SPIN

$$J = L + S$$

TOTAL ANGULAR MOMENTUM — ORBITAL ANGULAR MOM — SPIN

NEITHER L NOR S COMMUTE WITH H, BUT J DOES

→ TOTAL ANGULAR MOMENTUM IS CONSERVED

$$H \begin{pmatrix} u_A \\ u_B \end{pmatrix} = (\underline{\alpha} \cdot \underline{p} + \beta m) \begin{pmatrix} u_A \\ u_B \end{pmatrix}$$

$$= E \begin{pmatrix} u_A \\ u_B \end{pmatrix}$$

$$= \begin{pmatrix} m\mathbb{1} & \sigma \cdot p\,\mathbb{1} \\ \sigma \cdot p\,\mathbb{1} & -m\,\underline{\mathbb{1}} \end{pmatrix} \begin{pmatrix} u_A \\ u_B \end{pmatrix}$$

COMMUTATOR BETWEEN DIRAC HAMILTONIAN AND $\sigma \cdot p$

$$\begin{pmatrix} m\mathbb{1} & \sigma \cdot p\,\mathbb{1} \\ \sigma \cdot p\,\mathbb{1} & -m\mathbb{1} \end{pmatrix} \begin{pmatrix} \sigma \cdot p\,\mathbb{1} & 0 \\ 0 & \sigma \cdot p\,\mathbb{1} \end{pmatrix} -$$

$$- \begin{pmatrix} \sigma \cdot p\,\mathbb{1} & 0 \\ 0 & \sigma \cdot p\,\mathbb{1} \end{pmatrix} \begin{pmatrix} m\mathbb{1} & \sigma \cdot p\,\mathbb{1} \\ \sigma \cdot p\,\mathbb{1} & -m\mathbb{1} \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

SO $\sigma \cdot p$ IS A CONSERVED QUANTITY

DEFINE HELICITY

$$H = \frac{1}{2} \sigma \cdot \hat{p} = \frac{1}{2} \frac{\sigma \cdot p}{|p|}$$

H IS PROJECTION OF SPIN ALONG DIRECTION OF MOTION

EIGENVALUES $\pm \frac{1}{2}$

FOR A PARTICLE WITH MOMENTUM P:

$$\hat{P} = \sin\theta \cos\phi \,\hat{i} + \sin\theta \sin\phi \,\hat{j} + \cos\theta \,\hat{k}$$

SO, $\sigma \cdot \hat{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sin\theta\cos\phi +$

$$+ \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \sin\theta \sin\phi + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cos\theta$$

APPLY HELICITY TO SPINORS

$$\frac{\sigma \cdot \hat{P}}{2} \begin{pmatrix} U_A \\ U_B \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & -\cos\theta \end{pmatrix} \begin{pmatrix} U_A \\ U_B \end{pmatrix}$$

SEARCH FOR EIGENVALUES

$$\frac{1}{2}\begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & -\cos\theta \end{pmatrix}\begin{pmatrix} U_A \\ U_B \end{pmatrix} = \lambda \begin{pmatrix} U_A \\ U_B \end{pmatrix}$$

$$\begin{vmatrix} \cos\theta - 2\lambda & \sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & -\cos\theta - 2\lambda \end{vmatrix} = 0$$

$$(+\cos\theta - 2\lambda)(-\cos\theta - 2\lambda) - \sin^2\theta = 0$$

$$-\cos^2\theta + 4\lambda^2 - \sin^2\theta = 0$$

$$\lambda = \pm 1/2$$

SO IT GIVES THE EXPECTED EIGENVALUES

γ^5 MATRIX

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$\gamma^{5\dagger} = \gamma^5$$

$$(\gamma^5)^2 = \mathbb{1}$$

$$\gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = 0$$

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \quad \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix} \quad \gamma^5 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

APPLY γ^5 TO DIRAC EQN

$$\gamma^5 \begin{pmatrix} U_A \\ U_B \end{pmatrix} = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \begin{pmatrix} \chi \\ \frac{\sigma \cdot p}{E+m} \chi \end{pmatrix}$$

$$\chi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\gamma^5 \begin{pmatrix} U_A \\ U_B \end{pmatrix} = \begin{pmatrix} \frac{\sigma \cdot p}{E+m} \chi \\ \chi \end{pmatrix}$$

TAKE APPROX $E \gg m \rightarrow m \sim 0$

$E \sim |p|$

$$\gamma^5 \begin{pmatrix} U_A \\ U_B \end{pmatrix} \simeq \begin{pmatrix} \sigma \cdot \hat{p} \, \chi \\ \mathbb{1} \chi \end{pmatrix} =$$

BUT $(\sigma \cdot \hat{p})^2 = \underline{\mathbb{1}}$

$$= \begin{pmatrix} \sigma \cdot \hat{p} \, \chi \\ (\sigma \cdot \hat{p})^2 \chi \end{pmatrix} =$$

$$(\sigma \cdot \hat{p}) \begin{pmatrix} \chi \\ (\sigma \cdot \hat{p}) \chi \end{pmatrix} = (\sigma \cdot \hat{p}) \begin{pmatrix} \chi \\ \frac{\sigma \cdot \underline{p}}{E+m} \chi \end{pmatrix}$$

$$= (\sigma \cdot \hat{p}) \begin{pmatrix} U_A \\ U_B \end{pmatrix}$$

$$\Rightarrow \gamma^5 \begin{pmatrix} U_A \\ U_B \end{pmatrix} = \begin{pmatrix} \sigma \cdot \hat{p} & 0 \\ 0 & \sigma \cdot \hat{p} \end{pmatrix} \begin{pmatrix} U_A \\ U_B \end{pmatrix}$$

SO IN $m \sim 0$ APPROX, γ^5 IS HELICITY OPERATOR

DEFINE HELICITY PROJECTION OPERATORS

$$P_R = (1 + \gamma^5)/2$$

$$P_L = (1 - \gamma^5)/2$$

COMPLETENESS RELATION

$$\sum_{s=1,2} u_s(p)\,\bar{u}_s(p) =$$

$$= \sum_{s=1,2} N^* N \begin{pmatrix} \chi_s \\ \frac{\sigma\cdot p}{E+m}\chi_s \end{pmatrix} \left(\chi_s^\dagger , -\frac{\sigma\cdot p}{E+m}\chi_s^\dagger \right)$$

$$= \sum_{s=1,2} N^* N \begin{pmatrix} \chi_s^\dagger \chi_s & -\frac{(\sigma\cdot p)^\dagger}{E+m}\chi_s^\dagger \chi_s \\ \frac{\sigma\cdot p}{E+m}\chi_s^\dagger \chi_s & -\frac{E^2-m^2}{(E+m)^2}\chi_s^\dagger \chi_s \end{pmatrix}$$

$$= \sum_{s=1,2} N^* N \begin{pmatrix} \mathbb{1} & -\frac{(\sigma\cdot p)^\dagger}{E+m} \\ \frac{\sigma\cdot p}{E+m} & -\frac{E^2-m^2}{(E+m)^2} \end{pmatrix} \chi_s^\dagger \chi_s$$

$$\sum_{s=1,2} \chi_s^\dagger \chi_s = \begin{pmatrix} 1 \\ 0 \end{pmatrix}(1,0) + \begin{pmatrix} 0 \\ 1 \end{pmatrix}(0,1) =$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

IN UNIT VOLUME $N^* N = E + m$

$$\sum_{s=1,2} u_s u_s^{+} = \begin{pmatrix} \mathbb{1} & -\frac{\sigma\cdot p}{E+m} \\ \frac{\sigma\cdot p}{E+m} & -\frac{E^2-m^2}{(E+m)^2} \end{pmatrix}(E+m)$$

$$= \begin{pmatrix} (E+m)\,\mathbb{1} & -\frac{\sigma\cdot p}{E+m} \\ \frac{\sigma\cdot p}{E+m} & (m-E)\,\mathbb{1} \end{pmatrix}$$

$$\cancel{p} + m = \gamma^\mu p_\mu + m\,\mathbb{1} =$$

$$= \gamma^0 E - \gamma^k p_k + m\,\mathbb{1} =$$

$$= \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} E - \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix} p_k + \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} m$$

$$= \begin{pmatrix} E+m & -\sigma\cdot p \\ \sigma\cdot p & (m-E)\,\mathbb{1} \end{pmatrix}$$

$$\xrightarrow{\text{so}} \quad \sum_{s=1,2} u_s \bar{u}_s = \cancel{p} + m$$

$$\sum_{s=1,2} v_s \bar{v}_s = \cancel{p} - m$$

FORMS OF INTERACTIONS

- SCALAR $\bar{U} U$ EVEN PARITY
- VECTOR $\bar{U} \gamma^{\mu} U$ ODD PARITY

  (EX: WEAK INTERACTIONS)
- PSEUDO-SCALAR $\bar{U} \gamma^{5} U$

  ODD PARITY
- AXIAL INTERACTION (PSEUDO-VECTOR) $\bar{U} \gamma^{5} \gamma^{\mu} U$

  (ALSO IN WEAK INTS)
- TENSOR INTERACTION

  $$\bar{U} \sigma^{\mu\nu} U$$

TRACE THEOREMS

- $\mathrm{Tr}(\mathbb{1}) = 4$
- $\mathrm{Tr}[\not{a}\,\not{b}] = \mathrm{Tr}[\not{b}\,\not{a}]$

So

$$\mathrm{Tr}[\not{a}\,\not{b}] = \frac{1}{2}\,\mathrm{Tr}[\not{a}\,\not{b} + \not{b}\,\not{a}] =$$

$$= \frac{1}{2}\,\mathrm{Tr}[\gamma^\mu \gamma^\nu a_\mu b_\nu + \gamma^\mu \gamma^\nu b_\mu a_\nu]$$

- $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu}\,\mathbb{1}$

$$\mathrm{Tr}[\not{a}\,\not{b}] = \frac{1}{2}\,\mathrm{Tr}(\mathbb{1})\,2\,a \cdot b =$$

$$4\,a \cdot b$$

$$\mathrm{Tr}(\not{a}\not{b}\not{c}\not{d}) = \mathrm{Tr}[\gamma^\mu \gamma^\nu \gamma^\delta \gamma^\sigma a_\mu b_\nu c_\delta d_\sigma]$$

$$\mathrm{Tr}[\gamma^\mu \gamma^\nu \gamma^\delta \gamma^\sigma] = -\mathrm{Tr}[\gamma^\nu \gamma^\mu \gamma^\delta \gamma^\sigma] + \mathrm{Tr}[2g^{\mu\nu} \gamma^\delta \gamma^\sigma]$$

$$= \mathrm{Tr}[\gamma^\nu \gamma^\delta \gamma^\mu \gamma^\sigma] - \mathrm{Tr}[2g^{\mu\delta} \gamma^\nu \gamma^\sigma] + \mathrm{Tr}[2g^{\mu\nu} \gamma^\delta \gamma^\sigma]$$

$$= -\mathrm{Tr}[\gamma^\nu \gamma^\delta \gamma^\sigma \gamma^\mu] + \mathrm{Tr}[2g^{\mu\sigma} \gamma^\nu \gamma^\delta] - \mathrm{Tr}[2g^{\mu\delta} \gamma^\nu \gamma^\sigma] + \mathrm{Tr}[2g^{\mu\nu} \gamma^\delta \gamma^\sigma]$$

IS EQUAL TO

$\mathrm{Tr}[\gamma^\mu \gamma^\nu \gamma^\delta \gamma^\sigma]$ BECAUSE OF CYCLICAL PROPERTY

$$\rightarrow \mathrm{Tr}[\gamma^\mu \gamma^\nu \gamma^\delta \gamma^\sigma] = \mathrm{Tr}(g^{\mu\sigma} \gamma^\nu \gamma^\delta) - \mathrm{Tr}(g^{\mu\delta} \gamma^\nu \gamma^\sigma) + \mathrm{Tr}[g^{\mu\nu} \gamma^\delta \gamma^\sigma]$$

$$\mathrm{Tr}[\gamma^\mu \gamma^\nu] = 4 g^{\mu\nu}$$

$$\mathrm{Tr}[\gamma^\mu \gamma^\nu \gamma^\delta \gamma^\sigma] = 4 (g^{\mu\sigma} g^{\nu\delta} - g^{\mu\delta} g^{\nu\sigma} + g^{\mu\nu} g^{\delta\sigma})$$

$$\rightarrow \mathrm{Tr}[\not a \not b \not c \not d] = 4 [(a \cdot d)(b \cdot c) - (a \cdot c)(b \cdot d) + (a \cdot b)(c \cdot d)]$$

$$- \gamma_\mu \gamma^\nu \gamma^\mu = -2 \gamma^\nu$$

$$\gamma_\mu \gamma^\nu \gamma^\mu = -\gamma_\mu \gamma^\mu \gamma^\nu + 2 g^{\mu\nu} \gamma_\mu$$

$$= -4 \gamma^\nu + 2 \gamma^\nu = -2 \gamma^\nu$$

$$\gamma_\mu \not{a} \gamma^\mu = -2\not{a}$$

$$-\gamma_\mu \gamma^\rho \gamma^\sigma \gamma^\mu =$$

$$= -\gamma^\rho \gamma_\mu \gamma^\sigma \gamma^\mu + 2 g^\rho_\mu \gamma^\sigma \gamma^\mu$$

$$= +2\gamma^\rho \gamma^\sigma + 2\gamma^\sigma \gamma^\rho =$$

$$= 4 g^{\rho\sigma}$$

$$\gamma_\mu \not{a} \not{b} \gamma^\mu = 4\, a \cdot b$$

$$* \quad \gamma_\mu \not{a} \not{b} \not{c} \gamma^\mu =$$

$$= -\gamma_\mu \not{a} \not{b} \gamma^\mu \gamma^\nu c_\nu +$$

$$+ \gamma_\mu \not{a} \not{b} \, 2 g^{\mu\nu} c_\nu$$

$$= -4(a \cdot b)\not{c} + 2 \not{c} \not{a} \not{b}$$

$$= -4(a \cdot b)\not{c} + 2 \not{c} \gamma^\alpha \gamma^\beta a_\alpha b_\beta$$

$$= \cancel{-4(a \cdot b)} \not{c} - 2 \not{c} \gamma^\beta \gamma^\alpha a_\alpha b_\beta +$$

$$+ \cancel{4 \not{c} g^{\alpha\beta} a_\alpha b_\beta}$$

$$= -2 \not{c} \not{a} \not{b}$$

$$-\mathrm{Tr}\left[\gamma^5\gamma^\mu\right]$$

$$\gamma^5\gamma^\mu+\gamma^\mu\gamma^5=0$$

$$\mathrm{Tr}\left[\gamma^5\gamma^\mu\right]=-\mathrm{Tr}\left[\gamma^\mu\gamma^5\right]$$

$$=-\mathrm{Tr}\left[\gamma^5\gamma^\mu\right]$$

$$\Rightarrow \mathrm{Tr}\left[\gamma^5\gamma^\mu\right]=0$$

$$-\mathrm{Tr}\left[\gamma^5\gamma^\mu\gamma^\nu\right]$$

- CASE $\mu=\nu$

$$\mathrm{Tr}\left[\gamma^5 (\gamma^\mu)^2\right] = \mathrm{Tr}\left[\gamma^5 \mathbb{1}\right]$$

$$= \mathrm{Tr}\begin{bmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{bmatrix} = 0$$

= CASE $\mu = 1, \nu = 2$

$$\mathrm{Tr}\left[i \gamma^0 \gamma^1 \gamma^2 \gamma^3 \gamma^1 \gamma^2\right] =$$

$$= \mathrm{Tr}\left[i \gamma^0 (\gamma^1)^2 \gamma^2 \gamma^3 \gamma^2\right]$$

$$= -\mathrm{Tr}\left[i \gamma^0 (\gamma^2)^2 \gamma^3\right] =$$

$$= -i \,\mathrm{Tr}\left[\gamma^0 \gamma^3\right] = -i \,\mathrm{Tr}\left[\gamma^3 \gamma^0\right]$$

$\Rightarrow$ HAS TO BE 0

- TRACE ODD NUMBER OF γ MATRICES IS 0

$$\mathrm{Tr}\left(\not{a}_1 \not{a}_2 \cdots \not{a}_m\right) =$$

$$\mathrm{Tr}\left[\not{a}_1 \not{a}_2 \cdots \not{a}_m \gamma^5 \gamma^5\right]$$

BACK-PROPAGATE γ^5

$$= (-1)^m \mathrm{Tr}\left[\gamma^5 \not{a}_1 \cdots \not{a}_m \gamma^5\right]$$

USING CYCLICAL RULE

$$= (-1)^m \mathrm{Tr}\left[\not{a}_1 \cdots \not{a}_m \cancel{\gamma^5 \gamma^5}\right]$$

IF m IS ODD, ONLY VALID IF ZERO

$$-\mathrm{Tr}\left[\gamma^5\gamma^\mu\gamma^\nu\gamma^\delta\gamma^\sigma\right]$$

CONSIDER CASE

$$\mathrm{Tr}\left[\gamma^5\underbrace{\gamma^0\gamma^1\gamma^2\gamma^3}_{+\frac{1}{i}\gamma 5}\right] =$$

$$= \pm\frac{1}{i}\mathrm{Tr}\left[(\gamma^5)^2\right]$$

$$\frac{1}{i} = 4i$$

$$\mathrm{Tr}\left[\gamma^5\gamma^\mu\gamma^\nu\gamma^\delta\gamma^\sigma\right] = 4i\,\epsilon_{\mu\nu\delta\sigma}$$

$$\epsilon_{\mu\nu\delta\sigma} = \begin{cases} 1 & \text{EVEN PERMUT} \\ -1 & \text{ODD ''} \\ 0 & \text{REPEATED SYMBOL} \end{cases}$$

ELECTRON - MUON SCATTERING WITH SPIN

$$(\alpha \cdot p + \beta m)\psi = E\psi$$

FOR EM INTERACTIONS

$$p^\mu \rightarrow p^\mu + eA^\mu$$

$$E \rightarrow E + eV$$

$$p^k \rightarrow p^k + eA^k$$

$$\rightarrow \left(\alpha_k p^k + \beta m + e(\alpha_k A^k - V)\right)\psi = E\psi$$

DIRAC EQUATION IN EM FIELD

$$V_{DIRAC} = e\left(\alpha_k A^k - V\mathbb{1}\right)$$

$$T_{fi} = -i\int d^4x\, \psi_f^{\dagger} V_{DIRAC} \psi_i =$$

$$= -ie\int d^4x\, \psi_f^{\dagger}\left(-A_4\mathbb{1} + \alpha^k A_k\right)\psi_i$$

$$= -ie\int d^4x\, \bar{\psi}_f\left(-\gamma^0 A_0 + \gamma^k A_k\right)\psi_i$$

$$= ie\int d^4x\, \bar{\psi}_f \gamma^\mu A_\mu \psi_i =$$

$$J^\mu_{fi} = -e\,\bar{\psi}_f \gamma^\mu \psi_i$$

$$= -ie\int J^\mu_{fi} A_\mu\, d^4x$$

FOR PLANAR WAVE

$$J^{\mu}_{fi} = -e\,\bar{u}_f \gamma^{\mu} e^{i(p_f - p_i)x}$$

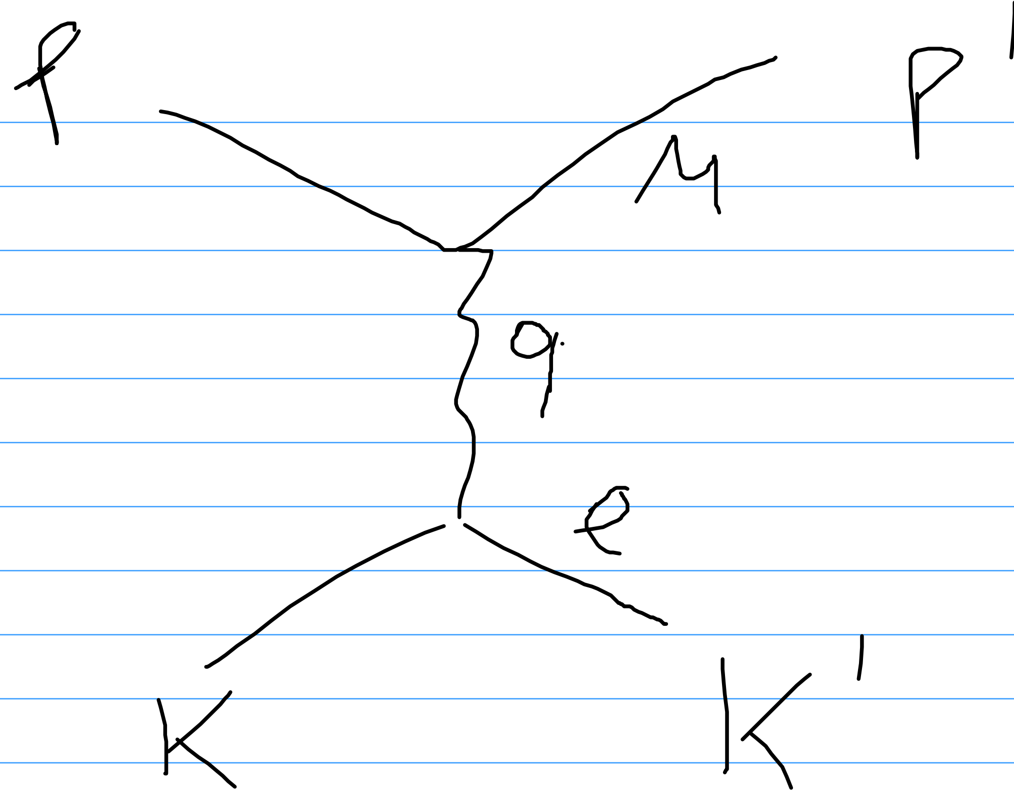
[IN SPINLESS CASE WAS $J^{\mu}_{fi} = -e(p_f - p_i)^{\mu} e^{i(p_f - p_i)}$

TRANSITION AMPLITUDE

$$T_{fi} = -i\int J_{\mu 1}\left(\frac{1}{q^2}\right) J^{\mu}_{2}\, d^4x$$

$$= ie\int d^4x\, \bar{u}(k')e^{ik'x}\gamma_{\mu}u(k)e^{-ikx}\left(\frac{-1}{q^2}\right)$$

$$(-e)\,\bar{u}(p')e^{ip'x}\gamma^{\mu}u(p)e^{-ipx}$$



$$|T_{fi}|^2 = \frac{e^4}{q^4}\left[\bar{u}(k')\gamma_\mu u(k)\right]\left[\bar{u}(p')\gamma^\mu u(p)\right]\left[\bar{u}(k')\gamma_\nu u(k)\right]^\dagger\left[\bar{u}(p')\gamma^\nu u(p)\right]^\dagger$$

HERMITIAN CONJUGATES

$$\left(u^\dagger(p')\gamma^0\gamma^\mu u(p)\right)^\dagger =$$

$$= u^\dagger(p)\gamma^{\mu\dagger}\gamma^{0\dagger}u(p')$$

$$= -u^\dagger(p)\gamma^\mu\gamma^0 u(p')$$

$$= u^\dagger(p)\gamma^0\gamma^\mu u(p')$$

$$= \bar{u}(p)\gamma^\mu u(p')$$

$$|T_{fi}|^2 = \frac{e^4}{q^4}\left[\bar{u}(k')\gamma_\mu u(k)\right]\left[\bar{u}(k)\gamma_\nu u(k')\right]$$

$$\left[\bar{u}(p')\gamma^\mu u(p)\right]\left[\bar{u}(p)\gamma^\nu u(p')\right]$$

DEFINE ELECTRON TENSOR

$$^{e}L_{\mu\nu} = \left[\bar{u}(k')\gamma_\mu u(k)\right]\left[\bar{u}(k)\gamma_\nu u(k')\right]$$

AND MUON TENSOR

$$^{\mu}L_{\mu\nu}$$

$$\rightarrow |T_{fi}|^2 = \frac{e^4}{q^4}\, {}^{e}L_{\mu\nu}\, {}^{\mu}L^{\mu\nu}$$

SUM OVER SPIN STATES

$${}^{0}L_{\mu\nu} = \frac{1}{2}\sum_{s}\sum_{s'} \bar{u}(k')\gamma_{\mu}u(k)\,\bar{u}(k)\gamma_{\nu}u(k')$$

$$= \frac{1}{2}\sum_{s}\sum_{s'} \bar{u}(k')_{\alpha}\gamma_{\mu}^{\alpha\beta}u(k)_{\beta}\,\bar{u}(k)_{\gamma}\gamma_{\nu}^{\gamma\delta}u(k')_{\delta}$$

$$= \frac{1}{2}\sum_{s}\sum_{s'} u(k')_{\delta}\,\bar{u}(k')_{\alpha}(\gamma_{\mu}^{\alpha\beta})u(k)_{\beta}\,\bar{u}(k)_{\gamma}\gamma_{\nu}^{\gamma\delta}$$

USING COMPLETENESS RELATIONS

$$= \left(\cancel{k}' + m\right)_{\delta\alpha} \left(\gamma^{\alpha\beta}_{\mu}\right) \left(\cancel{k} + m\right)_{\beta\gamma} \left(\gamma^{\gamma\delta}_{\nu}\right)$$

$${}^{e}L_{\mu\nu} = \frac{1}{2} \mathrm{Tr}\left[(\cancel{k}' + m)\gamma_\mu (\cancel{k} + m)\gamma_\nu\right]$$

$${}^{\mu}L_{\mu\nu} = \frac{1}{2} \mathrm{Tr}\left[(\cancel{p}' + m)\gamma_\mu (\cancel{p} + m)\gamma_\nu\right]$$

CALL $m_e \equiv m$

$m_\mu \equiv M$

$$|\mathcal{M}_{\mu i}|^2 = \frac{e^4}{q^4}\frac{1}{2}\mathrm{Tr}\left[(\not{k}'+m)\gamma_\mu(\not{k}+m)\gamma_\nu\right]$$

$$\frac{1}{2}\mathrm{Tr}\left[(\not{p}'+M)\gamma^\mu(\not{p}+M)\gamma^\nu\right]$$

ALL TERMS CONTAINING ODD NUMBER OF γ MATRICES ARE 0