MASSIVE SPIN-1 PARTICLES

FOR MASSLESS PHOTONS

$$\Box^2 A^\mu - \partial^\mu \partial_\nu A^\nu = J^\mu$$

$$(E^2 + p^2) A^\mu - \partial^\mu \partial_\nu A^\nu = 0$$

FOR MASSIVE PARTICLES, $E^2 = p^2 + m^2$

ADD BY HAND AN m^2 TERM: PROCA

$$\Box^2 A^\mu + m^2 A^\mu - \partial^\mu \partial_\nu A^\nu = \begin{cases} 0 & \text{REAL} \\ J^\mu & \text{VIRTUAL} \end{cases}$$

DIFFERENTATE WRT ∂_μ

$$\partial_\mu \Box^2 A^\mu + m^2 \partial_\mu A^\mu - \partial_\mu \partial^\mu \partial_\nu A^\nu = \begin{cases} 0 \\ \partial_\mu \partial^\mu \end{cases}$$

$$\partial_\mu \cancel{\partial_\mu \partial^\mu} A^\mu + m^2 \partial_\mu A^\mu - \cancel{\partial_\mu \partial^\mu \partial_\nu} A^\nu = \begin{cases} 0 \\ \partial_\mu J^\mu \end{cases}$$

FOR REAL PARTICLES, THE FREE PROCA EQUATION SATISFIES LORENTZ GAUGE CONDITION

THE SOLUTION, AS FOR PHOTON, IS

$$A^\mu = \varepsilon^\mu e^{-ip_\mu x^\mu}$$ WITH ε^μ POLARIS. VECT.

LORENTZ CONDITION:

$\partial_\mu A^\mu = 0 \implies i p_\mu \varepsilon^\mu = 0$

DEALING WITH MASSIVE PARTICLE

$\implies$ CAN GO TO ITS REST FRAME

THERE

$$p^\mu = (m, 0, 0, 0)$$

SPIN VECTORS

$$\varepsilon_1 = (0, 1, 0, 0)$$
$$\varepsilon_2 = (0, 0, 1, 0)$$
$$\varepsilon_3 = (0, 0, 0, 1)$$

SATISFY COND.

$$p_\mu \varepsilon^\mu = 0$$

BOOST PARTICLE ALONG Z. $\varepsilon_1, \varepsilon_2$ REMAIN SAME

THE 4-VECTOR BECOMES $(E, 0, 0, -P_z)$

REQUIRING LORENTZ CONDITION

$$(P_z, 0, 0, E) \cdot \frac{1}{m}(E, 0, 0, -P_z) = 0$$

SO, $\varepsilon_3 = \frac{1}{m}(P_z, 0, 0, E)$ SATISFIES LORENTZ COND.

CONSIDER COMPLETENESS RELATIONS

$$\sum_i \varepsilon_i \varepsilon_i^* = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}(0,1,0,0) + \begin{pmatrix}0\\0\\1\\0\end{pmatrix}(0,0,1,0) +$$

$$+\frac{1}{m^2}\begin{pmatrix}p_z\\0\\0\\E\end{pmatrix}(p_z,0,0,E) = \begin{pmatrix} p_z^2/m^2 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & E^2/m^2 \end{pmatrix}$$

$$= -g^{\mu\nu} + \frac{p^\mu p^\nu}{m}$$

TERM 00

$$-g^{00} + \frac{p^0 p^0}{m^2} = -1 + \frac{E^2}{m^2} = \frac{E^2 - m^2}{m^2} = p_z^2 / m^2$$

IT WORKS

TERM 33

$$-g^{33} + \frac{p^3 p^3}{m^2} = 1 + \frac{p_z^2}{m^2} = \frac{p_z^2 + m^2}{m^2} = \frac{E^2}{m^2}$$

IT WORKS AS WELL . . .

FOR VIRTUAL PHOTONS

$$q^\mu = (\nu, 0, 0, q_z)$$

WHERE ν IS <u>NOT</u> E

$$q^2 = q_\mu q^\mu = \nu^2 - q_z^2$$

$$q_z^2 = \nu^2 - q^2 = \nu^2 + Q^2$$

$$Q^2 = -q^2$$

$$q^\mu_z = (\nu, 0, 0, \sqrt{\nu^2 + Q^2})$$

FOR VIRTUAL PHOTONS

$$\left\{ \begin{array}{l} \varepsilon_1 = (0, 1, 0, 0) \\ \varepsilon_2 = (0, 0, 1, 0) \\ \varepsilon_3 = \dfrac{(\sqrt{\nu^2 + \hat{Q}}, 0, 0, \nu)}{Q^2} \end{array} \right.$$

PROPAGATOR FOR MASSIVE VECTOR BOSON

$$(\Box^2+m^2)A^\mu-\partial^\mu\partial_\nu A^\nu=J^\mu$$

WE HAVE SEEN THAT $\partial_\mu\Box^2 A^\mu=\partial_\mu\partial^\mu\partial_\nu A^\nu$

$$\Longrightarrow \quad m^2\partial_\nu A^\nu=\partial_\mu J^\mu$$

$$\partial_\nu A^\nu=\frac{1}{m^2}\partial_\mu J^\mu$$

BACK TO PROCA

$$(\Box^2+m^2)A^\mu-\frac{1}{m^2}\partial^\mu\partial_\mu J^\mu=J^\mu$$

$$(\Box^2+m^2)A^\mu=\frac{1}{m^2}\partial^\mu\partial_\nu J^\nu+g^{\mu\nu}J_\nu=\frac{1}{m^2}\partial^\mu\partial^\nu J_\nu+g^{\mu\nu}J_\nu$$

USING $\partial^\nu J_\nu = -iq J_\nu$

$$(\Box^2 + m^2) A^\mu = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m^2}\right) J_\nu$$

SO PROPAGATOR IS

$$\frac{g^{\mu\nu} - \frac{q^\mu q^\nu}{m^2}}{-q^2 + m^2}$$

VALID FOR A PARTICLE EXCHANGED IN AN INTERACTION

FOR A REAL PARTICLE THAT DECAYS (e.g. Z BOSON)

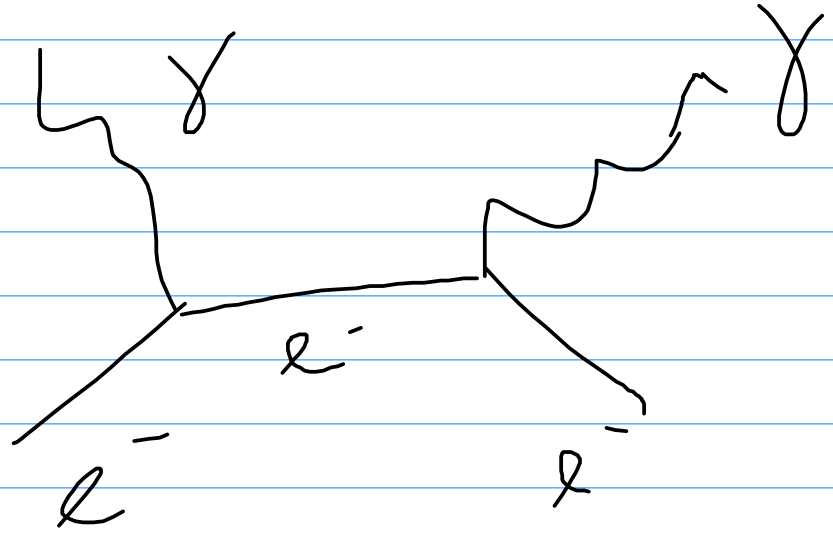
$$\psi = e^{-iMt} e^{-\frac{\Gamma t}{2}} \quad \text{AT REST}$$

$$|\psi|^2 = \psi^* \psi = e^{-\Gamma t}$$

SO IN PROPAGATOR, REPLACE $-iM$ WITH

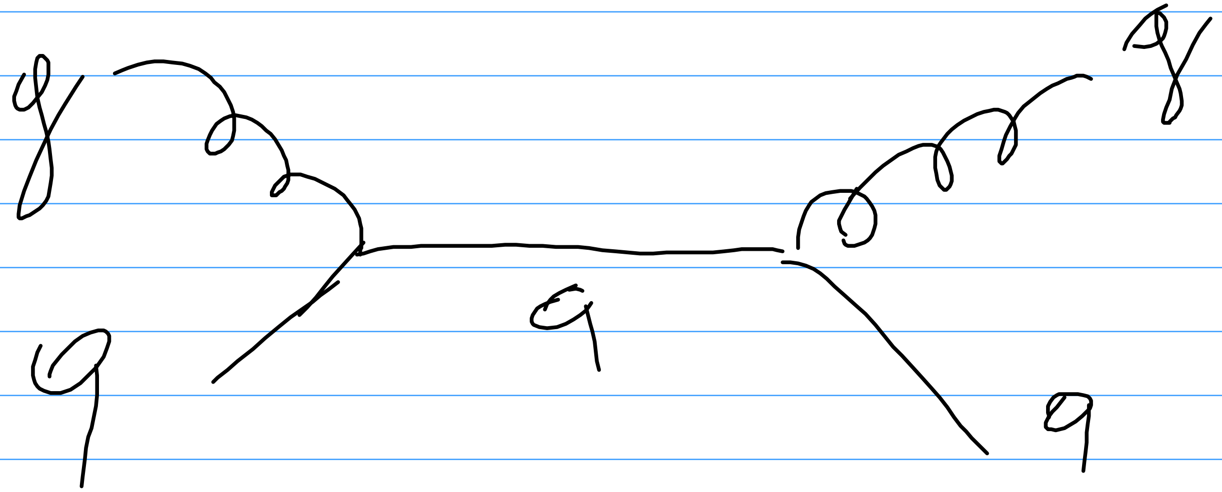
$$-iM - \Gamma/2 \longrightarrow \frac{-g^{\mu\nu} + q^\mu q^\nu/m^2}{q^2 - (m - \frac{i\Gamma}{2})^2} \simeq \frac{-g^{\mu\nu} + q^\mu q^\nu/m^2}{q^2 - m^2 - im\Gamma}$$

COMPTON SCATTERING

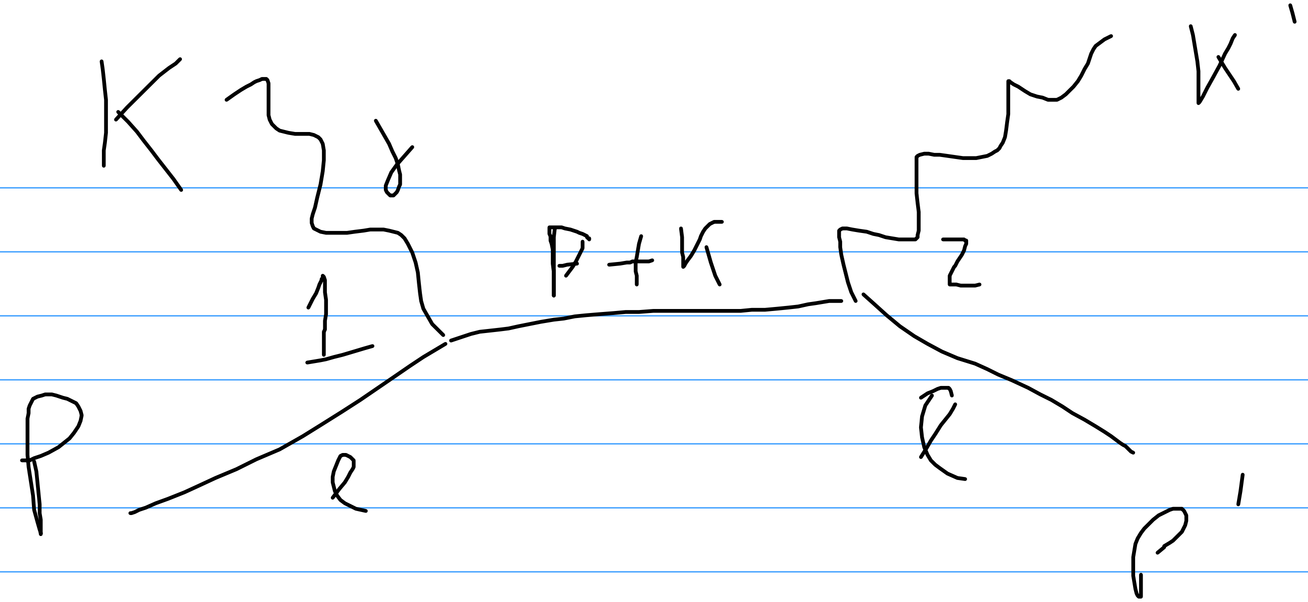


SIMILAR

SIMILAR DIAGRAM FOR QCD



COMPTON QCD



$$s = (K+P)^2$$

$$t = (K-K')^2$$

$$u = (K-p')^2$$

THERE ARE 2 VERTICES. PROPAGATOR APPROACH.

$$T_{fi} = -i \int d^4x_1 \int d^4x_2\, \phi^*(2) V(2)\, G_0(2,1)\, V(1)\, \phi(1) =$$

$$= -i \int d^4x_1 \int d^4x_2\, e\, \bar{u}\, e^{ip'x_2}\, e^{ik'x_2}\, \varepsilon^*_\nu \gamma^\nu \frac{1}{(2\pi)^4} e^{-i(p+k)(x_2-x_1)}\; i\, \frac{\not{p}+\not{k}+m}{(p+k)^2-m^2}$$

$$(-e)\, \varepsilon_\mu\, e^{-iKx_2}\, \gamma^\mu\, u\, e^{\cdot ipx_1}$$

INTEGRATING, AND REPLACING THE δ'S

$$T_{fi} = i\,\bar{u}(p')(-e)\,\epsilon^{*}_{\nu}\gamma^{\nu}\left(\frac{\cancel{p}+\cancel{k}+m}{(p+k)^2-m^2}\right)(-e)\,\epsilon_{\mu}\gamma^{\mu}u(p)$$

NEGLECTING m

$$= -i\frac{e^2}{s}\,\bar{u}(p')\,\epsilon^{*}_{\nu}\gamma^{\nu}(\cancel{p}+\cancel{k})\,\epsilon_{\mu}\gamma^{\mu}u(p)$$

$$|T_{fi}|^2 = \frac{e^4}{s^2}\Big(\epsilon^{*}_{\mu'}\epsilon_{\nu'}\epsilon^{*}_{\nu}\epsilon_{\mu}\,\bar{u}(p)\gamma^{\mu'}(\cancel{p}+\cancel{k})\gamma^{\nu'}u(p')\bar{u}(p')\gamma^{\nu}$$
$$(\cancel{p}+\cancel{k})\gamma^{\mu}u(p)\Big)$$

AVERAGE OVER INITIAL AND FINAL SPINS

$$\sum \epsilon^*_\mu \epsilon_{\mu'} = -g_{\mu\mu'}$$

FOR ELECTRONS USE COMPLETENESS RELATIONS

$$|T_{fi}|^2 = \frac{e^4}{4s^2} g_{\mu\mu'} g_{\nu\nu'} \mathrm{Tr}\left[(\not{p}'+m)\gamma^{\mu'}(\not{p}+\not{k})\gamma^{\nu'}(\not{p}+m)\gamma^{\nu}(\not{p}+\not{k})\gamma^{\mu}\right]$$

$$= \frac{e^4}{4s^2} \mathrm{Tr}\left[\gamma^\mu \not{p}' \gamma_\mu (\not{p}+\not{k}) \gamma_\nu \not{p} \gamma^\nu (\not{p}+\not{k})\right] =$$

$$= \frac{e^4}{4s^2} \mathrm{Tr}\left[\not{p}'(\not{p}+\not{k})\not{p}(\not{p}+\not{k})\right]$$

TERMS LIKE $\not{p}\not{p}$ OR $\not{p}'\not{p}' = m^2$

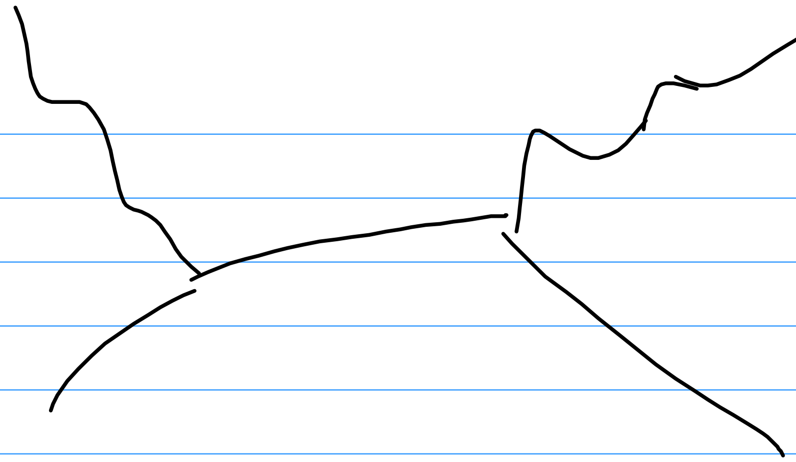
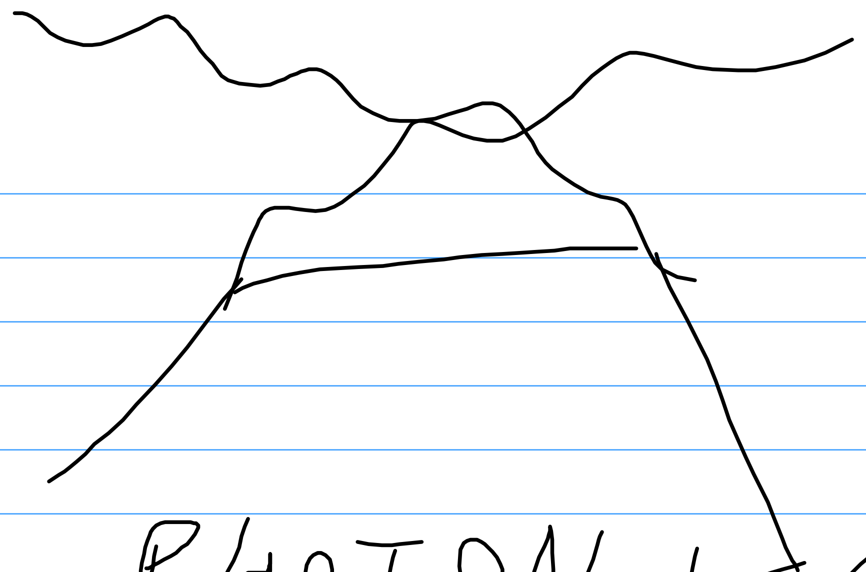
NEGLECTED

$$|T_{fi}|^2 = \frac{e^4}{s^2} \mathrm{Tr}\left[\not{p}'\not{k}\not{p}\not{k}\right] =$$

$$= \frac{e^4}{s^2}\left((p'\cdot k)(p\cdot k) - (p'\cdot p)(k\cdot k) + (p'\cdot k)(k\cdot p)\right)$$

$$k\cdot k = m_\gamma^2 = 0$$

$$= \frac{2e^4}{s^2}\left(2(p'\cdot k)2(p\cdot k)\right) = \frac{2e^4}{s^2}(-us) = -2e^4\left(\frac{u}{s}\right)$$

$|T_{fi}|^2 = \Big|$  $+$  $\Big|^2$

PHOTON LEGS ARE SWAPPED

$$|T_{fi}|^2 = -2e^4\left(\frac{s}{u}\right)$$

$$T_{fi} = -i \frac{e^2}{u} \bar{u}(p') \varepsilon_\mu \gamma^\mu (\not{p} - \not{k}) \varepsilon^*_\nu \gamma^\nu u(p)$$

$$|T^{I}_{fi} + T^{II}_{fi}|^2 = |T^{I}_{fi}|^2 + |T^{II}_{fi}|^2 + T^{*I}_{fi} T^{II}_{fi} + T^{*II}_{fi} T^{I}_{fi}$$

$$T^{*II}_{fi} T^{I}_{fi} = T^{+II}_{fi} T^{I}_{fi} = \frac{e^2}{u} \varepsilon_\nu \varepsilon^*_{\mu'} \bar{u}(p) \gamma^{\nu'} (\not{p} - \not{k}') \gamma^{\mu'} u(p') \times$$

$$\times \frac{e^2}{s} \varepsilon^*_\nu \varepsilon_\mu \bar{u}(p') \gamma^\nu (\not{p} + \not{k}) \gamma^\mu u(p)$$

$$= \frac{e^4}{5U} \frac{1}{4} g_{\mu\mu'} g_{\nu\nu'} \sum_{\substack{\text{SPIN} \\ \text{ELECTRON}}} \bar{U}(p) \gamma^{\nu'} (\cancel{P} - \cancel{K}) \gamma^{\mu'} U(p') \bar{U}(p') \gamma^{\nu} (\cancel{P} + \cancel{K}) \gamma^{\mu} U(p)$$

$$= \frac{e^4}{4U5} T_n \left[\cancel{P} \gamma_\nu (\cancel{P} - \cancel{K}') \gamma_\mu \cancel{P}' \gamma^\nu (\cancel{P} + \cancel{K}) \gamma^\mu \right]$$

USING $\gamma_\mu \cancel{a} \cancel{b} \cancel{c} \gamma^\mu = -2 \cancel{c} \cancel{b} \cancel{a}$

AND CYCLICAL PROPERTY OF TRACES

$$= \frac{e^4}{4U5} T_n \left[-2 (\cancel{P} - \cancel{K}') \gamma_\nu \cancel{P} \cancel{P}' \gamma^\nu (\cancel{P} + \cancel{K}) \right] = \text{AGAIN}$$

$$= \frac{e^4}{4U5} T_n \left[\cancel{P} \cancel{P}' 4 (P + K) (P - K') \right] =$$

$$-\frac{2e^4}{us}(p\cdot p')(p+k)(p-k') =$$

$$= \frac{4e^4 t}{su}\left(p\cdot p + k\cdot p - p\cdot k' - k\cdot k'\right) =$$

$$= \frac{4e^4 t}{su}\left(\cancel{m^2} + \frac{s}{2} + \frac{u}{2} + \frac{t}{2}\right) = \frac{2e^4 t}{us}(s+t+u)$$

$$s+t+u = 2(m_e^2 + m_\gamma^2)$$

FOR REAL PHOTONS, $m_\gamma^2 = 0$ INTERF. NEGLIGIBLE

FOR VIRTUAL PHOTONS,

$$s + t + u = Q^2$$

ZERO FOR REAL γs

SO IN GENERAL

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} 2e^4 \left(\frac{-u}{s} - \frac{s}{u} + \frac{2tQ^2}{su} \right)$$

LIMIT FOR $s \gg t, u, Q^2$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} 2e^4 \left(\frac{-u}{s} - \frac{s}{u} \right) =$$

$$= \frac{2e^4}{64\pi^2} \left(-\frac{1}{u} \right) \qquad u \simeq -2p \cdot k'$$

$$\frac{d\sigma}{d\Omega} = \frac{\cancel{2} e^4}{64\pi^2} \frac{1}{\cancel{2} p \cdot k'} = \frac{e^4}{64\pi^2 \, p \cdot k'}$$

IN COM SYSTEM

$$P \cdot K' = P_e E_\gamma \left(1 + \cos\theta + \frac{1}{2} \frac{m_e^2}{2 P_e^2} + \cdots \right)$$

USING

$$E_e = \sqrt{P_e^2 + m_e^2} = P_e \sqrt{1 + \frac{m_e^2}{P_e^2}} \simeq$$

$$\simeq P_e \left(1 + \frac{m_e^2}{2 P_e^2} + \cdots \right)$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{(4\pi)^2} \frac{e^4}{\frac{s}{4} \left(1 + \cos\theta + \frac{2 m_e^2}{s} \right)}$$

INTEGRATE OVER φ

$$d\sigma = \frac{2\pi\alpha^2}{s} \frac{d\cos\theta}{1+\cos\theta + \frac{2m_e^2}{s}}$$

$$l = 1+\cos\theta + 2m_e^2/s$$

$$\sigma = \frac{2\pi\alpha^2}{s}\int \frac{dl}{l} = \frac{2\pi\alpha^2}{s}\Big[1+\cos\theta + 2m_e^2/s\Big]_{-1}^{1}$$

$$= \frac{2\pi\alpha^2}{s} \ln\left(\frac{2\left(1+\frac{m_e^2}{s}\right)}{2\,\frac{m_e^2}{s}}\right) =$$

$$= \frac{2\pi\alpha^2}{s} \ln\left(\frac{s}{m_e^2}\right)$$