WEAK INTERACTIONS

EXAMPLE: β DECAY $n \to p e^- \bar{\nu}_e$

CHARGED CURRENTS, SO OF THE FORM: $\bar{u}_p \gamma^\mu u_n$

MATRIX ELEMENT

$$M = A \, j^\mu_{pn} \, g_{\mu\nu} \, j^\nu_{e\bar{\nu}_e}$$

$$j^\mu_{pn} = \bar{u}_p \gamma^\mu u_n$$

$$j^\nu_{e\nu} = \bar{u}_e \gamma^\nu u_{\bar{\nu}_e}$$

$$\mathcal{L}_{FERMI} = A\,\bar{\psi}_p(x)\gamma^\mu\psi_n(x)\,\bar{\psi}_e(x)\gamma_\mu\psi_{\bar{\nu}_e}(x)$$

$$+ A\,\bar{\psi}_n(x)\gamma^\mu\psi_p(x)\,\bar{\psi}_{\nu_e}(x)\gamma_\mu\psi_e(x)$$

SOLUTIONS OF DIRAC'S EQUATION

$$u_p = N\begin{pmatrix} 1 \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \end{pmatrix}\phi_p$$

FOR PARTICLES AT REST, $m \gg p$

$$u_p \to N\begin{pmatrix} 1 \\ 0 \end{pmatrix}\phi_p$$

EVALUATE CURRENT IN NON-RELATIVISTIC LIMIT

$$\bar{u}_p \gamma^\mu u_n = u_p^\dagger \gamma_0 \gamma^\mu u_n = u_p^\dagger \beta (\beta, \beta \underline{\alpha}) u_n =$$

$$= u_p^\dagger (1, \underline{\alpha}) u_n = u_p^\dagger \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & \underline{\sigma} \\ \underline{\sigma} & 0 \end{pmatrix} \right) u_n$$

$$= \left(u_p^\dagger \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} u_n, \; u_p^\dagger \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix} u_n \right) = \text{NON-REL LIMIT}$$

$$= N^2 \left((\phi_p, 0) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \phi_n \\ 0 \end{pmatrix}, (\phi_p^\dagger \; 0) \begin{pmatrix} 0 & \underline{\sigma} \\ \underline{\sigma} & 0 \end{pmatrix} \begin{pmatrix} \phi_n \\ 0 \end{pmatrix} \right)$$

$$= N^2 \left((\phi_p^+ \; 0) \begin{pmatrix} \phi_n \\ 0 \end{pmatrix}, (\phi_p^+ \; \underline{0}) \begin{pmatrix} 0 \\ \underline{0} \; \phi_n \end{pmatrix} \right) =$$

$$= N^2 \left(\phi_p^+ \phi_n , \underline{0} \right)$$

$\longrightarrow$ THIS IS ALSO ZERO IF SPIN IS DIFFERENT

HOWEVER, SOME DECAYS WERE OBSERVED WITH SPIN FLIP $\Delta J = \pm 1$

MORE COMPLETE LAGRANGIAN (GAMOW-TELLER)

$$\mathcal{L}_{GT}^{\Delta J=0} \propto \bar{\psi}_p(x)\,\psi_n(x)\,\bar{\psi}_e(x)\,\psi_{\nu_e}(x) +$$

$$\mathcal{L}_{GT}^{\Delta J=1} \propto \bar{\psi}_p(x)\,\sigma^{\mu\nu}\,\psi_n(x)\,\bar{\psi}_e(x)\,\sigma_{\mu\nu}\,\psi_{\nu_e}(x)$$

$$\sigma^{\mu\nu} = \frac{i}{2}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)$$

TO ACCOUNT FOR SPIN-FLIP

PARITY VIOLATION

- 1957 WU USED ^{60}Co $(J=5)$ ALIGNED WITH EXTERNAL B-FIELD

$$^{60}Co \rightarrow {}^{60}Ni\ (J=4)\ e^-\ \bar{\nu}_e$$

DIRECTION OF OUTGOING ELECTRONS WRT $\langle J \rangle$

$$I(\theta) = 1 - \langle \bar{J} \rangle \cdot \frac{\bar{P}}{E} = 1 - P\nu \cos\theta$$

$\bar{P}$ ELECTRON MOMENTUM

E " ENERGY

P POLARISATION

A PARITY TRANSFORMATION SWAPS $\underline{P} \leftrightarrow -\underline{P}$ BUT LEAVES $\underline{J}$ UNCHANGED

APPLY PARITY TO PREVIOUS RESULT

$$I(\theta) = 1 + \langle \bar{J} \rangle \bar{P}/E = 1 + PN \cos\theta$$

SO, WEAK INTERACTIONS VIOLATE PARITY, SINCE RESULT IS A SUM OF A PARITY-EVEN TERM (1) AND PARITY ODD ($\bar{J} \cdot \bar{P}/E$)

TO ACCOMMODATE THAT, THE CURRENT MUST CONTAIN A VECTOR AND AXIAL COMPONENT

$$\bar{u}\gamma^{\mu}u \longrightarrow \bar{u}\gamma^{\mu}(1-r\gamma^{5})u$$

$r = 1$ FOR THE $e\nu$ CURRENT

$r \sim 1.2$ FOR NUCLEAR CURRENT

DEVIATION FROM 1 DUE TO QUARK MIXING; IN REALITY, $R=1$ FOR QUARKS AS WELL

FOR LEPTONIC PROCESSES LIKE MUON DECAYS

$$\mathcal{M} = j^{\mu}_{\nu_\mu \mu} \, 2\sqrt{2}\, G_F \, g_{\mu\nu} \, j^{\nu}_{e \nu_e}$$

CURRENTS

LEFT-HANDED

$$\begin{cases} j^{\mu}_{\nu_\mu \mu} = \bar{u}_{\nu_\mu} \gamma^{\mu} \left(\frac{1-\gamma^5}{2}\right) u_{\mu} \\ j^{\nu}_{e, \nu_e} = \bar{u}_{e} \gamma^{\nu} \left(\frac{1-\gamma^5}{2}\right) v_{\bar{\nu}_e} \end{cases}$$

WEAK INTERACTIONS MEDIATED BY W'S

$$\mathcal{M} = i \, j^{\mu}_{\nu_\mu \mu} \left(\frac{-i g_W}{\sqrt{2}} \right) \left(\frac{-i(g_{\mu\nu} - p_\mu p_\nu / m_W^2)}{p^2 - m_W^2} \right) \left(\frac{-i g_W}{\sqrt{2}} \right) j^{\nu}_{e \nu_e}$$

IN NON-RELATIVISTIC LIMIT $m_W^2 >> p^2$

$$\mathcal{M} \rightarrow j^{\mu}_{\nu_\mu \mu} \left(\frac{g_W^2}{2 m_W^2} \right) g_{\mu\nu} \, j^{\nu}_{e\nu}$$

$$\Rightarrow \frac{g_W^2}{2 m_W^2} = 2\sqrt{2} \, G_F$$

LEFT-HANDED CURRENTS

$$P_L = \frac{1-\gamma^5}{2} \qquad\qquad u = \left(\frac{1-\gamma^5}{2}\right)u + \left(\frac{1+\gamma^5}{2}\right)u =$$

$$P_R = \frac{1+\gamma^5}{2} \qquad\qquad = u_L + u_R$$

$$P_{L/R}\, u = \frac{1}{2}(1 \mp \gamma^5)\, u_L + \frac{1}{2}(1 \mp \gamma^5)\, u_R =$$

$$= \frac{1}{2}(1 \mp \gamma^5)\left(\frac{1-\gamma^5}{2}\right)u + \frac{1}{2}(1 \mp \gamma^5)\left(\frac{1+\gamma^5}{2}\right)u =$$

$$= \frac{1}{4}\left(1-\gamma^5 \mp \gamma^5(1-\gamma^5)\right)u + \frac{1}{4}\left(1+\gamma^5 \mp \gamma^5(1+\gamma^5)\right)u$$

$$\frac{1}{4}\left(1-\gamma^5 \mp (\gamma^5-\gamma^5)^2\right)u + \frac{1}{4}\left(1+\gamma^5 \mp (\gamma^5+\gamma^5)^2\right)v =$$

$$= \frac{1}{4}\left(1-\gamma^5 \pm (1-\gamma^5)\right)u + \frac{1}{4}\left(1+\gamma^5 \mp (1+\gamma^5)\right)v =$$

$$= \frac{1}{4}\left((1-\gamma^5)(1\pm 1)\right)u + \frac{1}{4}\left((1+\gamma^5)(1\mp 1)\right)v =$$

$$= \frac{1}{2}(1\pm 1)\,u_L + \frac{1}{2}(1\mp 1)\,u_R$$

$$P_L P_L P_L \cdots u = u_L \qquad P_L P_R u = 0$$

$$j^{\mu}_{\nu_\mu \mu} = \bar{u}_{\nu_\mu} \gamma^{\mu} \frac{(1-\gamma^5)}{2} u_\mu =$$

$$= \bar{u}_{\nu_\mu} \gamma^{\mu} P_L u_\mu = \bar{u}_{\nu_\mu} \gamma^{\mu} P_L P_L u_\mu =$$

$$= \bar{u}_{\nu_\mu} \gamma^{\mu} P_L u_{\mu,L} = u^{\dagger}_{\nu_\mu} \gamma^0 \gamma^{\mu} P_L u_{\mu,L}$$

FROM ANTICOMMUTATOR RELATION

$$\{\gamma_\mu, \gamma_5\} = 0$$

$$\gamma_0 \gamma^\mu P_L = \gamma_0 \gamma^\mu \left(\frac{1-\gamma^5}{2}\right) =$$

$$= \gamma_0 \left(\frac{1+\gamma^5}{2}\right) \gamma^\mu =$$

$$= \left(\frac{1-\gamma^5}{2}\right) \gamma_0 \gamma^\mu = P_L \gamma^0 \gamma^\mu$$

$\gamma^5 = \gamma^{5\dagger}$ so $\gamma^0 \gamma^\mu P_L = P_L^\dagger \gamma^0 \gamma^\mu$

CURRENT:

$$J^{\mu}_{\nu_\mu \mu} = u^{\dagger}_{\nu_\mu} P^{\dagger}_{L} \gamma_0 \gamma^{\mu} u_{\mu, L} =$$

$$= (P^{\dagger}_{L} u^{\dagger}_{\nu_\mu}) \gamma_0 \gamma^{\mu} u_{\mu, L} =$$

$$= u^{\dagger}_{\nu_\mu, L} \gamma_0 \gamma^{\mu} u_{\mu, L} = \bar{u}_{\nu_\mu, L} \gamma^{\mu} u_{\mu, L}$$

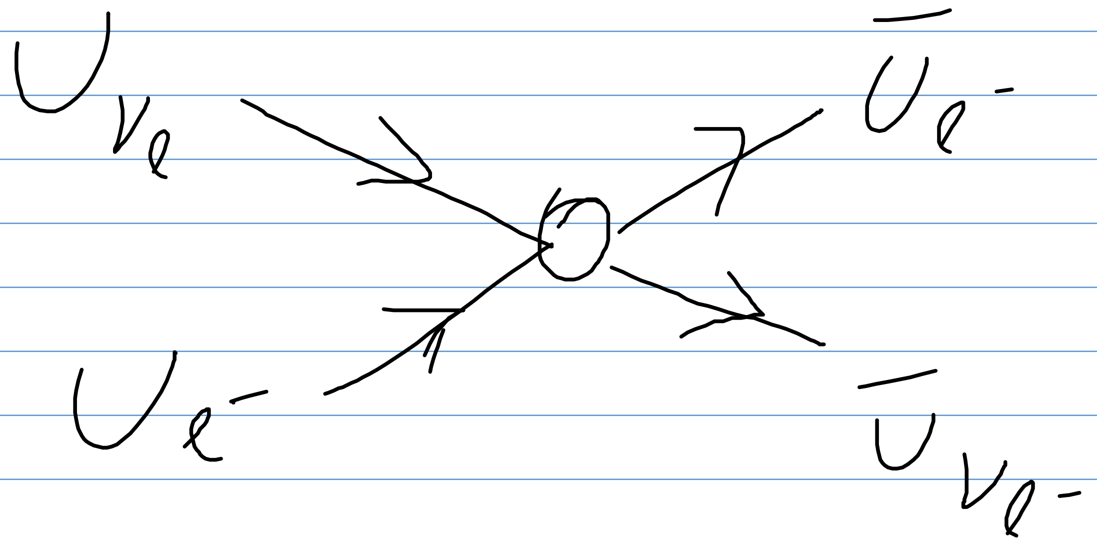
ISOSPIN

LEFT-HANDED FERMIONS HAVE
ISOSPIN $\frac{1}{2}$

WEAK BOSONS HAVE
ISOSPIN 1

IN SM, LEFT-HANDED FERMIONS ARE ORGANISED IN ISOSPIN DOUBLETS $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad e^-_R$

NEUTRINO-ELECTRON SCATTERING



IN POINT-LIKE APPROXIMATION

$$M = j^{\mu}_{e\nu_e,L} \; 2\sqrt{2}\, G_F \, g_{\mu\nu} \, J^{\nu}_{\nu_e e,L}$$

$$J^{\mu}_{e\nu_\ell, L} = \bar{u}_e(q_\ell)\,\gamma^{\mu}\left(\frac{1-\gamma^5}{2}\right) u_{\nu_e}(p_{\nu_\ell})$$

$$J^{\nu}_{\nu_\ell, e, L} = \bar{u}_{\nu_\ell}(q_{\nu_\ell})\,\gamma^{\nu}\left(\frac{1-\gamma^5}{2}\right) u_e(p_\ell)$$

$$M^* = J^{\alpha *}_{e,\nu_e, L}\, 2\sqrt{2}\, G_F\, g_{\alpha\beta}\, J^{\beta *}_{\nu_e, eL}$$

p: INITIAL STATE MOMENTA

q: FINAL STATE MOMENTA

CONJUGATES OF CURRENTS:

$$J^{\alpha *}_{e.\nu_e L} = \left(\bar{u}_e(q_e) \gamma^\alpha \frac{(1-\gamma^5)}{2} u_{\nu_e}(p_{\nu_e}) \right)^* =$$

$$= \left(\bar{u}_e(q_e) \gamma^\alpha \left(\frac{1-\gamma^5}{2} \right) u_{\nu_e}(p_{\nu_e}) \right)^\dagger =$$

$$= u^\dagger_{\nu_e}(p_{\nu_e}) \left(\frac{1-\gamma^5}{2} \right)^\dagger \gamma^{\alpha\dagger} \bar{u}_e^\dagger(q_e) =$$

$$= u^\dagger_{\nu_e}(p_{\nu_e}) \left(\frac{1-\gamma^5}{2} \right) \gamma^0 \gamma^0 \gamma^{\alpha\dagger} \gamma^0 u_e(q_e)$$

$$= u^{+}_{\nu_\ell}(p_{\nu_\ell})\left(\frac{1-\gamma^5}{2}\right)\gamma^0\gamma^\alpha u_e(q_e) =$$

$$= \bar{u}_{\nu_\ell}(p_{\nu_\ell})\left(\frac{1+\gamma^5}{2}\right)\gamma^\alpha u_\ell(q_\ell) =$$

$$= \bar{u}_{\nu_\ell}(p_{\nu_\ell})\gamma^\alpha\left(\frac{1-\gamma^5}{2}\right)u_\ell(q_\ell)$$

SIMILARLY,

$$\int_{\nu_\ell, e, L}^{\beta *} = \bar{u}_\ell(p_\ell)\gamma^\beta\left(\frac{1-\gamma^5}{2}\right)u_{\nu_\ell}(q_{\nu_\ell})$$

So

$$|M|^2 = J^{\mu}_{e\nu_e,L}\, 2\sqrt{2}\, G_F\, g_{\mu\nu}\, J^{\nu}_{\nu_e e,L}\, J^{\alpha *}_{e\nu_e,L}\, 2\sqrt{2}\, G_F\, g_{\alpha\beta}\, J^{\beta *}_{\nu_e e,L}$$

$$= 8\, G_F^2\, g_{\alpha\beta}\, g_{\mu\nu} \left(J^{\mu}_{e,\nu_e,L}\, J^{\alpha *}_{e\nu_e,L} \right) \left(J^{\nu}_{\nu_e e,L}\, J^{\beta *}_{\nu_e,e,L} \right)$$

$$J^{\mu}_{\ell\nu_\ell} J^{\alpha *}_{\ell\nu_\ell} = \bar{u}_\ell(q_\ell)\gamma^\mu P_L u_{\nu_\ell}(p_{\nu_\ell}) \bar{u}_{\nu_\ell}(p_{\nu_\ell})\gamma^\alpha P_L u_\ell(q_\ell)$$

$$= \mathrm{Tr}\left(\bar{u}_\ell(q_\ell)\gamma^\mu P_L u_{\nu_\ell}(p_{\nu_\ell}) \bar{u}_{\nu_\ell}(p_{\nu_\ell})\gamma^\alpha P_L u_\ell(q_\ell)\right) =$$

$$= \mathrm{Tr}\left(u_\ell(q_\ell)\bar{u}_\ell(q_\ell)\gamma^\mu P_L (u_{\nu_\ell}(p_{\nu_\ell})\bar{u}_{\nu_\ell}(p_{\nu_\ell}))\gamma^\alpha P_L\right)$$

$$J^{\nu}_{\nu_\ell \ell} J^{\beta *}_{\nu_\ell \ell} = \mathrm{Tr}\left((u_{\nu_\ell}(q_{\nu_\ell})\bar{u}_{\nu_\ell}(q_{\nu_\ell}))\gamma^\nu P_L (u_\ell(p_\ell)\bar{u}_\ell(p_\ell))\gamma^\beta P_L\right)$$

APPLY $\displaystyle\sum_s u(p,s)\,\bar{u}(p,s) = \not{p} + m$

$$\sum_s J^{\mu}_{\ell \nu_\ell} \, J^{\alpha *}_{\ell \nu_\ell} = \mathrm{Tr}\Big(\big(u_\ell(q_\ell)\,\bar{u}_\ell(q_\ell) \big) \gamma^{\mu} P_L \big(v_{\nu_\ell}(p_{\nu_\ell})\,\bar{v}_{\nu_\ell}(p_{\nu_\ell}) \big) \gamma^{\alpha} P_L \Big)$$

$$= \mathrm{Tr}\left(\not{q}_\ell \, \gamma^{\mu} P_L \, \not{p}_{\nu_\ell} \, \gamma^{\alpha} P_L \right) =$$

$$= \mathrm{Tr}\left(\not{q}_\ell \, \gamma^{\mu} \, \not{p}_{\nu_\ell} \, \gamma^{\alpha} P_L \right)$$

SIMILARLY

$$\sum_s J^{\nu}_{\nu_\ell \ell} J^{\beta *}_{\nu_\ell \ell} = \mathrm{Tr}\left[\not{p}_{\nu_\ell} \gamma^{\nu} \not{p}_{\ell} \gamma^{\beta} P_L\right]$$

$$\sum_s |M|^2 = \sum_s 8 G_F^2 g_{\alpha\beta} g_{\mu\nu} \left(J^{\mu}_{\ell \nu_\ell} J^{\alpha *}_{\ell \nu_\ell}\right)\left(J^{\nu}_{\nu_\ell \ell} J^{\beta *}_{\nu_\ell \ell}\right)$$

$$= 8 G_F^2 \mathrm{Tr}\left[\not{p}_{\ell} \gamma^{\mu} \not{p}_{\nu_\ell} \gamma^{\alpha} P_L\right] \mathrm{Tr}\left[\not{p}_{\nu_e} \gamma_{\mu} \not{p}_{e} \gamma_{\alpha} P_L\right]$$

TRACE RELATION

$$\mathrm{Tr}\left[\gamma^{\mu}\gamma^{\nu}\gamma^{\delta}\gamma^{\sigma}(1-\gamma^{5})\right] = 4g^{\mu\nu}g^{\delta\sigma} + 4g^{\mu\sigma}g^{\nu\sigma} -$$

$$- 4g^{\mu\rho}g^{\nu\sigma} + 4i\epsilon^{\mu\nu\rho\sigma}$$

$$\mathrm{Tr}\left[\not{q}_\ell \gamma^\mu \not{p}_{\nu_\ell} \gamma^\alpha P_L\right] \mathrm{Tr}\left[\not{q}_{\nu_\ell} \gamma_\mu \not{p}_\ell \gamma_\alpha P_L\right] =$$

$$= \frac{1}{4}\left(4 q_\ell^\mu p_{\nu_\ell}^\alpha + 4 q_\ell^\alpha p_{\nu_\ell}^\mu - 4 g^{\mu\alpha} (q_\ell p_{\nu_\ell}) - 4 i \epsilon^{\mu\alpha\rho\sigma} q_{\ell,\rho} p_{\nu_\ell,\sigma}\right)$$

$$+ \ \ldots \ - \ -$$

$$= \frac{1}{4}\left(4 q_\ell^\mu p_{\nu_\ell}^\alpha + \ldots \ldots 4 g^{\mu\alpha} (q_\ell p_{\nu_\ell})\right)\Big) \times$$

$$\times \left(4 q_{\nu_\ell} \ldots - \ - \ - 4 g_{\mu\alpha} (q_\nu p_\ell)\right)\Big) - \frac{1}{4}\left(16 \epsilon^{\mu\alpha\rho\sigma} \epsilon\, q p q_\rho\right)$$

$$= \frac{1}{4} \left(\qquad \right) \left(\qquad \right) + $$

$$+ \frac{1}{4} \left(32 \left(\delta^{\kappa}_{k} \delta^{\sigma}_{\lambda} - \delta^{\rho}_{\lambda} \delta^{\sigma}_{k} \right) q^{\kappa} p^{\lambda} q_{\rho} q_{\sigma} \right)$$

$$= \frac{1}{4} \left(\qquad \right) \left(\qquad \right) + $$

$$+ 8 \left((p_{\ell} \; p_{\nu_\ell})(q_{\nu_\ell} \; q_{\ell}) - (p_{\ell} \; q_{\ell})(p_{\nu_\ell} \; q_{\nu_\ell}) \right)$$

$= \;-\;-\;-\;-$

$$= 16\,(p_\ell\, p_\nu)(q_\ell\, q_\nu)$$

$$\sum_{sp} |M|^2 = 64\, G_F^2\, (p_\ell\, p_\nu)(q_\ell\, q_\nu)$$

$$s = (p_\ell + p_\nu)^2 = \cancel{p_\ell^2} + \cancel{p_\nu^2} + 2\, p_\ell\, p_\nu = 2\, p_\ell\, p_\nu$$

$$s = (q_\ell + q_\nu)^2 = 2\, q_\ell\, q_\nu$$

$$\sum_{sp} |M|^2 = 16\, G_F^2\, s^2$$

LORENTZ-INVARIANT PHASE-SPACE FACTOR (LIPS)

$$d(LIPS) = \frac{1}{4\pi} \frac{p_f}{4\sqrt{s}} \cdot d\Omega$$

FLUX FACTOR $\quad F = 4 p_i \sqrt{s}$

$$s = 2 p_e \cdot p_{\nu_e} = 2\left(E_e E_\nu - |p_1||p_\nu| \cos\theta_{e\nu}\right)$$

$$= 2\left(|p_e||p_{\nu_e}|\right)\left(1 - \cos\theta\right)$$

IN COM FRAME $\theta = \pi$

$$s = 4|p|^2$$

$$p_i = |p| = \frac{\sqrt{s}}{2} \quad \text{ALSO} \quad |p_f| = \frac{\sqrt{s}}{2}$$

$$d\sigma = \frac{1}{F} \sum_{s} |M|^2 \, d(LIPS) =$$

$$= \frac{1}{4 p_1 \sqrt{s}} \, 16 \, G_F^2 \, s^2 \, \frac{1}{4\pi^2} \, \frac{p_1}{4\sqrt{s}} \, d\Omega$$

$$= \frac{G_F^2 \, s}{4\pi^2} \, d\Omega$$