Brunel University
Queen Mary, University of London
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Intercollegiate post-graduate course in High Energy Physics

Paper 1 : The Standard Model

Tuesday, 31 January 2012

Time allowed for Examination : 3 hours

Answer 6 from 8 questions

Books and notes may be consulted

Question 1 (20 marks)

At a collider, two high energy particles, A and B, with energies E_A and E_B which are much greater than their rest masses, collide head on. Derive the expression for the centre-of-mass energy. Using this expression, what would be the centre-of-mass energy of a proposed future facility ("LHeC") which will collide 7 TeV protons with 70 GeV electrons ? **[3]**

Now consider particle B (the proton) to be at rest. Derive the formula for the centre-of-mass energy of such a fixed-target experiment. What electron beam energy would be required in the fixed-target experiment in order to achieve the same centre-of-mass energy as in the proposed LHeC facility ? **[3]**

One of the motivations for an eP collider is to search for leptoquarks. Discuss. **[4]**

The invariant mass of two massless jets, M^{jj}, can be written in terms of their transverse energies, E_T^{jet1} and E_T^{jet2}, pseudorapidities, η^{jet1} and η^{jet2}, and azimuthal angles, ϕ^{jet1} and ϕ^{jet2} :

$$M^{jj} = \sqrt{2E_T^{\text{jet1}} E_T^{\text{jet2}} \left[\cosh\left(\eta^{\text{jet1}} - \eta^{\text{jet2}}\right) - \cos\left(\phi^{\text{jet1}} - \phi^{\text{jet2}}\right)\right]} .$$

For two jets back-to-back in ϕ and with equal E_T^{jet}, show that :

$$M^{jj} = \frac{2E_T^{\text{jet}}}{\sqrt{1 - \cos^2\theta^*}} ,$$

where θ^*, the angle between the jet-jet axis and the beam axis in the two-jet centre-of-mass system is given by :

$$\cos\theta^* = \tanh\left(\frac{\eta^{\text{jet1}} - \eta^{\text{jet2}}}{2}\right) .$$

(Recall: $\cos 2\theta = 2\cos^2\theta - 1$, $\text{sech}^2\theta + \tanh^2\theta = 1$.) **[4]**

The cross-section dependence for a spin-1 propagator is $\propto (1 - |\cos\theta^*|)^{-2}$ and for a spin$-\frac{1}{2}$ propagator is $\propto (1 - |\cos\theta^*|)^{-1}$. Draw two Feynman diagrams representing parton collisions at the LHC, one of which has a spin-1 and the other a spin$-\frac{1}{2}$ propagator. **[2]**

Many Feynman diagrams exist already at leading order in QCD at a hadron collider such as the LHC. Draw the four diagrams for the partonic process, $gg \to gg$. **[4]**

Question 2 (20 marks)

Given the cross section for the scattering of an electron from a fixed Coulomb potential of point charge Ze :

$$\frac{d\sigma}{d\Omega} = \frac{2(Z\alpha)^2 m^2}{|\mathbf{q}|^4} \mathrm{Tr}\left[\gamma_0 \frac{\not{p}_i + m}{2m} \gamma_0 \frac{\not{p}_f + m}{2m}\right] ,$$

where p_i and p_f are the initial and final momenta and $\mathbf{q} = \mathbf{p}_f - \mathbf{p}_i$, determine the Mott cross section :

$$\frac{d\sigma}{d\Omega} = \frac{(Z\alpha)^2}{4(\gamma\beta^2)^2(mc^2)^2 \sin^4\theta/2}\left(1 - \beta^2 \sin^2\frac{\theta}{2}\right) .$$

Trace theorems used should be explicitly stated. **[15]**

Show that in the non-relativistic limit

$$\frac{d\sigma}{d\Omega} = \frac{(Z\alpha)^2}{16E^2 \sin^4\frac{\theta}{2}}$$

and in the extreme-relativistic limit

$$\frac{d\sigma}{d\Omega} = \frac{(Z\alpha)^2 \cos^2\frac{\theta}{2}}{4E^2 \sin^4\frac{\theta}{2}} .$$

[5]

Question 3 (20 marks)

In the decay of a π^- at rest, $\pi^- \to e^- + \bar{\nu}_e$, show that

$$\frac{1}{2}\left(1 - \frac{v_e}{c}\right) = \frac{m_e^2}{m_\pi^2 + m_e^2} .$$

where v_e is the velocity of the electron. **[5]**

To lowest order, the partial decays rate for pions are :

$$\frac{1}{\tau(\pi \to e\bar{\nu}_e)} = \frac{\alpha_\pi^2}{4\pi}\left(1 - \frac{v_e}{c}\right) p_e^2 E_e , \qquad \frac{1}{\tau(\pi \to \mu\bar{\nu}_\mu)} = \frac{\alpha_\pi^2}{4\pi}\left(1 - \frac{v_\mu}{c}\right) p_\mu^2 E_\mu .$$

where α_π is an effective coupling constant and E_e, E_μ and p_e, p_μ are the charged lepton's energy and momentum. Hence show :

$$\frac{\tau(\pi \to \mu\bar{\nu}_\mu)}{\tau(\pi \to e\bar{\nu}_e)} = \frac{m_e^2(m_\pi^2 - m_e^2)^2}{m_\mu^2(m_\pi^2 - m_\mu^2)^2} .$$

[5]

Use the analogue of the above equation for the decay of the K^- to estimate the ratio

$$\frac{\tau(K \to \mu\bar{\nu}_\mu)}{\tau(K \to e\bar{\nu}_e)}$$

and compare with the observed value $(2.4 \pm 0.1) \times 10^{-5}$.
Given the lifetimes $\tau(K \to \mu\bar{\nu}_\mu) = 1.948 \times 10^{-8}\,$s and $\tau(\pi \to \mu\bar{\nu}_\mu) = 2.603 \times 10^{-8}\,$s, estimate α_K/α_π.
($m_K = 493.67\,$MeV, $m_\pi = 139.57\,$MeV, $m_\mu = 105.66\,$MeV, $m_e = 0.511\,$MeV .) **[5]**

Draw quark model diagrams for the decays $\pi^- \to \mu^- + \bar{\nu}_\mu$ and $K^- \to \mu^- + \bar{\nu}_\mu$, stating which element of the CKM matrix is involved in each.

Neglecting masses, the ratio of the CKM elements is equal to α_K/α_π. Hence estimate $\sin\theta_{12}$.
[5]

Question 4 (20 marks)

The covariant derivative,

$$D^\mu = \partial^\mu + ieA^\mu,$$

in $U(1)$ satisfies the commutation relation

$$[D^\mu, D^\nu]\psi = ieF^{\mu\nu}\psi\,.$$

Hence determine $F^{\mu\nu}$. **[5]**

In $SU(2)$, using the definition of the covariant derivative,

$$D^\mu = \partial^\mu + ig\,\boldsymbol{\tau}\cdot\frac{\mathbf{W}^\mu}{2},$$

show that

$$[D^\mu, D^\nu]\psi = \frac{ig}{2}\boldsymbol{\tau}\cdot(\partial^\mu\mathbf{W}^\nu - \partial^\nu\mathbf{W}^\mu - g\mathbf{W}^\mu\times\mathbf{W}^\nu)\psi\,.$$

[7]

In $SU(3)$, the covariant derivative,

$$D^\mu = \partial^\mu + \frac{ig_s}{2}\boldsymbol{\lambda}\cdot\mathbf{A}^\mu$$

transforms as

$$D'^\mu\psi' = \left(1 + \frac{ig_s}{2}\boldsymbol{\lambda}\cdot\boldsymbol{\eta}(x)\right)D^\mu\psi$$

where

$$\psi' = \left(1 + \frac{ig_s}{2}\boldsymbol{\lambda}\cdot\boldsymbol{\eta}(x)\right)\psi\,.$$

Hence show that $\mathbf{A}'^\mu = \mathbf{A}^\mu + \delta\mathbf{A}^\mu$ is given by

$$A'^\mu_a = A^\mu_a - \partial^\mu\eta_a(x) - g_s f_{abc}\eta_b(x)A^\mu_c\,.$$

[8]

Question 5 (20 marks)

The dependency of the electric charge on an arbitrary scale, μ, is (to one-loop order) :

$$\mu \frac{de_\mu}{d\mu} = \frac{e_\mu^3}{12\pi^2} \,.$$

Given that at some scale, $\mu = M$, $e_\mu = e_M$, solve the above to show that

$$\alpha_\mu = \frac{\alpha_M}{1 - \frac{\alpha_M}{3\pi} \ln(\mu^2/M^2)}$$

where $\alpha = e^2/4\pi$. **[8]**

Sketch a plot of α_s versus scale, Q, and describe the features and their physical implications. **[4]**

Give a brief description of a variable you could use to extract a value of α_s . **[2]**

The DGLAP equations are:

$$\frac{dQ_i(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[Q_i(y,Q^2) P_{qq}\left(\frac{x}{y}\right) + G(y,Q^2) P_{qg}\left(\frac{x}{y}\right) \right]$$

$$\frac{dG(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[Q_i(y,Q^2) P_{gq}\left(\frac{x}{y}\right) + G(y,Q^2) P_{gg}\left(\frac{x}{y}\right) \right]$$

Explain the functions Q_i and G and the four P_{ij} functions. **[6]**

Question 6 (20 marks)

Draw a Feynman diagram of a decay of a τ lepton. **[2]**

Explain why we can write the branching ratio as

$$\Gamma(\tau^- \rightarrow e^-\overline{\nu}_e + \nu_\tau) \sim \frac{1}{2 + N_c}$$

where N_c is the number of colours. Is this consistent with the experimental measure of 18% ? **[4]**

The branching ratio, $\Gamma(W^- \rightarrow e^- + \overline{\nu}_e)$, can be written in a similar form; what is this ? How does the result of this expression compare with the experimental value of 10.8% ? **[3]**

Draw quark diagrams for the following and explain the difference in the measured branching ratios :

$$\begin{aligned} \Gamma(D^+ \rightarrow K^+ + \pi^0) &= 1.83 \times 10^{-4} \\ \Gamma(D^+ \rightarrow K^+ + K^0) &= 2.83 \times 10^{-3} \end{aligned}$$

and using the CKM matrix,

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97383 & 0.2272 & 0.004 \\ 0.2271 & 0.97296 & 0.042 \\ 0.008 & 0.042 & 0.9991 \end{pmatrix},$$

predict the ratio of the two to within 20%. **[6]**

Draw an example parton-level Feynman diagram of top production at the Tevatron. Why could $t\bar{t}$ production not be observed at LEP or at HERA ? Explain why the cross section for single-top production at LEP and HERA is small and draw a Feynman diagram for its production at both colliders. **[5]**

Question 7 (20 marks)

The unpolarised cross section, $e^+e^- \to \mu^+\mu^-$ is given by

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left[A(1+\cos^2\theta) + B\cos\theta\right]$$

where θ is the scattering angle in the centre-of-mass frame. Given the forward–backward asymmetry is defined as

$$A_{\rm FB} = \frac{N_{\rm F} - N_{\rm B}}{N_{\rm F} + N_{\rm B}}$$

where $N_{\rm F}$ is the number scattered into the forward hemisphere, $0 \le \cos\theta \le 1$, and $N_{\rm B}$ that into the backward hemisphere, $-1 \le \cos\theta < 0$, determine $A_{\rm FB}$ in terms of A and B. **[4]**

Given the relation of constants in electroweak theory,

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}\,,$$

predict the masses of the W and Z bosons.
($G_F = 1.16637 \times 10^{-5}\,\text{GeV}^{-2}$, $\alpha = 7.297 \times 10^{-3}$, $\sin^2\theta_W \sim 0.23$.) **[4]**

From the Lagrangian

$$\frac{1}{8}\left[g_W^2(v+h)^2(W_\mu^1 - iW_\mu^2)(W_\mu^1 + iW_\mu^2) - (v+h)^2(g'B_\mu - g_W W_\mu^3)(g'B^\mu - g_W W_3^\mu)\right]$$

derive the WWH and $WWHH$ couplings and the ZZH and $ZZHH$ couplings. (Simplify your answer to remove dependencies on both v and g'.) **[9]**

The Higgs Boson was searched for in e^+e^- collisions at LEP. Draw the Feynman diagram of the process for Higgs production. Masses of up to about 114 GeV were ruled out; hence give the approximate maximum centre-of-mass energy of LEP. **[3]**

Question 8 (20 marks)

Draw Feynman diagrams of the lowest-order (electroweak) process in electron-proton scattering. **[3]**

The matrix-element squared for lowest-order electromagnetic electron-proton scattering (under the assumption of the proton being a structureless, point-like Dirac particle) with given four-momentum, $e^-(p_i)\,p(P_i) \to e^-(p_f)\,p(P_f)$, is

$$|T_{\rm fi}|^2 = \left(\frac{e^2}{q^2}\right)^2 \frac{1}{4}\,{\rm Tr}\left[\frac{\not p_f + m}{2m}\gamma^\mu \frac{\not p_i + m}{2m}\gamma^\nu\right]\,{\rm Tr}\left[\frac{\not P_f + M}{2M}\gamma_\mu \frac{\not P_i + M}{2M}\gamma_\nu\right].$$

Evaluate the traces to show that

$$|T_{\rm fi}|^2 = \frac{e^4}{2m^2M^2q^4}\left[(p_f\cdot P_f)\,(p_i\cdot P_i) + (p_f\cdot P_i)\,(p_i\cdot P_f) - M^2\,(p_f\cdot p_i) - m^2\,(P_f\cdot P_i) + 2m^2M^2\right]$$

where m is the mass of the electron, M is the mass of the proton and $q = p_f - p_i$. **[8]**

Assuming four-vectors,

$$p_i = (E, \mathbf{p})\,, \quad p_f = (E', \mathbf{p}')\,, \quad P_i = (M, \mathbf{0})\,, \quad P_f = (E_f, \mathbf{P_f})\,,$$

conserve energy and momentum to show that for $m \ll E$,

$$\frac{E - E'}{M} = -\frac{q^2}{2M^2}\,.$$

[5]

Hence show that the cross section,

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{4\pi^2}\frac{E'/E}{1 + (2E/M)\sin^2\theta/2}|T_{\rm fi}|^2\,,$$

where

$$|T_{\rm fi}|^2 = \frac{16\pi^2\alpha^2 EE'}{m^2q^4}\left[1 + \frac{q^2}{4EE'}\left(1 + \frac{E'-E}{M}\right) + \frac{m^2}{2EE'}\left(\frac{E'-E}{M}\right)\right]$$

can be simplified, in the limit $E \gg m$ but $E \ll M$, to

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\cos^2\frac{\theta}{2}$$

where θ is the angle between the outgoing and incoming electron. **[4]**

[Total Marks = 120]

END OF PAPER