$$|T_{fi}|^2 = \frac{8e^4}{q^4}\Big[(k'\cdot p')(k\cdot p) + (k'\cdot p)(k\cdot p') - \cancel{M^2}(p'\cdot p)$$
$$- \cancel{m^2}(k'\cdot k) + \cancel{m^2 M^2}\Big)$$

AT HIGH ENERGY,
MASSES SMALL COMPARED TO MOMENTA

MENDELSTRAN VARIABLES:

$$s = (k+p)^2 \sim 2k\cdot p \sim 2k'p'$$
$$t = q^2 = (k-k')^2 = (p-p')^2 \sim 2\,k\cdot k' \sim 2p\cdot p'$$
$$u = (k-p')^2 \sim -2p'\cdot k \sim 2k'\cdot p$$

S DOES NOT MIX BEFORE AND AFTER COLLISION

t DOES NOT MIX e AND μ

U MIXES BOTH!

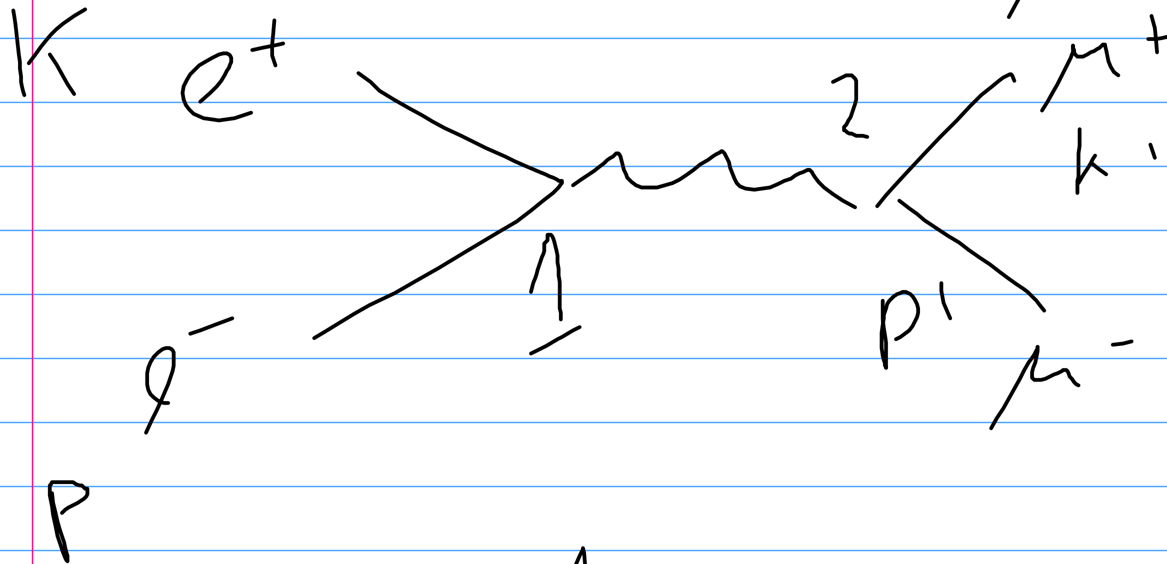
$$|T_{fi}|^2 = \frac{8e^4}{t^2}\left(\frac{s}{2}\frac{s}{2} + \left(-\frac{u}{2}\right)\left(-\frac{u}{2}\right)\right) =$$

$$= \frac{2e^4}{t^2}\left(s^2+u^2\right) = 2e^4\left(\frac{s^2+u^2}{t^2}\right)$$

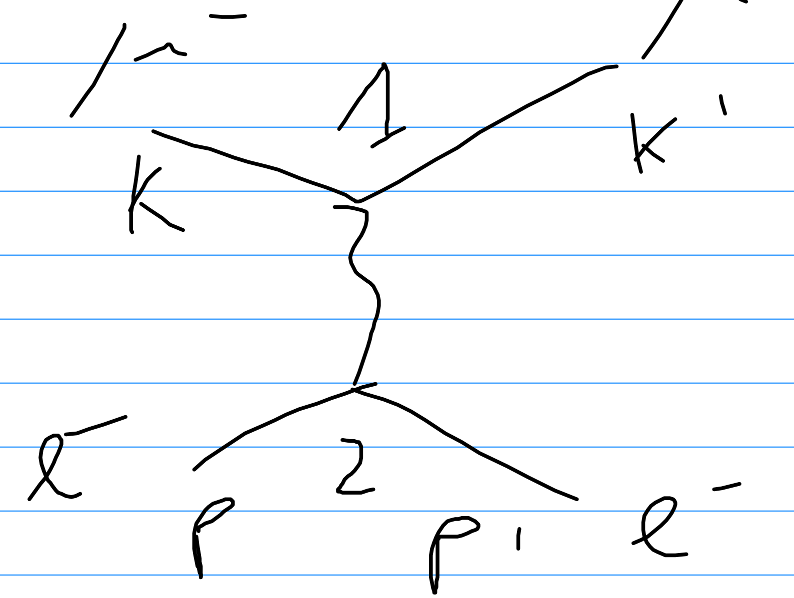
DIFF. CROSS SECTION

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|T_{fi}|^2 = \frac{e^4}{32\pi^2 s}\left(\frac{s^2+u^2}{t^2}\right)$$

$e^+ e^- \to \mu^+ \mu^-$



$\Longleftrightarrow$



IN $e\mu$ $\quad k \to k' \quad 1$

$\quad p \to p' \quad 2$

IN e^+e^- $\quad k \to -p \quad 1$

$\quad -k' \to p' \quad 2$

VERICES

CAN USE FINAL RESULT FROM $e\mu$.

$$|T_{fi}|^2 = \frac{8e^4}{q^4}\left((k'\cdot p')(k\cdot p) + (k'\cdot p)(k\cdot p')\right)$$

WITH $q^4 = 4(k\cdot k')^2$

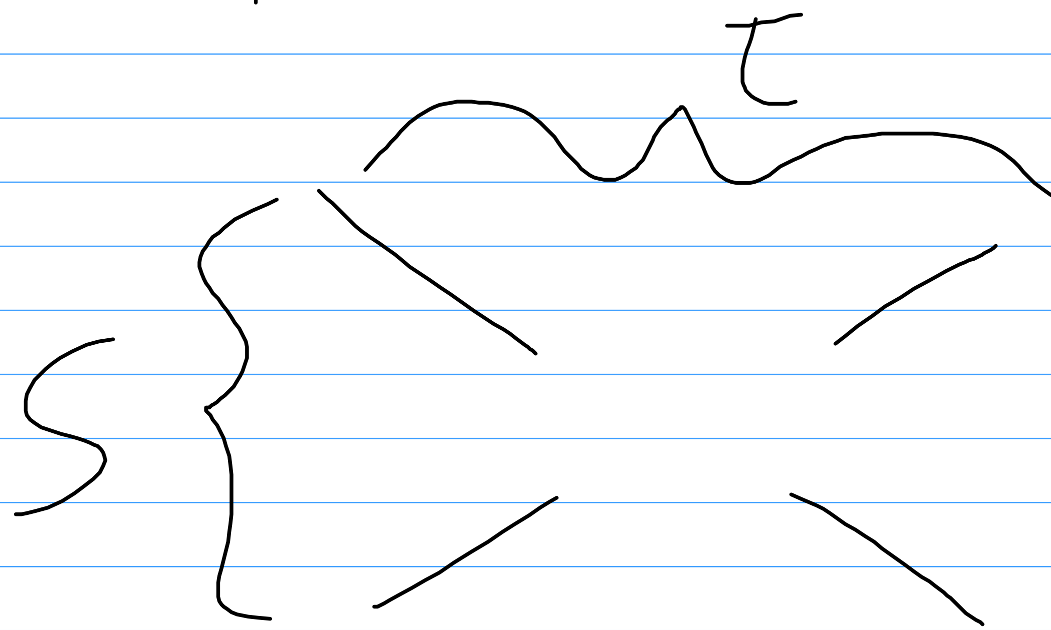
FOR e^+e^-:

$$|T_{fi}|^2 = \frac{8e^4}{4(k\cdot p)^2}\left((p\cdot p')(k\cdot k') + (p\cdot k')(k\cdot p')\right) =$$

$$= \frac{2e^4}{\left(\frac{s}{2}\right)^2}\left(\left(-\frac{t}{2}\right)\left(-\frac{t}{2}\right) + \left(-\frac{u}{2}\right)\left(-\frac{u}{2}\right)\right)$$

$$|T_{p,1}|^2 = \frac{2e^4}{64\pi^2}\left(\frac{t^2 + u^2}{s^2}\right)$$

WITH RESPECT TO $e\mu$, $s \longleftrightarrow t$



TOTAL CROSS-SECTION

DERIVE s, t, u FOR MASSLESS PARTICLES

$$s \sim 2K \cdot P = 4 E_{e^-} E_{e^+}$$

$$t = 4(K \cdot K')^2 = 4\left(E_{e^-} E_{e^+} - \vec{p}_{e^-} \cdot \vec{p}_{e^+}\right)^2 =$$

$$= 4 E_{e^-} E_{e^+} (1 - \cos\theta)^2$$

$$u = 4(K \cdot P')^2 = 4 E_{e^-} E_{\mu^-} (1 + \cos\theta)^2$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \, 2e^4 \, \frac{4\left(\left(E_{e^-}E_{\mu^+}(1-\cos\theta)\right)^2 + \left(E_{e^-}E_{\mu^-}(1+\cos\theta)\right)^2\right)}{16\, E_{e^+} E_{e^-}}$$

CALCULATE IN COM SYSTEM

$$\Rightarrow \quad E_{e^-} = E_{e^+} = E_{\mu^-} = E_{\mu^+}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \, e^4 \, (1+\cos^2\theta)$$

INTEGRATE OVER φ: $d\Omega = 2\pi \, d\cos\theta$

DEFINE $\quad \alpha_{EM} = \dfrac{e^2}{4\pi}$

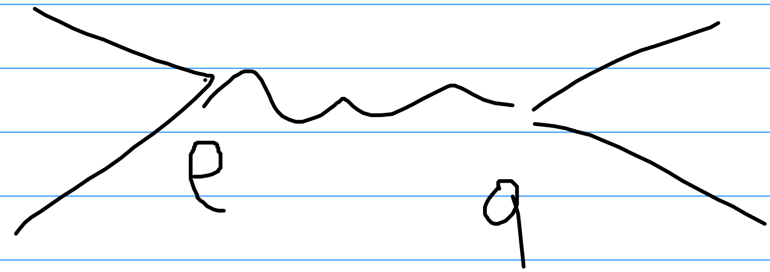
$$\sigma = \int_0^{\pi} \frac{\alpha^2}{4s}\left(1+\cos^2\theta\right) d\cos\theta$$

$$x = \cos\theta$$

$$= \frac{2\pi\alpha^2}{4s}\left[-\cos\theta - \frac{1}{3}\cos^3\theta\right]_0^{\pi} =$$

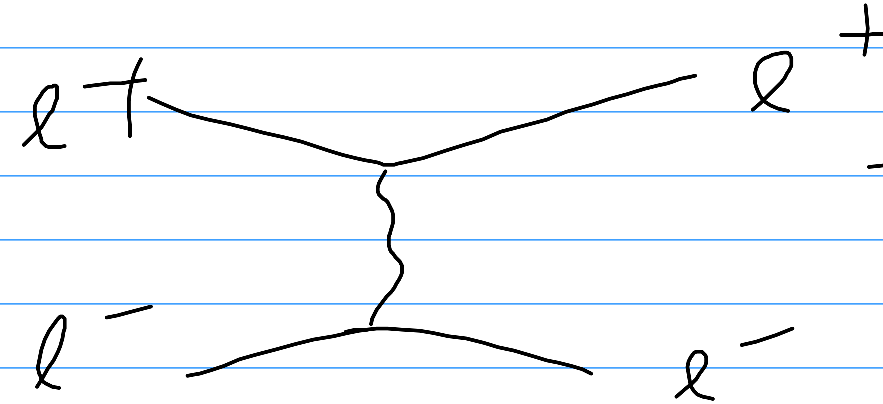
$$= \frac{4\pi\alpha^2}{s}$$

GENERALISATION: $e^+e^- \to X^+X^-$



ALSO VALID FOR PRODUCTION OF τ'S AND QUARKS

FOR FINAL-STATE ELECTRONS, MORE COMPLICATED



T-CHANNEL

$$R = \frac{\sigma(e^+e^- \to \text{HADRONS})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

FOR A GENERATION OF QUARKS.

CHARGES ARE $\frac{2}{3}$, $-\frac{1}{3}$

$$\sigma \propto e^4 = e^2 e^2$$

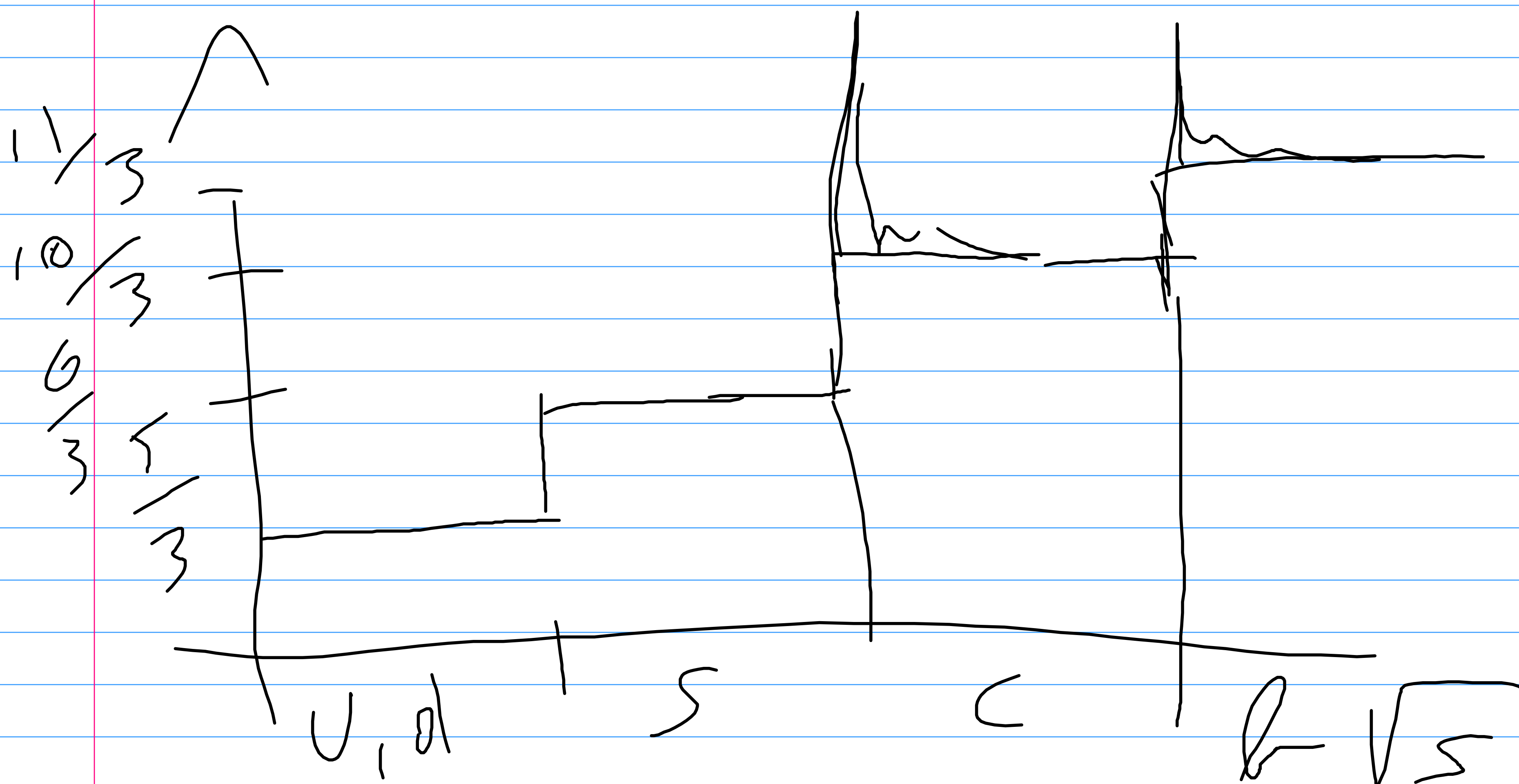
e^2 ↙ INITIAL STATE e^+e^- VERTEX

e^2 → FINAL STATE VERTEX

FOR EACH QUARK GENERATION

$$R = \frac{\left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right)e^2}{e^2} \times 3$$

COLOR FACTOR



HELICITIES

$$u_L = \frac{1}{2}(1-\gamma^5)u$$

$$u_R = \frac{1}{2}(1+\gamma^5)u$$

$$\bar{u}_L = u_L^\dagger \gamma^0 = \frac{1}{2}\ u^\dagger (1-\gamma^5)^\dagger \gamma^0 = \frac{1}{2} u^\dagger (1-\gamma^5)\gamma^0$$

$$= \frac{1}{2} u^\dagger \gamma^0 (1+\gamma^5) = \frac{1}{2}\bar{u}\,(1+\gamma^5)$$

CONSIDER E.M. CURRENT

$$\bar{u}\,\gamma^\mu u = (\bar{u}_L + \bar{u}_R)\,\gamma^\mu\,(u_L + u_R)$$

CAN LEFT-HANDED AND RIGHT-HANDED TERMS MIX?

$$\bar{u}_L \gamma^\mu u_R = \frac{1}{2}\bar{u}\,(1+\gamma^5)\,\gamma^\mu \frac{1}{2}(1+\gamma^5)\,u =$$

$$= \frac{1}{4}\bar{u}\,\gamma^\mu (1-\gamma^5)(1+\gamma^5)\,u =$$

$$= \frac{1}{4}\bar{u}\,\gamma^\mu \left(1-(\gamma^5)^2\right) u = 0$$

$\Rightarrow$ ELECROMAGNETISM CONSERVES HELICITY

$\Rightarrow$ OPPOSITE HELICITY

MASSLESS SPIN-1 PARTICLES

PHOTONS

MAXWELL'S EQUATIONS

$$\begin{cases} \underline{\nabla} \cdot \underline{E} = \rho \\ \underline{\nabla} \cdot \underline{B} = 0 \\ \underline{\nabla} \times \underline{E} = -\dot{\underline{B}} \\ \underline{\nabla} \times \underline{B} = \underline{J} + \dot{\underline{E}} \end{cases}$$

POTENTIAL $\underline{A}$

$$\underline{B} = \underline{\nabla} \times \underline{A}$$

$$\underline{\nabla} \times \underline{E} = -\frac{\partial\, \underline{\nabla} \times A}{\partial t} =$$

$$= -\underline{\nabla} \times \left(\frac{\partial \underline{A}}{\partial t}\right)$$

$$\Rightarrow \underline{\nabla} \times \left(E + \frac{\partial \underline{A}}{\partial t}\right) = 0$$

SOLUTION

$$\underline{E} + \frac{\partial \underline{A}}{\partial t} = -\underline{\nabla}\phi$$

BECAUSE GRADIENT OF SCALAR HAS ZERO CURL.

COMBINE WITH FIRST EQ. $(\underline{\nabla} \cdot \underline{E} = \rho)$

$$-\nabla^2 \phi - \frac{\partial (\underline{\nabla} \cdot \underline{A})}{\partial t} = \rho$$

FROM 4TH EQ.

$$\underline{\nabla} \times \underline{B} = \underline{J} + \frac{\partial E}{\partial t} \qquad \text{BUT } \underline{B} = \underline{\nabla} \times \underline{A}$$

$$\underline{\nabla} \times \underline{\nabla} \times \underline{A} = \underline{J} + \frac{\partial E}{\partial t} \qquad \text{But } \underline{E} = -\nabla\phi - \frac{\partial A}{\partial t}$$

$$= \underline{J} + \frac{\partial}{\partial t}\left(-\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t}\right) = \underline{J} - \underline{\nabla}\frac{\partial \phi}{\partial t} - \frac{\partial^2 \underline{A}}{\partial t^2}$$

$$\underline{\nabla} \times \underline{\nabla} \times = \underline{\nabla} \cdot (\underline{\nabla} \cdot \underline{A}) - \nabla^2 \underline{A}$$

SO

$$\nabla^2 \underline{A} - \frac{\partial^2 \underline{A}}{\partial t^2} - \underline{\nabla}(\underline{\nabla} \cdot \underline{A}) - \frac{\partial \nabla \phi}{\partial t} = \underline{J}$$

RECALL: FOR SCALAR POTENTIAL

$$\nabla^2 \phi - \frac{\partial^2 \phi}{\partial t} + \frac{\partial}{\partial t} \underline{\nabla} \cdot \underline{A} = -\rho$$

DEFINE

$$\Box^2 = \left(\frac{\partial^2}{\partial t^2} - \nabla^2 \right)$$

CURRENT $\quad J^\mu = (\rho, \underline{J})$

POTENTIAL $\quad A^\mu = (\phi, \underline{A})$

$$\Rightarrow \quad \Box^2 A^\mu - \partial^\mu \partial_\nu A^\nu = J^\mu$$

PHOTONS TRAVELLING ALONG Z

4 DIFFERENT POLARISATIONS

$|1,0,0,0\rangle$ TIME-LIKE

$|0,1,0,0\rangle$ X-POLARISED

$|0,0,1,0\rangle$ Y "

$|0,0,0,1\rangle$ Z "

CONSIDER LORENTZ GAUGE

$$\partial_\mu A^\mu = 0$$

MAXWELL'S EQUATIONS SIMPLIFY TO

$$\Box^2 A^\mu = J^\mu$$

FOR FREE PHOTONS $J^\mu = 0$

$$\Box^2 A^\mu = 0$$

SOLUTIONS

$$A^\mu = \varepsilon^\mu_i \, e^{-iq\cdot x}$$

APPLY LORENTZ CONDITION ON THE SOLUTIONS

$$\partial_\mu A^\mu = \partial_\mu \, \xi_i^\mu \, e^{-i q_\mu x^\mu} = 0$$

$$\Rightarrow -i q_\mu \, \xi_i^\mu \, e^{-i q_\mu x^\mu} = 0$$

$$\Rightarrow q_\mu \, \xi_i^\mu = 0$$

$$q_0 \, \xi_i^0 = q_K \, \xi_i^K$$

E AND B FIELDS DO NOT CHANGE

FOR
$$A'^{\mu} \longrightarrow A^{\mu} + \partial^{\mu} \Lambda$$

$\Lambda \to$ SCALAR FIELD

WE CAN USE
$$\Lambda = i a e^{-iq_{\mu} x^{\mu}}$$

LORENTZ CONDITION $\Longrightarrow$ $\partial_{\mu} \partial^{\mu} \Lambda = (-iq_{\mu})(-iq^{\mu}) i a e^{iqx}$

$$= -i a q^2 e^{iqx} = 0$$

BUT $q^2 = E^2 - p^2$ SATISFIED BY REAL PHOTONS

RECALL $A^\mu = \varepsilon_i^\mu e^{-iqx}$

$$A'^\mu = A^\mu + \partial^\mu \Lambda = \varepsilon_i^\mu e^{-iqx} + (-iq^\mu) i a e^{-iqx}$$

SO FOR SPINORS, GAUGE TRANSFORM MEANS

$$\varepsilon'^\mu = \varepsilon^\mu + a q^\mu$$

SAME PHOTON DESCRIBED BY SPINORS

THAT DIFFER BY A MULTIPLE OF q^μ

SO $\epsilon_0 = 0$

LORENTZ CONDITION BECOMES

$$\underline{\epsilon} \cdot \underline{q} = 0$$

$\underline{\epsilon}$ VECTORS OF POLARISATIONS ARE PERPENDICULAR TO PHOTON DIRECTION

COMBINATION OF

$$\varepsilon_1 = (0, 1, 0, 0)$$

$$\varepsilon_2 = (0, 0, 1, 0)$$

OR

$$\varepsilon_R = \frac{1}{\sqrt{2}} (\varepsilon_1 + i \varepsilon_2)$$

$$\varepsilon_L = \frac{1}{\sqrt{2}} (\varepsilon_1 - i \varepsilon_2)$$

FOR REAL PHOTONS

PROPAGATORS

$$\Box^2 A^\mu = J^\mu = g^{\mu\nu} J_\nu$$

WE HAVE SEEN THAT A SOLUTION IS

$$A^\mu = -\frac{g^{\mu\nu}}{q^2} J_\nu$$

IN A PROPAGATOR APPROACH:

$$A^\mu(x') = \int G(x', x) J^\mu(x) \, d^4x$$

$$\Box^2 A^\mu(x') = \int \Box^2 G(x', x) J^\mu(x) \, d^4x = J^\mu$$

$$\Box^2 A^\mu(x') = J^\mu(x') = \int d^4x\, \delta(x-x')\, J^\mu(x)$$

COMPARING ABOVE EXPRESSIONS

$$\Rightarrow \Box^2 G(x,x') = \delta(x-x')$$

MAKE FOURIER TRANSFORM

$$\frac{1}{(2\pi)^4}\int d^4q\, (-iq)^2\, G(q)\, e^{-iq(x-x')} = \frac{1}{(2\pi)^4}\int d^4q\, e^{-iq(x-x')}$$

$$\Rightarrow G(q) = -\frac{1}{q^2}$$

SO $$A^\mu(x) = -\frac{J^\mu(x)}{q^2} = -\frac{g^{\mu\nu} J_\nu(x)}{q^2}$$

PROPAGATOR IS INVERSE OF EXPRESSION DESCRIBING PARTICLE IN FREE SPACE

FOR KLEIN-GORDON EQ.

$$i(\Box^2 + m^2)\phi = -iV\phi$$

PROPAGATOR IS $$\frac{1}{i(\Box^2+m^2)} = \frac{-i}{\Box^2+m^2}$$

$$\square^2 = \partial_\mu \partial^\mu = \frac{i\partial_\mu \, i\partial^\mu}{i^2} = -p^2$$

SO PROPAGATOR FOR KLEIN-G.

IS

$$\frac{i}{p^2 - m^2}$$

PROPAGATOR FOR DIRAC PARTICLES

$$\left[(\alpha p + \beta m) + e\left(\alpha A - A^0 \mathbb{1}\right)\right]\psi = E\psi$$

MULTIPLY BY β

$$\left[\beta\alpha p + \beta^2 m + e\left(\beta\alpha A - \beta A^0 \mathbb{1}\right)\right]\psi = \beta E\psi$$

$$\left(\beta E - \beta\alpha p - \beta^2 m\right)\psi = e\left(\beta\alpha A - \beta A^0\right)\psi$$

$$\left(\gamma^0 E - \gamma^k p_k - \mathbb{1} m\right)\psi = -e\left(\beta A^0 - \gamma^k A_k\right)\psi$$

$$(\not{p} - m)\psi = -e\not{A}\psi$$

$$-i(\not{p}-m)\psi = ie\not{A}\psi = -iV\psi$$

PROPAGATOR IS

$$\frac{1}{-i(\not{p}-m)} = \frac{i(\not{p}+m)}{(\not{p}-m)(\not{p}+m)} = \frac{i(\not{p}+m)}{p^2-m^2} =$$

COMPLETENESS RELATION

$$= \frac{i\sum u\bar{u}}{p^2-m^2}$$

PHOTON EXCHANGE BETWEEN E.M. CURRENTS

$$J_\mu^A(x)\left(\frac{-g^{\mu\nu}}{q^2}\right)J_\nu^B(x) = -J_\mu^A(x)\frac{1}{q^2}J^{B\mu}(x)$$

$$= -\frac{1}{q^2}\left(J_1^A J^{1B} + J_2^A J^{2B} + J_3^A J^{3B} - J_0^A J^{0B}\right)$$

$$J_\mu = \bar{u}_f\, e^{ip_f x}\, \gamma^\mu\, u_i\, e^{-ip_i\cdot x}$$

$$q = p_f - p_i$$

SO $\partial_\mu J^\mu = q_\mu J^\mu$

SINCE E.M. CURRENT IS CONSERVED

$\partial_\mu J^\mu = 0 \Rightarrow q_\mu J^\mu = 0$

DEFINE x_3 AXIS PARALLEL TO q

$q_1 = q_2 = 0$, $q_3 J^1 = q_3 J^2 = 0$

WE ARE LEFT WITH

$$q_\mu J^\mu = q_0 J^0 - q_3 J^3$$

$$J^3 = q_0 J^0 / q^3$$

AMPLITUDE

$$\frac{1}{q_3^2 \, q^2} q_0^2 J_0^A J^{0B} = \frac{1}{q^2} J_0^A J^{0B} = \frac{1}{q^2} J_0^A(k) J^{0B}(k) \left(\frac{q_0^2 - q^2}{q_3^2} \right)$$

BUT $q^2 = q_0^2 - q_3^2$

SO AMPLITUDE IS

$$\frac{J_{\mu}^{A}(k)\, J^{\mu B}(k)}{|q|^2}$$

IF WE TAKE FOURIER TRANSFORM

$\longrightarrow$ COULOMB'S LAW