Symmetries & Conservations Laws – Exam Question – Feb 2010

SJH

Answer all the questions. Time 1.5 hours. Total 20 marks.
Hint: Roughly every key step or point corresponds to 0.5 marks.

1) Translations in Quantum Mechanics [5 marks]

Demonstrate that the momentum operator in QM is the generator of translations (throughout this question, just consider 1 dimension, ie translations along the x-axis).

Using the generator representation of translation operators, show that the set of these operators forms an Abelian group (with the normal operation of "follows"). Explain why it is a Lie group.

2) Proton wave-function [6 marks]

Sketch out the derivation of the proton wave-function from the description of 3 quarks, explaining the spin, flavour and colour components. There is no need to write the complete, fully symmetrised form.
Illustrate this with the Young Tableaux corresponding to SU(2) and SU(3) for 3 quarks. Mark the Tableaux with their multiplicities.

3) Baryon magnetic moments [5 marks]

Baryon magnetic moments can be derived by using the full SU(6) symmetrised spin-flavour wave-functions and focussing on a single quark – as we did in the problem class.
Alternatively, they can be derived by using simplified wave-functions along with the magnetic moment operators for all three quarks (ref. Lichtenberg).
The question which follows is much easier than what we did in the problem class!

Using $p = uud\frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - (\uparrow\downarrow + \downarrow\uparrow)\uparrow)$ and a magnetic moment operator $M = q^1 S_z^1 + q^2 S_z^2 + q^3 S_z^3$,

where q^i is the charge of the ith quark and S_z^i is its z-component of spin, calculate an expression for the proton's magnetic moment $\langle p|M|p\rangle$. (Use q_u, q_d, q_s for the quark charges – do not express them as numbers.)
Use isospin symmetry to calculate the neutron's magnetic moment.
Using $\Lambda = uds\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow$, calculate the magnetic moment of the Λ baryon.

4) Young Tableaux [4 marks]

Consider 4 spin ½ particles. Identify all the possible Young Tableaux, with their corresponding multiplicities corresponding to the group SU(2). You should check the multiplicities sum to $2^4 = 16$.

Why are none of these 4 quark states observed in nature?