Brunel University
Queen Mary, University of London
Royal Holloway, University of London
University College London

Intercollegiate post-graduate course in High Energy Physics

Paper 1: The Standard Model

Wednesday, 4 February 2009

Time allowed for Examination: 3 hours

Answer **ALL** questions

Books and notes may be consulted

The Standard Model

Question 1 (5 marks)

At a collider, two high energy particles, A and B with energies E_A and E_B, which are much greater than their rest masses, collide at a crossing angle, θ. Derive the general expression for the centre-of-mass energy in terms of the energies and crossing angle. **[2]**

At HERA, a 27.5 GeV electron beam collided with a 920 GeV proton beam at zero crossing angle. Evaluate the total centre-of-mass energy and show that a fixed-target electron accelerator would require a beam energy of approximately 5×10^4 GeV to achieve the same total centre of mass energy **[3]**

Question 2 (8 marks)

The invariant mass of two massless jets, M^{jj}, can be written in terms of their transverse energies, E_T^{jet1} and E_T^{jet2}, pseudorapidities, η^{jet1} and η^{jet2} and azimuthal angles, ϕ^{jet1} and ϕ^{jet2}:

$$M^{jj} = \sqrt{2E_T^{\text{jet1}} E_T^{\text{jet2}} \left[\cosh\left(\eta^{\text{jet1}} - \eta^{\text{jet2}}\right) - \cos\left(\phi^{\text{jet1}} - \phi^{\text{jet2}}\right)\right]}.$$

For two jets back-to-back in ϕ and with equal E_T^{jet}, show that:

$$M^{jj} = \frac{2E_T^{\text{jet}}}{\sqrt{1 - \cos^2\theta^*}},$$

where θ^*, the angle between the jet-jet axis and the beam axis in the two-jet centre-of-mass, system is given by:

$$\cos\theta^* = \tanh\left(\frac{\eta^{\text{jet1}} - \eta^{\text{jet2}}}{2}\right).$$

(Recall: $\cos 2\theta = 2\cos^2\theta - 1$, $\text{sech}^2\theta + \tanh^2\theta = 1$.) **[2]**

The cross-section dependence for a spin-1 propagator is $\propto (1 - |\cos\theta^*|)^{-2}$ and for a spin$-\frac{1}{2}$ propagator is $\propto (1 - |\cos\theta^*|)^{-1}$. Draw two Feynman diagrams representing parton collisions at the LHC, one of which has a spin-1 and the other a spin$-\frac{1}{2}$ propagator. **[2]**

Many Feynman diagrams exist already at leading order in QCD at a hadron collider such as the LHC. Draw the four diagrams for the partonic process, $gg \to gg$. **[4]**

Question 3 (6 marks)

State what is meant by local and global gauge transformations. **[2]**

From the Lagrangian

$$\frac{1}{8}\left[g_W^2(v+h)^2(W_\mu^1 - iW_\mu^2)(W_\mu^1 + iW_\mu^2) - (v+h)^2(g'B_\mu - g_W W_\mu^3)(g'B^\mu - g_W W_3^\mu)\right]$$

derive the ZZH and ZZHH couplings. (Simplify your answer to remove any dependency on v or g'.) **[4]**

Question 4 (10 marks)

An electron of 4-momentum p scatters elastically off a μ^- of 4-momentum k to produce a final state electron of 4-momentum p' and a μ^- of 4-momentum k'. The square of the matrix element is given by:

$$|T_{\mathrm{fi}}^2| = \frac{8e^4}{q^4}\left[(p\cdot k)(p'\cdot k') + (p\cdot k')(p'\cdot k) - M^2 p\cdot p'\right],$$

where M is the muon mass, $q = p' - p$ and the electron mass is set to zero.

Show (from $q = p' - p$) that $q^2 = -2p\cdot p'$ and (from $k - q = k'$) that $q^2 = 2k\cdot q$. **[1]**

Hence, after using energy-momentum conservation to eliminate k', show that $|T_{\mathrm{fi}}^2|$ can be re-written as:

$$|T_{\mathrm{fi}}^2| = \frac{8e^4}{q^4}\left[2(p\cdot k)(p'\cdot k) + \frac{1}{4}(q^2)^2 + \frac{1}{2}q^2M^2\right].$$

[4]

The initial μ^- is at rest and:

$$p = (E, \vec{p}), \quad k = (M, \vec{0}), \quad p' = (E', \vec{p'}).$$

The electron is scattered through an angle θ. Hence deduce that $q^2 = -2EE'(1-\cos\theta)$ and

$$|T_{\mathrm{fi}}^2| = \frac{8e^4}{q^4}\cdot 2M^2EE'\left[\cos^2(\theta/2) - \frac{q^2}{2M^2}\sin^2(\theta/2)\right].$$

(Recall: $\cos 2\theta = 1 - 2\sin^2\theta$) **[5]**

Question 5 (8 marks)

The decays $B^0 \to D^-\pi^+$ and $B^0 \to D^-K^+$ have branching fractions $(2.68 \pm 0.13) \times 10^{-3}$ and $(2.0 \pm 0.6) \times 10^{-4}$, respectively. Assuming the spectator model, draw diagrams for the two decays and briefly explain the difference in the rates of the two channels. **[3]**

Using, the CKM matrix,

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97419 & 0.2257 & 0.00359 \\ 0.2256 & 0.97334 & 0.0415 \\ 0.00874 & 0.0407 & 0.999133 \end{pmatrix},$$

show that the measured ratio of rates can be predicted to within just over one standard deviation of the experimental uncertainty. (NB. remember the decay rate is $\sim |T_{\rm fi}|^2$.) **[5]**

Question 6 (15 marks)

At leading order, the amplitude for the decay of the Z^0 boson into a fermion of 4-momentum p and anti-fermion of 4-momentum p' is:

$$T_{\rm fi} = -i\frac{g_W}{\cos\theta_W}\bar{u}(p)\gamma^\mu\frac{1}{2}(c_V - c_A\gamma^5)v(p')\epsilon^\mu,$$

where g_W and $\cos\theta_W$ are the usual constants, c_V and c_A are the vector and vector-axial couplings for a given fermion and ϵ^μ the polarization of the boson.

Show that:

$$|T_{\rm fi}|^2 = \frac{1}{12}\frac{g_W^2}{\cos^2\theta_W}(c_V^2 + c_A^2)4\left[2p\cdot p' + \frac{2}{M_Z^2}(p\cdot q)(p'\cdot q) - \frac{1}{M_Z^2}(q\cdot q)(p\cdot p')\right],$$

showing all your working. **[8]**

In the rest frame of the Z^0 boson,

$$q = (M_Z; 0, 0, 0), \quad p = \frac{M_Z}{2}(1; 0, 0, 1), \quad p' = \frac{M_Z}{2}(1; 0, 0, -1),$$

and using,

$$\Gamma = \frac{1}{64\pi^2 M_Z}\int |T_{\rm fi}|^2 d\Omega,$$

show that the decay width is,

$$\Gamma(Z^0 \to f\bar{f}) = \frac{1}{6\sqrt{2}\pi}G_F M_Z^3(c_V^2 + c_A^2)$$

[6]

and to electrons is,

$$\Gamma(Z^0 \to e^+e^-) = \frac{G_F M_Z^3}{12\sqrt{2}\pi}(1 - 4\sin^2\theta_W + 8\sin^4\theta_W).$$

[1]

Question 7 (5 marks)

Contrast the advantages and disadvantages of e^+e^- and pp colliders. Use two headline measurements or major discoveries to justify your answer. **[5]**

Question 8 (6 marks)

At the PETRA collider ($\sqrt{s} = 34\,\text{GeV}$), the unpolarized cross section, $e^+e^- \to \mu^+\mu^-$, was consistent with the form:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left[A_0(1+\cos^2\theta) + A_1\cos\theta\right]$$

where θ is the scattering angle in the centre-of-mass frame. Explain the origin of the two terms in the square brackets. **[4]**

Integrate the differential formula over the solid angle to derive the total cross section in terms of constants and A_0. **[2]**

Question 9 (6 marks)

Considering only QED, draw an example Feynman diagram for each of s-, t- and u-channel processes. **[3]**

Write down the form of their cross sections in terms of the Mandelstam variables. **[3]**

Question 10 (8 marks)

The first measurements of the proton structure function, F_2, showed that the quantity "scaled", i.e. was constant, with Q^2, the square of the 4-momentum transfer of the probe. Later results showed a "violation" of this scaling with different values of x, the fraction of the proton's momentum carried by the struck parton. Draw a sketch of F_2 versus Q^2, indicating the range in Q^2 currently measured and specify low and high values of x. **[3]**

Explain the different trends at high and low x stating the dominant partonic density. **[2]**

The highest energy scales for measurements of the parton densities in the proton are from HERA. However, this is significantly below the LHC energy: briefly explain how predictions of the proton structure can be made for the LHC and highlight an inadequacy in the approach. **[3]**

Question 11 (5 marks)

The muon decay rate, Γ, to an electron and neutrino pair and the muon lifetime, τ, are given by:

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) \equiv \frac{1}{\tau} = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

Calculate the lifetime of the muon. **[2]**

Given that the branching fraction for $(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$ is 17.85%, use the above formula to predict the lifetime of the τ lepton. **[2]**

State whether your prediction agrees with the measured value, $(290.6 \pm 1.0) \times 10^{-15}$ s. **[1]**

($G_F = 1.16637 \times 10^{-5}\,\mathrm{GeV}^{-2}$, $h = 6.6261 \times 10^{-34}\,\mathrm{J\,s}$, $e = 1.6022 \times 10^{-19}\,\mathrm{C}$;
$m_\mu = 105.658367\,\mathrm{MeV}$, $m_\tau = 1776.84\,\mathrm{MeV}$)

[Total Marks = 82]