Brunel University
Queen Mary, University of London
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Intercollegiate post-graduate course in High Energy Physics

Paper 1 : The Standard Model

Tuesday, 1 February 2011

Time allowed for Examination : 3 hours

Answer 6 from 8 questions

Books and notes may be consulted

Question 1 (20 marks)

For Bhabha scattering, $e^+(q)\,e^-(p) \rightarrow e^+(q')\,e^-(p')$, with four momenta as given, determine

$$\begin{array}{rcl} p\cdot q & (= & p'\cdot q') \\ p\cdot p' & (= & q\cdot q') \\ p\cdot q' & (= & p'\cdot q) \end{array}$$

in terms of the energy, E, of the electron and of the positron and in terms of half the scattering angle, $\theta/2$, in the centre-of-mass system. **[5]**

Draw the two lowest order diagrams for Bhabha scattering, stating whether they are s-, t- or u-channel. **[3]**

The lowest order cross section can be written as

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2}\left[\frac{T_1}{(p-p')^4} + \frac{T_2}{(p+q)^4} - \frac{2T_3}{(p-p')^2(p+q)^2}\right],$$

where α is the fine structure constant and

$$\begin{aligned} T_1 &= \mathrm{Tr}\left[\frac{\not{p}'+m}{2}\gamma_\mu\frac{\not{p}+m}{2}\gamma_\nu\right]\mathrm{Tr}\left[\frac{-\not{q}+m}{2}\gamma^\mu\frac{-\not{q}'+m}{2}\gamma^\nu\right], \\ T_2 &= \mathrm{Tr}\left[\frac{-\not{q}+m}{2}\gamma_\mu\frac{\not{p}+m}{2}\gamma_\nu\right]\mathrm{Tr}\left[\frac{\not{p}'+m}{2}\gamma^\mu\frac{-\not{q}'+m}{2}\gamma^\nu\right], \\ T_3 &= \mathrm{Tr}\left[\frac{\not{p}'+m}{2}\gamma_\mu\frac{\not{p}+m}{2}\gamma_\nu\frac{-\not{q}+m}{2}\gamma^\mu\frac{-\not{q}'+m}{2}\gamma^\nu\right]. \end{aligned}$$

Hence show that

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8E^2}\left[\frac{1+\cos^4\theta/2}{\sin^4\theta/2} - \frac{2\cos^4\theta/2}{\sin^2\theta/2} + \frac{1+\cos^2\theta}{2}\right]$$

in the high-energy limit. Trace theorems and identities for γ matrices need not be derived, but should be quoted. **[12]**

Question 2 (20 marks)

Draw the two lowest order diagrams for Møller scattering, $e^-e^- \to e^-e^-$, stating whether they are s-, t- or u-channel. **[3]**

The cross section for Møller scattering in the high-energy limit is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8E^2}\left[\frac{1+\cos^4\theta/2}{\sin^4\theta/2} + \frac{2}{\sin^2\theta/2\cos^2\theta/2} + \frac{1+\sin^4\theta/2}{\cos^4\theta/2}\right]. \qquad (1)$$

Show that this can be simplified to

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2}\frac{\left(3+\cos^2\theta\right)^2}{\sin^4\theta}.$$

where E is the energy of both electrons and θ is the scattering angle in the centre-of-mass frame. **[7]**

Also starting from the original cross section given (Eq. 1), derive the dependence on the Mandelstam variables, rather than angle θ. **[6]**

Use this result to write down, with a brief explanation, the cross sections in terms of the Mandelstam variables for $e^-\mu^- \to e^-\mu^-$ and $e^+e^- \to \mu^+\mu^-$. **[4]**

Question 3 (20 marks)

Draw Feynman diagrams of the lowest-order (electroweak) process in electron-proton scattering. **[3]**

The matrix-element squared for lowest-order electromagnetic electron-proton scattering (under the assumption of the proton being a structureless, point-like Dirac particle) with given four-momentum, $e^-(p_i)\, p(P_i) \to e^-(p_f)\, p(P_f)$, is

$$|T_{\rm fi}|^2 = \left(\frac{e^2}{q^2}\right)^2 \frac{1}{4}\,\mathrm{Tr}\left[\frac{\not{p}_f + m}{2m}\gamma^\mu \frac{\not{p}_i + m}{2m}\gamma^\nu\right] \mathrm{Tr}\left[\frac{\not{P}_f + M}{2M}\gamma_\mu \frac{\not{P}_i + M}{2M}\gamma_\nu\right] .$$

Evaluate the traces to show that

$$|T_{\rm fi}|^2 = \frac{e^4}{2m^2M^2q^4}\left[(p_f \cdot P_f)\,(p_i \cdot P_i) + (p_f \cdot P_i)\,(p_i \cdot P_f) - M^2\,(p_f \cdot p_i) - m^2\,(P_f \cdot P_i) + 2m^2M^2\right]$$

where m is the mass of the electron, M is the mass of the proton and $q = p_f - p_i$. **[8]**

Assuming four-vectors,

$$p_i = (E, \mathbf{p})\,, \qquad p_f = (E', \mathbf{p}')\,, \qquad P_i = (M, 0)\,, \qquad P_f = (E_f, \mathbf{P_f})\,,$$

conserve energy and momentum to show that for $m \ll E$,

$$\frac{E - E'}{M} = -\frac{q^2}{2M^2} .$$

[5]

Hence show that the cross section,

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{4\pi^2}\frac{E'/E}{1 + (2E/M)\sin^2\theta/2}|T_{\rm fi}|^2 ,$$

where

$$|T_{\rm fi}|^2 = \frac{16\pi^2\alpha^2 EE'}{m^2q^4}\left[1 + \frac{q^2}{4EE'}\left(1 + \frac{E' - E}{M}\right) + \frac{m^2}{2EE'}\left(\frac{E' - E}{M}\right)\right]$$

can be simplified, in the limit $E \gg m$ but $E \ll M$, to

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\cos^2\frac{\theta}{2}$$

where θ is the angle between the outgoing and incoming electron. **[4]**

Question 4 (20 marks)

Contrast the advantages and disadvantages of e^+e^- and pp colliders. Use two headline measurements of major discoveries to justify your answer. **[6]**

Draw Feynman diagram(s) for $e^+e^- \to \mu^+\mu^-$, stating whether it (they) are s-, t-, or u-channel. **[2]**

At the PETRA collider ($\sqrt{s} = 34\,\text{GeV}$), the unpolarized cross section, $e^+e^- \to \mu^+\mu^-$, was consistent with the form:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left[A_0(1+\cos^2\theta) + A_1\cos\theta\right]$$

where θ is the scattering angle in the centre-of-mass frame. Explain the origin of the two terms in the square brackets. **[4]**

Integrate the differential formula over the solid angle to derive the total cross section in terms of constants and A_0. Hence show that the total cross section can be written in approximate form as:

$$\sigma \sim \frac{20\,A_0\ (\text{nb})}{E^2_{\text{beam}}\ (\text{in GeV}^2)},$$

where E_{beam} is the beam energy (of both the e^+ and e^-).
$\left(\alpha = \frac{1}{137.036}\right)$ **[5]**

Draw a (Feynman-like) diagram showing Drell-Yan production and write down the total lowest-order partonic cross section. **[3]**

Question 5 (20 marks)

Sketch quark diagrams for the decays :

$$\begin{array}{lll} \text{(a)}\ K^+ & \rightarrow & \mu^+ + \nu_\mu \\ \text{(b)}\ D^+ & \rightarrow & \overline{K}^0 + e^+ + \nu_e \\ \text{(c)}\ B^+ & \rightarrow & \overline{D}^0 + \pi^+ \end{array}$$

and identify the quark vertices according to the CKM matrix. **[9]**

Explain the difference in the measured branching ratios :

$$\begin{array}{lll} \Gamma(D^0 \rightarrow K^- + \pi^+) & = & 3.80\,\% \\ \Gamma(D^0 \rightarrow K^+ + \pi^-) & = & 1.43 \times 10^{-4} \end{array}$$

and using the CKM matrix,

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97383 & 0.2272 & 0.004 \\ 0.2271 & 0.97296 & 0.042 \\ 0.008 & 0.042 & 0.9991 \end{pmatrix},$$

predict the ratio of the two to within 30%. **[6]**

Draw an example parton-level Feynman diagram of top production at the Tevatron. Why could $t\bar{t}$ production not be observed at LEP or at HERA ? Explain why the cross section for single-top production at LEP and HERA is small and draw a Feynman diagram for its production at both colliders. **[5]**

Question 6 (20 marks)

Draw all Feynman diagrams at lowest order in QCD for the hard scatters $qq' \to qq'$ and $qq \to qq$. **[4]**

By comparing to QED processes, write down the forms of the (partonic) cross sections for $qq' \to qq'$ and $qq \to qq$ in terms of the Mandelstam variables s, t and u, associating each term with the relevant Feynman diagram **[6]**

The DGLAP equations are:

$$\frac{dQ_i(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[Q_i(y,Q^2)P_{qq}\left(\frac{x}{y}\right) + G(y,Q^2)P_{qg}\left(\frac{x}{y}\right)\right]$$

$$\frac{dG(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[Q_i(y,Q^2)P_{gq}\left(\frac{x}{y}\right) + G(y,Q^2)P_{gg}\left(\frac{x}{y}\right)\right]$$

Explain the functions Q_i and G and the four P_{ij} functions. **[6]**

Discuss the importance of understanding the structure of the proton for future measurements at the LHC. **[4]**

Question 7 (20 marks)

State what are meant by global and local gauge transformations. **[2]**

Explain the four terms in the Lagrangian of QED :

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi + e\bar{\psi}\gamma_\mu A^\mu \psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

[4]

Using the Euler-Lagrange equation,

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \left(\partial_\mu \phi\right)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0\,,$$

i. substitute the Lagrangian,

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - \frac{1}{2}m^2\phi^2\,,$$

and show that this gives the Klein-Gordon equation. **[5]**

ii. substitute the Lagrangian,

$$\mathcal{L} = i\overline{\psi}\gamma_\mu \partial^\mu \psi - m\overline{\psi}\psi$$

and show that this gives the Dirac equation. **[3]**

From the Lagrangian

$$\frac{1}{8}\left[g_W^2(v+h)^2(W_\mu^1 - iW_\mu^2)(W_\mu^1 + iW_\mu^2) - (v+h)^2(g'B_\mu - g_W W_\mu^3)(g'B^\mu - g_W W_3^\mu)\right]$$

derive the ZZH and $ZZHH$ couplings. (Simplify your answer to remove dependencies on both v and g'.) **[6]**

Question 8 (20 marks)

The highest energy cosmic rays have $E \sim 10^{20}\,\text{eV}$. Assuming the cosmic ray collides with a proton in the atmosphere, determine how much higher in centre-of-mass energy such a collision is compared to that at the LHC. **[4]**

Show that the sum of the three Mandelstam variables in the collision $AB \to CD$ is given by

$$s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$$

where m_i are the rest masses of the particles. **[3]**

Taking $e^-e^+ \to e^-e^+$ to be the s-channel process, verify that

$$\begin{aligned} s &= 4(k^2 + m^2) \\ t &= -2k^2(1 - \cos\theta) \\ u &= -2k^2(1 + \cos\theta) \end{aligned}$$

where θ is the centre-of-mass scattering angle and $k = |\mathbf{k_i}| = |\mathbf{k_f}|$, where $\mathbf{k_i}$ and $\mathbf{k_f}$ are, respectively, the momenta of the incident and scattered electrons in the centre-of-mass frame. **[6]**

Considering combinations of the Pauli matrices,

$$\{\sigma_1, \sigma_2\}\,, \quad \{\sigma_1, \sigma_3\}\,, \quad \{\sigma_2, \sigma_3\}\,, \quad \{\sigma_1, \sigma_1\}\,,$$

determine them explicitly to show that

$$\{\sigma_i, \sigma_j\} = 2\delta_{ij} I\,,$$

where I is the 2×2 identity matrix. **[4]**

Consider a momentum $\mathbf{p}$ in the direction specified by the polar coordinates θ and ϕ

$$\hat{\mathbf{p}} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)\,.$$

Show that

$$\boldsymbol{\sigma} \cdot \hat{\mathbf{p}} = \begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & -\cos\theta \end{pmatrix}$$

[3]

[Total Marks = 120]

END OF PAPER