition for polarization vectors $\epsilon^\mu\,(p_\pm) \cdot p_\pm = 0$ and canonical normalization $\epsilon\,(p_\pm)\,.\epsilon\,(p_\pm)^* = -1$.
The amplitude for the decay $H \to W^+W^-$, where the $W^\pm$ bosons are longitudinally polarized is given by

$$-i\mathcal{M}_L = ig_W\, M_W\, g_{\mu\nu}\, \epsilon_L^\mu\,(p_+)\, \epsilon_L^\nu\,(p_-)\ .$$

Compute this amplitude in terms of g_W, M_W and M_H, and hence the corresponding squared amplitude $|\mathcal{M}_L|^2$.

[8]

(c) Comparing the expressions derived for $\sum_{W^\pm\text{pols}} |\mathcal{M}|^2$, in part (a), and $|\mathcal{M}_L|^2$ in part (b), what do you infer about the decay $H \to W^+W^-$ in the limit $M_H \gg M_W$?

[2]

3. Abelian gauge invariance for a complex scalar field theory.

The Lagrangian density for a complex scalar (ϕ) field theory is given by

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - V(\phi, \phi^*)$$

$$V(\phi, \phi^*) = -m^2 \phi^* \phi - \lambda (\phi^* \phi)^2 \, .$$

(a) Determine how the potential $V(\phi, \phi^*)$ changes under a local $U(1)$ symmetry transformation $\phi \to U\phi$, $U = e^{iq\Lambda}$, $\Lambda = \Lambda(x)$.

[3]

(b) Determine how the derivative term $\partial_\mu \phi$ changes under the same $U(1)$ transformation.

[4]

(c) Defining the covariant derivative as

$$D_\mu = \partial_\mu + iqA_\mu \, ,$$

with A^μ transforming as

$$\begin{aligned} A^\mu \to A'^\mu &= UA^\mu U^\dagger + \frac{i}{q} (\partial^\mu U) U^\dagger \\ &= A^\mu - \partial^\mu \Lambda \, , \end{aligned}$$

determine the result of the same $U(1)$ transformation applied to $D_\mu \phi$.

[6]

(d) Hence show that

$$\mathcal{L}_{\text{gauged}} = (D_\mu \phi) \left(D^{\dagger\mu} \phi^* \right) - V(\phi, \phi^*) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\nu \, .$$

is $U(1)$ gauge invariant.

[7]

4. Goldstone bosons

Unless otherwise stated you should assume the usual summation convention applies: repeated indices are implicitly summed over.

Consider a simple scalar field theory, of three *real* scalar fields ϕ_i, $i = 1, 2, 3$, with Lagrangian

$$\mathcal{L}(\phi_i, \partial_\mu \phi_i) = \frac{1}{2}(\partial_\mu \phi_i)(\partial^\mu \phi_i) - V(\phi_i \phi_i) \,, \qquad V(\phi_i \phi_i) = \frac{1}{2}\mu^2 \phi_i \phi_i + \lambda (\phi_i \phi_i)^2 \,.$$

(a) Given that U is a global $\mathrm{SO}(3)$ transformation, determine/write down $U_{ij} U_{ik}$.

[3]

(b) Compute the effect of the global $\mathrm{SO}(3)$ transformation $\phi_i \to \phi_i' = U_{ij}\phi_j$ on $\phi_i \phi_i$.

[3]

(c) Compute the effect of the global $\mathrm{SO}(3)$ transformation $\phi_i \to \phi_i' = U_{ij}\phi_j$ on $\mathcal{L}$.

[2]

(d) Write the potential in terms of $|\phi| = \sqrt{\phi_i \phi_i}$ and, assuming the parameter μ^2 is negative, show the potential minima has extrema located along $|\phi| = a = \sqrt{-\frac{\mu^2}{4\lambda}}$. State whether these extrema are minima or maxima of V.

[4]

(e) Assume the vacuum state of the theory is at $(\phi_1, \phi_2, \phi_3)|_{\text{vacuum}} = (0, 0, a)$, i.e. ϕ_3 acquires a non-zero vacuum expectation value $\langle \phi_3 \rangle_{\text{vacuum}} = a$, with a as given in part (d). Determine the part of the potential which is quadratic/bilinear in ϕ_1, ϕ_2, and the shifted field χ where $\phi_3 = \chi + a$. What are the masses of ϕ_1, ϕ_2 and χ?

[6]

(f) Given that the vacuum state of the theory, $(\phi_1, \phi_2, \phi_3) = (0, 0, a)$, is itself invariant under $\mathrm{SO}(2)$ rotations about the ϕ_3 direction in isospin space, how many Goldstone bosons do you expect to find based on Goldstone's theorem and why?

[2]

5. Hadron collider cross section

Throughout this question you should assume the masses of the colliding hadrons (protons) are zero.

At the LHC the main mechanism by which the Higgs boson is produced, at parton level, is the so-called gluon fusion process, $g + g \to H$, in which the colliding gluons and outgoing Higgs boson are connected to one another via a triangular loop of top-quarks, as depicted in fig. 1.

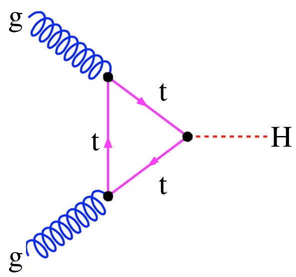

Figure 1: Higgs production via gluon fusion at parton level.

The colliding hadrons travelling in the $+/-$ z-directions are labelled $h_\oplus$ and $h_\ominus$ respectively, and their momenta are denoted by $p_\oplus$ and $p_\ominus$. The fraction of $h_\oplus$'s momentum carried by the colliding gluon inside it is denoted $\eta_\oplus$ in the hadronic centre-of-mass frame. Similarly, the fraction of $h_\ominus$'s momentum carried by the colliding gluon inside it is $\eta_\ominus$. The total hadronic centre-of-mass energy squared is denoted $S = (p_\oplus + p_\ominus)^2$. The Higgs boson momentum is labelled p_H.

(a) Using momentum conservation, $\eta_\oplus p_\ominus + \eta_\ominus p_\ominus = p_H$, and assuming the Higgs boson is produced on-shell, with mass m_H, determine m_H^2 in terms of $\eta_\oplus$, $\eta_\ominus$, and S.

[3]

(b) Determine the rapidity of the Higgs boson, y_H, in terms of $\eta_\oplus$ and $\eta_\ominus$.

[3]

(c) Determine the momentum fractions $\eta_\oplus$ and $\eta_\ominus$ in terms of m_H, y_H and S.

[2]

(d) Compute the flux factor, F, in terms of m_H, where F is in general given by

$$F = 4\sqrt{\left((\eta_\oplus p_\oplus) \,.\, (\eta_\ominus p_\ominus)\right)^2 - (\eta_\oplus p_\oplus)^2 \,(\eta_\ominus p_\ominus)^2}\,.$$

[3]

(e) The one body phase space factor for the final-state Higgs boson is

$$dLIPS = 2\pi\,\delta\left(\eta_{\oplus}\eta_{\ominus}S - m_H^2\right) .$$

The squared matrix element for $g + g \to H$, in the infinite top-quark mass limit, summed over all gluon and Higgs polarizations, and colours, is given by

$$\sum_{\text{pols}} |\mathcal{M}|^2 = \left(\frac{2}{3}\,\frac{\alpha_S}{M_W}\,\frac{\sqrt{\alpha_W}}{\sqrt{\pi}}\right)^2 m_H^4 ,$$

where $\alpha_W = \frac{g_W^2}{4\pi}$ with g_W the weak coupling constant, and $\alpha_S = \frac{g_S^2}{4\pi}$ with g_S the strong coupling constant. Write down the *partonic cross section* for $g + g \to H$, $d\hat{\sigma}_{gg\to H}$, in terms of $\eta_{\oplus}$, $\eta_{\ominus}$, S, and the variables contained within the expression given here for $\sum_{\text{pols}} |\mathcal{M}|^2$.

[5]

(f) Determine the total $h_{\oplus}(p_{\oplus}) + h_{\ominus}(p_{\ominus}) \to H$ hadronic cross section, $\sigma_{h_{\oplus}h_{\ominus}\to H}$, in the form of a single integral over the Higgs boson's rapidity; you may use the relation $d\eta_{\oplus}d\eta_{\ominus} = \frac{1}{S}\,dm_H^2 dy_H$.

[4]

6. Decay width of the top quark

Throughout the question you should assume that the b-quark mass is entirely negligible.

In the following we denote the t, b and W particle momenta respectively as p_t, p_b, p_W.

(a) Draw the Feynman diagram for the two-body decay of a top quark into a b-quark and a W^+ boson, labelling the external particles by their momenta p_t, p_b, p_W. Take care to label the flow of fermion number appropriately with an arrow.

[4]

(b) The amplitude for the top quark decay to a b-quark and W^+ boson is

$$-i\mathcal{M} = \frac{-ig_W}{\sqrt{2}}\,\epsilon^{\mu}\left(p_W\right)\,j^{tb}_{\mu}\,, \qquad j^{tb}_{\mu} = \bar{u}\left(p_b\right)\gamma_{\mu}\frac{1}{2}\left(1-\gamma_5\right)u\left(p_t\right)\,,$$

where g_W is the weak coupling constant. Show that the complex conjugate of the amplitude is given by (recall $\gamma_{\mu}^{\dagger} = \gamma_0\gamma_{\mu}\gamma_0$)

$$i\mathcal{M}^{*} = \frac{ig_W}{\sqrt{2}}\,\epsilon^{\nu *}\left(p_W\right)\,j^{tb\,*}_{\nu}\,, \qquad j^{tb\,*}_{\nu} = \overline{u}\left(p_t\right)\gamma_{\nu}\frac{1}{2}\left(1-\gamma_5\right)u\left(p_b\right)\,.$$

[4]

(c) Hence determine that the matrix element squared is equal to

$$\left|\mathcal{M}\right|^2 \;=\; \frac{g_W^2}{2}\,\epsilon^{\mu}\left(p_W\right)\epsilon^{\nu *}\left(p_W\right)\,\mathrm{Tr}\left[\bar{u}\left(p_b\right)\gamma_{\mu}\frac{1}{2}\left(1-\gamma_5\right)u\left(p_t\right)\overline{u}\left(p_t\right)\gamma_{\nu}\frac{1}{2}\left(1-\gamma_5\right)u\left(p_b\right)\right]\,.$$

[4]

(d) Sum the matrix element squared over fermion spins (t and b) and show, *without the use of trace theorems*

$$\sum_{t,b\,\mathrm{spins}}\left|\mathcal{M}\right|^2 \;=\; \frac{g_W^2}{2}\,\epsilon^{\mu}\left(p_W\right)\epsilon^{\nu *}\left(p_W\right)\,\mathrm{Tr}\left[\not{p}_b\gamma_{\mu}\;\not{p}_t\gamma_{\nu}\frac{1}{2}\left(1-\gamma_5\right)\right]\,.$$

[4]

(e) The squared matrix element summed over fermion spins and gauge boson polarizations is

$$\sum_{\mathrm{spins,pols}}\left|\mathcal{M}\right|^2 \;=\; \frac{4G_F m_t^4}{\sqrt{2}}\left(1+\left(\frac{m_W}{m_t}\right)^2 - 2\left(\frac{m_W}{m_t}\right)^4\right)\,.$$

The Lorentz invariant phase space measure for the decay is

$$d\mathrm{LIPS} = \frac{1}{(4\pi)^2}\frac{1}{2}\left(1-\frac{m_W^2}{m_t^2}\right) d\cos\theta_b\, d\phi_b\,,$$

where θ_b and ϕ_b are the polar and azimuthal angles of the b-quark in the top quark rest frame. Show that the decay width $\Gamma\,(t \to bW)$ is

$$\Gamma\,(t \to bW) = \frac{G_F m_t^3}{8\pi\sqrt{2}}\left(1-\left(\frac{m_W}{m_t}\right)^2\right)\left(1+\left(\frac{m_W}{m_t}\right)^2 - 2\left(\frac{m_W}{m_t}\right)^4\right).$$

[4]