MESONS

$$2 = \begin{pmatrix} u \\ d \end{pmatrix} \qquad I = \frac{1}{2}$$

$$I_3 = \pm \frac{1}{2}$$

$$\bar{2} = \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}' = e^{-i\pi \tau_2/2} \begin{pmatrix} u \\ d \end{pmatrix} =$$

$$= \left(\cos\frac{\pi}{2} - i\frac{\tau_2}{2}\sin\frac{\pi}{2}\right)\begin{pmatrix} u \\ d \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} -d \\ u \end{pmatrix}$$

$$u' \longrightarrow -d$$

$$d' \longrightarrow u$$

$$\bar{2} = \begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix} \qquad \begin{aligned} \bar{d}' &\rightarrow -\bar{u} \\ \bar{u}' &\rightarrow \bar{d} \end{aligned}$$

$$\begin{pmatrix} \cdot & - & - \end{pmatrix} \begin{pmatrix} \cdot \\ \cdot \\ ' \\ \cdot \end{pmatrix} = m$$

$$\begin{pmatrix} \cdot \\ \cdot \\ ' \\ \cdot \end{pmatrix} \begin{pmatrix} \cdot & - & - \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & - \\ - & \cdot & - \; - \\ - & & - \end{pmatrix}$$

$$\begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} \begin{pmatrix} u & d \end{pmatrix} = \begin{pmatrix} -u\bar{d} & -d\bar{d} \\ u\bar{u} & d\bar{u} \end{pmatrix}$$

$$\pi^+ = -u\bar{d}$$

$$\pi^- = d\bar{u}$$

$$\pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$

$$3 = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad \bar{3} = \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \end{pmatrix}$$

$$\begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \end{pmatrix} \begin{pmatrix} u & d & s \end{pmatrix} = \begin{pmatrix} u\bar{u} & d\bar{u} & s\bar{u} \\ u\bar{d} & d\bar{d} & s\bar{d} \\ u\bar{s} & d\bar{s} & s\bar{s} \end{pmatrix}$$

$u\bar{d} = \pi^+$ $\qquad$ $u\bar{s} = K^+$

$d\bar{u} = \pi^-$ $\qquad$ $s\bar{u} = K^-$

$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) = \pi^0$ $\qquad$ $s\bar{d} = K^0$

$d\bar{s} = \bar{K}^0$

$$\eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

$$\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

NON-RELATIVISTIC QM

$$E = \frac{p^2}{2m}$$

$$E \rightarrow +i\frac{\partial}{\partial t} \qquad \nearrow \frac{\partial}{\partial y}$$

$$\underline{p} = -i\underline{\nabla} = -i(\partial_x, \partial_y, \partial_z)$$

$$\frac{(-i)^2 \nabla^2 \psi}{2m} = i\frac{\partial \psi}{\partial t}$$

$$(4.1) \qquad -\frac{\nabla^2 \psi}{2m} = i\frac{\partial \psi}{\partial t}$$

$\times \psi^*$

$$-\frac{\psi^* \nabla^2 \psi}{2m} = i\psi^* \frac{\partial \psi}{\partial t}$$

COMP. CONJ. OF 4.1

$$-\frac{\nabla^2 \psi^*}{2m} = -i\frac{\partial \psi^*}{\partial t}$$

$\times \psi$ $\quad -\dfrac{\psi \nabla^2 \psi^*}{2m} = -i \psi \dfrac{\partial \psi^*}{\partial t}$

$$-\frac{1}{2m}\left(\psi^* \nabla^2 \psi - \psi \nabla^2 \psi^*\right) = i\left(\psi^* \frac{\partial \psi}{\partial t} + \frac{\psi \partial \psi^*}{\partial t}\right)$$

$$-\frac{i}{2m}\left(\psi \nabla^2 \psi^* - \psi^* \nabla^2 \psi\right) = \frac{\partial (\psi^* \psi)}{\partial t}$$

$$\frac{\partial}{\partial t}\int_V (\psi^* \psi)\, dV + \frac{i}{2m}\int_V \underline{\nabla} \cdot \left(\psi \nabla \psi^* - \psi^* \nabla \psi\right) dV$$

$$= 0$$

$$\frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot \underline{J} = 0$$

$$\rho = |\psi|^2 \qquad \underline{j} = \frac{i}{2m}\left(\psi \underline{\nabla} \psi^* - \psi^* \underline{\nabla} \psi\right)$$

$$\frac{i}{2m} \int_S \left(\psi \underline{\nabla} \psi^* - \psi^* \underline{\nabla} \psi\right) \cdot \underline{\hat{n}}\, dS \underset{\text{THEOREM}}{\overset{\text{DIVERGENCE}}{=}}$$

$$= \frac{i \nabla}{2m} \int_V \left(\psi \underline{\nabla} \psi^* - \psi^* \underline{\nabla} \psi\right) dV$$

$$\psi = N e^{i(px - Et)}$$

$$\rho = |\psi|^2 = N e^{i(px-Et)} N^* e^{-i(px-Et)} =$$

$$= N^* N$$

$$\underline{J} = \frac{i}{2m}\left(N e^{i(px-Et)} \underline{\nabla} N^* e^{-i(px-Et)} - N^* e^{-i(px-Et)} \underline{\nabla} N e^{i(px-Et)}\right)$$

$$= \frac{i}{2m}\left(NN^*(-iP) - N^*N(iP)\right) =$$

$$= \frac{P\,|N|^2}{m}$$

THIS WAS ALL IN SCHRÖDINGER PICTURE

WAVE FUNCTIONS ARE TIME DEPENT

OPERATORS INDEPENDENT

IN HEISENBERG PICTURE:

$$i\frac{\partial \Psi}{\partial t} = H\Psi$$

AN OBSERVABLE A IS AN OPERATOR AVERAGED BY THE WAVE FUNCTION

$$\langle A \rangle = \int \Psi^*(\underline{r},t)\, A\, \Psi(\underline{r},t)\, d^3r$$

SOLVE SCHRÖDINGER EQ.

$$i\frac{\partial \Psi(\underline{r},t)}{\partial t} = H\,\Psi(\underline{r},t)$$

$$i\frac{d\Psi}{\Psi} = H\,dt$$

$$i\int_0^{t}{}^{\Psi} \frac{d\Psi(\underline{r},t')}{\Psi}dt' = \int_0^t H\,dt'$$

$$\ln(\psi(\underline{r},t)) - \ln(\psi(\underline{r},0)) = -iHt$$

$$\psi(\underline{r},t) = \psi(\underline{r},0)\, e^{-iHt}$$

$$\langle A\rangle = \int \psi^*(\underline{r},0)\, e^{iHt} A\, \psi(\underline{r},0)\, e^{-iHt}\, d^3r$$

$$A_H = e^{iHt} A\, e^{-iHt}$$

$$\frac{dA_H}{dt} = iH e^{iHt} A e^{-iHt} - e^{iHt} A (iH) e^{-iHt} =$$

$$= i(HA - AH) = i[H, A]$$

FOR SMALL ENERGY PERTURBATIONS

$$H = H_0 + H_1$$

DEFINE $H_1' = e^{iH_0 t} H_1 e^{-iH_0 t}$

$$\frac{dH_1'}{dt} = i[H_0, H_1]$$

HARMONIC OSCILLATOR

$$H = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}$$

$$q \equiv x$$

$$F = -kq$$

$$= m\frac{d^2 q}{dt^2}$$

$$V = -\int F\,dq = \int k q\,dq = \frac{kq^2}{2}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$V = \frac{m\omega^2 q^2}{2}$$

CREATION AND ANNIHILATION

$$a = \frac{1}{\sqrt{2}}\left(\sqrt{m\omega}\,q + \frac{ip}{\sqrt{m\omega}}\right)$$

$$a^{\dagger} = \frac{1}{\sqrt{2}}\left(\sqrt{m\omega}\,q - \frac{ip}{\sqrt{m\omega}}\right)$$

$[q, p] = i$ so

$$[a, a^\dagger] = \frac{1}{2}\left(m\omega \cancel{[q,q]} + \frac{i}{\cancel{\sqrt{m\omega}}}[p,q]\cancel{\sqrt{m\omega}} - i\cancel{\sqrt{m\omega}}\,[q,p]\frac{1}{\cancel{\sqrt{m\omega}}} + \cancel{\frac{1}{m\omega}[p,p]}\right)$$

$$= \frac{1}{2}\left(i(-i) - i(i)\right) = 1$$

$$a^\dagger a = \frac{1}{\sqrt{2}}\left(\sqrt{m\omega}\, q - \frac{i}{\sqrt{m\omega}} p\right)\frac{1}{\sqrt{2}}\left(\sqrt{m\omega}\, q + \frac{i p}{\sqrt{m\omega}}\right)$$

$$= \frac{1}{2}\left(m\omega q^2 + \frac{p^2}{m\omega} + i(qp - pq)\right)$$

$$a a^\dagger = \frac{1}{2}\left(m\omega q^2 + \frac{p^2}{m\omega} - i(qp - pq)\right)$$

$$H = \frac{a^+ a + a a^+}{2} \omega$$

SINCE $[a, a^+] = 1$

$$H = \left(a^+ a + \frac{1}{2}\right) \omega$$

- COMMUTATORS BETWEEN H AND a, a^+

$$[H, a] = \left[\left(a^+ a + \tfrac{1}{2}\right)\omega, a\right] =$$

$$= \left(a^+ a a - a a^+ a\right) \omega =$$

$$= [a^+, a] a \omega = -a \omega$$

$$[H, a^+] = a^+ \omega$$

AN EIGENSTATE $|m\rangle$ WITH ENERGY $H|m\rangle = E_m |m\rangle$

$$H a^{+} |m\rangle = (a^{+}\omega + a^{+}H)|m\rangle$$

$$= (a^{+}\omega + a^{+}E_m)|m\rangle =$$

$$= (E_m + \omega)\, a^{+} |m\rangle$$

SIMILARLY

$$H a |m\rangle = (E_m - \omega)\, a |m\rangle$$

THERE MUST BE A FUNDAMENTAL STATE:

$$a|0\rangle = 0$$

APPLY CREATION OPERATOR TO STATE $|0\rangle$

$$a^{+}|0\rangle = |1\rangle$$

$$\frac{(a^{+})^2}{\sqrt{2}}|0\rangle = |2\rangle$$

$$|m\rangle = \frac{(a^{+})^m}{\sqrt{m!}}|0\rangle$$

$$H|0\rangle = \left(a^\dagger a + \frac{1}{2}\right)\omega|0\rangle = \frac{\omega}{2}$$

$$Ha^\dagger|0\rangle = (a^\dagger\omega + a^\dagger H)|0\rangle =$$

$$= a^\dagger(\omega + H)|0\rangle =$$

$$= a^\dagger\left(\omega + \frac{\omega}{2}\right)|0\rangle = \left(1+\frac{1}{2}\right)\omega a^\dagger|0\rangle$$

$$H|m\rangle = \left(m+\frac{1}{2}\right)\omega|m\rangle$$

$$H = \left(a^\dagger a + \frac{1}{2}\right)\omega$$

$$a^\dagger a|m\rangle = m|m\rangle$$

$$|m\rangle = \frac{(a^\dagger)^m}{\sqrt{m!}}|0\rangle$$

$$|m+1\rangle = \frac{1}{\sqrt{(m+1)!}}(a^\dagger)^{m+1}|0\rangle =$$

$$= \frac{1}{\sqrt{(m+1)!}}a^\dagger\sqrt{m!}\,|m\rangle = \frac{a^\dagger|m\rangle}{\sqrt{m+1}}$$

$$|m-1\rangle = a|m\rangle/\sqrt{m}$$

ANHARMONIC OSCILLATOR

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} + \lambda x^3 = H_0 + \lambda H'$$

$$H_0 |m\rangle^0 = E_m^0 |m\rangle^0$$

$$E_m = E_m^0 + \lambda E_m^1 + \lambda^2 E_m^2 + \dots$$

$$|m\rangle = |m\rangle^0 + \lambda |m\rangle^1 + \lambda^2 |m\rangle^2 + \dots$$

WITH $|m\rangle^1, |m\rangle^2$ ETC. ORTHOGONAL TO $|m\rangle^0$

$$H|m\rangle = (H_0 + \lambda H')(|m\rangle^0 + \lambda |m\rangle^1 + \lambda^2 |m\rangle^2)$$

$$= E|m\rangle =$$

$$= (E_m^0 + \lambda E_m^1 + \lambda^2 E_m^2)(|m\rangle^0 + \lambda |m\rangle^1 + \lambda^2 |m\rangle^2)$$

FOR λ^0 TERMS

$$H_0 |m\rangle^0 = E_m^0 |m\rangle$$

λ^1

$$H_0 |m\rangle^1 - E_m^0 |m\rangle^1 + H_1 |m\rangle^0 - E_m^1 |m\rangle^0 = 0$$

$$(H_0 - E_m^0)|m\rangle^1 = -(H_1 - E_m^1)|m\rangle^0$$

MULTIPLY BY ${}^{0}\langle m|$

$${}^{0}\langle m|\cancel{H_0 - E_m^0}|m\rangle^1 + {}^{0}\langle m|H_1 - E_m^1|m\rangle^0 = 0$$

$$\langle E_M^1\rangle = {}^{0}\langle m|H_1|m\rangle^0$$

MULTIPLY BY ${}^{0}\langle m|$

$${}^{0}\langle m|m\rangle^0 = 0$$

$${}^{0}\langle m|H_0|m\rangle^1 - {}^{0}\langle m|E_m^0|m\rangle^1 +$$

$${}^{0}\langle m|H_1|m\rangle^{01} - {}^{0}\langle m|E_m^1|m\rangle^0 = 0$$

$${}^{0}\langle m|H_0 - E_m^0|m\rangle^1 + {}^{0}\langle m|H_1|m\rangle -$$

$$- {}^{0}\langle m|\cancel{E_m^1|m\rangle^0} = 0$$

$${}^0\langle m | E_m^0 - E_n^0 | n\rangle^1 + {}^0\langle m | H_1 | n\rangle^0 = 0$$

$${}^0\langle m | n\rangle^1 = \frac{{}^0\langle m | H_1 | n\rangle^0}{E_m^0 - E_n^0}$$

$$|n\rangle^1 = \sum_m |m\rangle^0 \frac{{}^0\langle m | H_1 | n\rangle^0}{E_n^0 - E_m^0}$$

$$a = \sqrt{\frac{m\omega}{2}}\, q - \frac{i}{\sqrt{2m\omega}}\, p$$

$$a + a^\dagger = \sqrt{2m\omega}\, q$$

$$q^3 = (a + a^\dagger)^3 (2m\omega)^{-3/2}$$

$$|m\rangle' = \frac{1}{(2m\omega)^{3/2}} \sum_{m_1} |m_1\rangle^{0} \frac{{}^{0}\langle m_1|(a+a^{\dagger})^{3}|m\rangle^{0}}{E^{0}_{m_1} - E^{0}_{m}}$$

$${}^{0}\langle m|(a+a^{\dagger})^{3}|m\rangle^{0} =$$

$${}^{0}\langle m|(a+a^{\dagger})(a+a^{\dagger})^{2}|m\rangle^{0}$$

$$= {}^{0}\langle m|(a^{2}+aa^{\dagger}+a^{\dagger}a+a^{\dagger 2})(a+a^{\dagger})|$$

$$[a, a^{\dagger}] = 1 \qquad a^{\dagger}a = m$$

$$a^{\dagger}|m\rangle = \sqrt{m+1}\,|m+1\rangle$$

$$a|m\rangle = \sqrt{m}\,|m-1\rangle$$

$$Y^3 = \ell \langle m | \sqrt{m(m-1)(m-2)}\, | m-3 \rangle^0$$

$$+ \sqrt{(m+1)(m+2)(m+3)}\, | m+3 \rangle^0$$

$$+ (3+2m)\sqrt{m}\, | m-1 \rangle^0$$

$$+ (1+3m)\sqrt{m+1}\, | m+1 \rangle^0$$

LAGRANGIAN

$$S = \int \mathcal{L}(q, \dot{q})\, dt$$

$$F = \frac{d(mv)}{dt}$$

$$S = \int_{t_1}^{t_2} \mathcal{L}(q, \dot{q})\, dt$$

$$\delta S = \int_{t_1}^{t_2} \left(\frac{\partial \mathcal{L}}{\partial q} \delta q + \frac{\partial \mathcal{L}}{\partial \dot{q}} \delta \dot{q} \right) dt$$

$$\delta \dot{q} = \frac{\delta\, dq}{dt} = \frac{d\,(dq)}{dt}$$

$$\delta S = \int_{t_1}^{t_2} \left[\frac{\partial \mathcal{L}}{\partial q} \delta q + \frac{\partial \mathcal{L}}{\partial \dot{q}} \frac{d(\delta q)}{dt} \right] dt$$

$$= \int_{t_1}^{t_2} \left[\frac{\partial \mathcal{L}}{\partial q} \delta q - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} \delta q \right] dt$$

$$+ \left[\frac{\partial \mathcal{L}}{\partial \dot{q}} \right]_{t_1}^{t_2}$$

$$\frac{\partial \mathcal{L}}{\partial q} \cancel{\delta q} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} \cancel{\delta q} = 0$$

EULER-LAGRANGE

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}}$$

FOR HARM OSC.

$$L = \frac{m\dot{q}^2}{2} - \frac{m\omega^2 q^2}{2}$$

$$\frac{\partial L}{\partial \dot{q}} = m\dot{q} \qquad \frac{\partial L}{\partial q} = m\omega^2 q$$

$$m\ddot{q} = -m\omega^2 q$$

IN QUANTUM MECHANICS:

$$P = \frac{\partial \mathcal{L}}{\partial \dot{q}}$$

$$[q, P] = i$$

$$\frac{dA}{dt} = i[H, A]$$

$$H = P \cdot \dot{q} - \mathcal{L}$$

$$\mathcal{L} = \frac{m\dot{q}^2}{2} - \frac{m\omega^2 q^2}{2}$$

$$H = p\dot{q} - \mathcal{L} =$$

$$= p\dot{q} - \frac{m\dot{q}^2}{2} + \frac{m\omega^2 q^2}{2}$$

$$p = \frac{\partial \mathcal{L}}{\partial \dot{q}} = m\dot{q}$$

$$\frac{m\dot{q}^2}{2} - \cancel{\frac{m\dot{q}^2}{2}} + \frac{m\omega^2 q^2}{2}$$

DIRAC δ FUNCTION

LIMIT $\sigma \to 0$

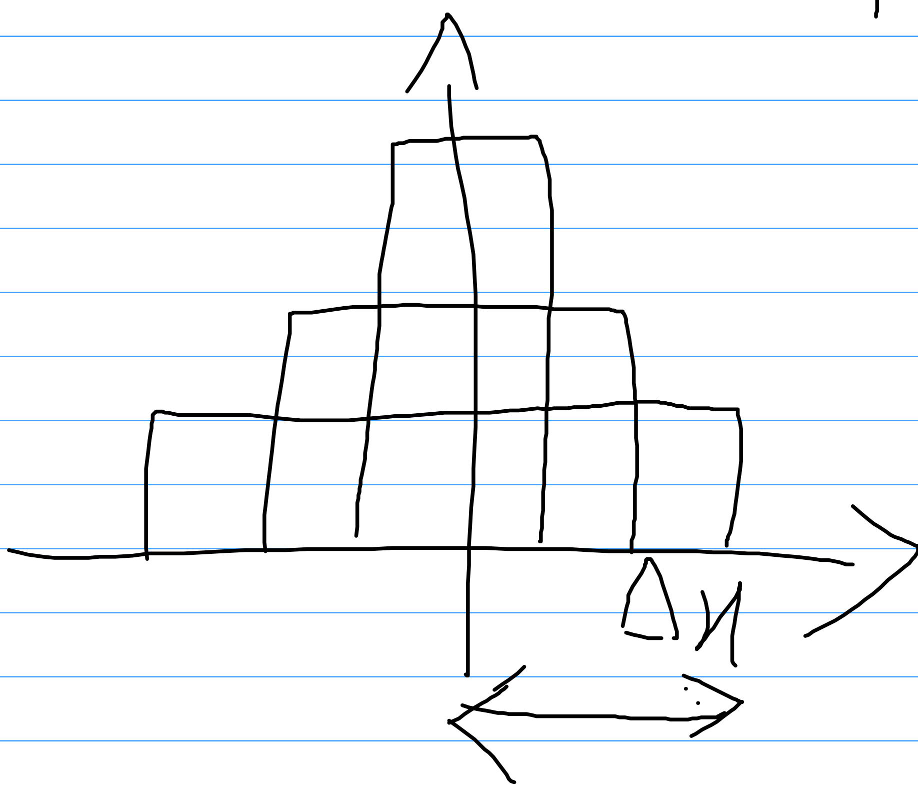
$$\delta(x - x_0) = \begin{cases} 0 & x \neq x_0 \\ \to \infty & x = x_0 \end{cases}$$

$$\int f(x)\,dx \cong \sum_i f(x_i)\,\Delta x$$

$$\int f(x)\,\delta(x - x_0)\,dx \cong \sum_i f(x_i)\,\Delta x\,\delta(x - x_i)$$

$$\delta(x_i - x_0) = \begin{cases} 0 & x_i \neq x_0 \\ \frac{1}{\Delta x} & x_i = x_0 \end{cases}$$

$$\int f(x)\,\delta(x - x_0)\,dx = f(x_0)$$



$$\delta(x) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \quad -\frac{\varepsilon}{2} < x < \frac{\varepsilon}{2}$$

$$\delta(x) = \lim_{\varepsilon \to 0} \frac{1}{\pi} \frac{\varepsilon}{x^2 + \varepsilon^2}$$

$$I = \int_{-\infty}^{+\infty} \frac{1}{\pi} \frac{\varepsilon}{x^2+\varepsilon^2}\, dx$$

$$x = \varepsilon \tan\theta$$

$$dx = \varepsilon \sec^2\theta = \frac{\varepsilon}{\cos^2\theta}$$

$$I = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cancel{\varepsilon\varepsilon}}{\cos^2\theta} \frac{1}{\cancel{\varepsilon^2}(1+\tan^2\theta)}\, d\theta =$$

$$= \frac{1}{\pi} \int_{-\pi/2}^{\frac{\pi}{2}} \frac{1}{\cancel{\cos^2\theta}} \frac{1}{\cancel{\cos\theta + \sin^2} / \cancel{\cos\theta^2}}\, \theta$$

FOURIER - δ

$$\delta(x - x_0) = \int_{-\infty}^{\infty} \frac{1}{2\pi} e^{ik(x - x_0)} \, dk$$

$$f(x) = \sum_{m=\infty}^{\infty} a_m e^{i \frac{2\pi m x}{L}}$$

$f(x)$ NON-ZERO BETWEEN $-\frac{L}{2}$ AND $\frac{L}{2}$

$$\int_{-L/2}^{L/2} f(x) e^{-i \frac{2\pi m x}{L}} \, dx = \int_{-L/2}^{L/2} a_m \, dx$$

$$a_n = \frac{1}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} f(x)\, e^{-i\frac{2\pi n x}{L}}\, dx$$

$$K = \frac{2\pi n}{L}$$

$$\Delta K = 2\pi \frac{\Delta n}{L}$$

$$f(x) = \sum_{-\infty}^{\infty} a_n e^{iKx} \frac{L}{2\pi} \Delta K$$

$$\xrightarrow{\Delta K \to 0} \int_{-\infty}^{\infty} e^{ikx} g(k)\, dk$$

$$g(k) = \int_{-\infty}^{+\infty} f(x)\, e^{-ikx}\, dx$$

$$f(x) = \int_{-\infty}^{\infty} \frac{1}{2\pi}\, g(k)\, e^{ikx}\, dk$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x')\, e^{-ikx'}\, e^{ikx}\, dk\, dx'$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x') \int_{-\infty}^{\infty} e^{ik(x-x')}\, dk\, dx'$$

So

$$\delta(x-x') = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ik(x-x')}\,dk$$

PROPERTIES

$$\delta(ax) = \frac{\delta(x)}{a} \qquad y = ax$$

$$dy = a\,dx$$

$$\int_{-\infty}^{\infty} \delta(ax)\,dx = \int_{-\infty}^{\infty} \delta(y)\frac{dy}{a}$$

$$= 1/a$$

$\delta(x) = \delta(-x)$

FOR A FUNCTION

$f(x)$

$$\delta(f(x)) = \sum_i \frac{\delta(x - a_i)}{\left(\frac{df}{dx}\right)_{x=a_i}}$$

a_i ARE THE POINTS WHERE $f(a_i) = 0$

$$f(x) = f(a_i) + (x - a_i)\frac{df}{dx}$$

$$\delta(f(x)) = \sum_i \delta\left((x_i - a_i)\left(\frac{df}{dx}\right)\right)$$

$$= \sum_i \frac{\delta(x - a_i)}{\left(\frac{df}{dx}\right)_{x=a_i}}$$

HEAVISIDE STEP FUNCT.

$$\theta(\tau) = \begin{cases} 1 & \tau > 0 \\ 0 & \tau < 0 \end{cases}$$

$$\frac{d\theta}{dx} = \delta(\tau)$$

$$\delta(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\omega\tau}\, d\omega$$

$$\theta = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\tau} e^{-i\omega\tau}\, d\tau\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{e^{-i\omega\tau}}{-i\omega}\, d\omega$$

$$\theta = \lim_{\varepsilon\to 0} \frac{-1}{2\pi i}\int_{-\infty}^{\infty} \frac{e^{-i\omega\tau}}{\omega + i\varepsilon}\, d\omega$$

SPECIAL RELATIVITY

$$x^\mu = (t, \underline{r})$$

$$x_\mu = (t, -\underline{r})$$

$$p^\mu = (E, \underline{p})$$

$$p_\mu = (E, -\underline{p})$$

$$x_\mu x^\mu = t^2 - x^2$$

$$p_\mu p^\mu = E^2 - p^2 = m^2$$

$$x_\mu = g_{\mu\nu} x^\nu$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$$

$$g_{\mu\nu} g^{\mu\nu} = \sum_{\mu} g_{\mu\mu}^{2} = 4$$

IN Q.M.

$$E = i \frac{\partial}{\partial t} \qquad \underline{P} = -i \underline{\nabla}$$

$$P^{\mu} = (E, \underline{P}) = i \left(\frac{\partial}{\partial t}, -\underline{\nabla} \right)$$

$$p^\mu = i\partial^\mu = \frac{i}{\partial x_\mu}$$

$$\partial_\mu = i\left(\frac{\partial}{\partial t}, \nabla\right) = \frac{i}{\partial x^\mu}$$

D'ALEMBERT

$$\partial^\mu \partial_\mu = \Box^2 = \frac{\partial^2}{\partial t^2} - \nabla^2$$

LORENTZ TRANSF.

$$t' = \gamma(t - v x_1)$$

$$x_1' = \gamma(x_1 - v t)$$

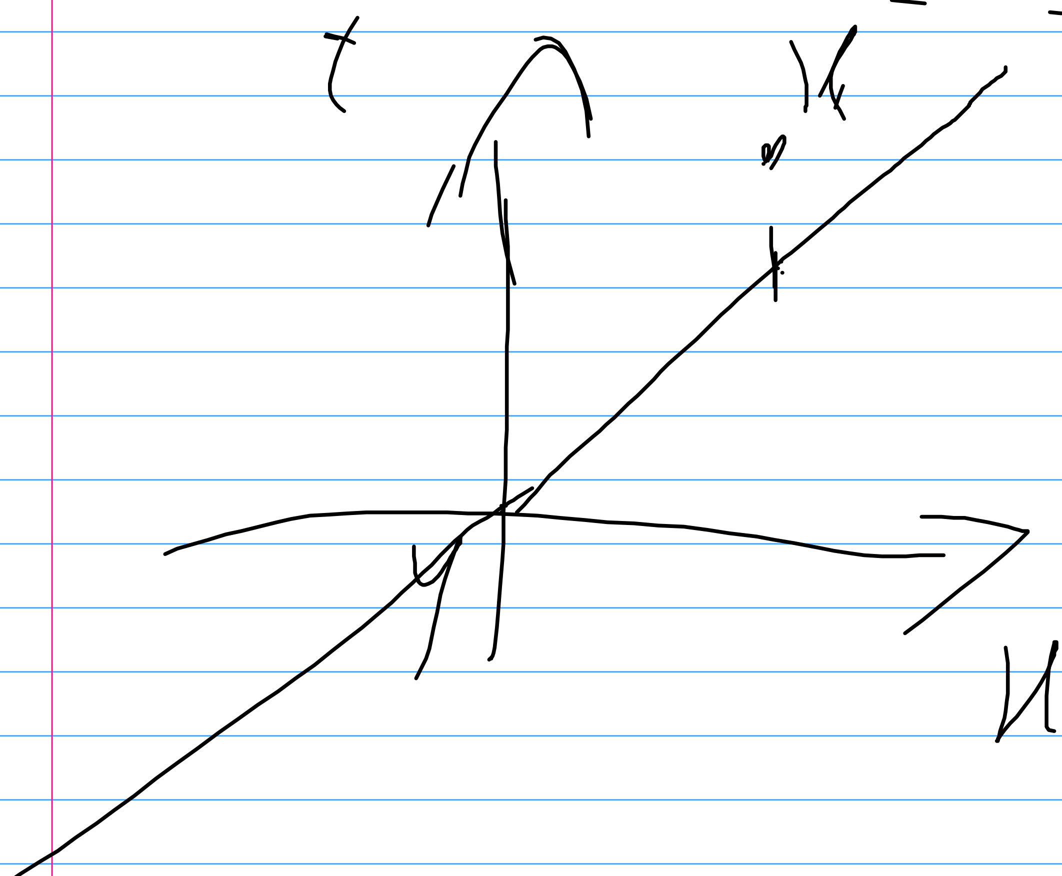
$$x_2' = x_2$$

$$x_3' = x_3 \qquad \gamma = \frac{1}{\sqrt{1 - v^2}}$$

x^μ $\qquad$ y^μ

$(x^0, \underline{x})$ $\qquad$ $(y^0, \underline{y})$

$$s = (x^\mu - y^\mu)^2 = (x_0 - y_0)^2 - (\underline{x} - \underline{y})^2$$

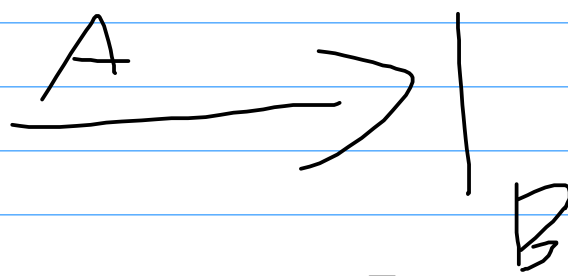


SCATTERING

$$A + B \to C + D$$

$$s = (p_A^\mu + p_B^\mu)^2 = (p_C^\mu + p_D^\mu)^2$$

IF B IS AT REST



$$s = (E_A + E_B)^2 - (\underline{p}_A - \underline{p}_B)^2 =$$

$$= (E_A + m_B)^2 - \underline{p}_A^2 =$$

$$= \cancel{p_A^2} + m_A^2 + 2E_A m_B + m_B^2 - \cancel{p_A^2}$$

$$\sqrt{s} = \sqrt{2E_A m_B}$$

CENTRE OF MASS FRAME

$$s = \left(p_A^{\mu} + p_B^{\mu}\right)^2$$

THE MOMENTUM IS OPPOSITE

$$s = \left(E_A^{CMS} + E_B^{CMS}\right)^2 - \cancel{\left(p_A^{CMS} + p_B^{CMS}\right)^2}$$

$$\sqrt{s} = E_A^{CMS} + E_B^{CMS}$$

$$\beta = \frac{|\underline{p}_A| + |\underline{p}_B|}{E_A + E_B}$$

$$\gamma = |\underline{p}_A| + |\underline{p}_B| / \sqrt{s}$$

$$p_A^{\mu} + p_B^{\mu} = \left(\sqrt{s}, \underline{0}\right)$$

$$p_{A\mu}\left(p_A^{\mu} + p_B^{\mu}\right) = \left(E_A^{CMS}, \underline{p}_A^{CMS}\right)\left(\sqrt{s}, \underline{0}\right)$$

$$= E_A^{CMS}\sqrt{s}$$

$$m_A^2 + p_{A\mu}p_B^{\mu} = E_A^{CMS}\sqrt{s}$$

$$s = \left(p_A^{\mu} + p_B^{\mu}\right)\left(p_{A\mu} + p_{B\mu}\right) =$$

$$= p_A p_A + p_B p_B + 2 p_A^{\mu} p_{B\mu}$$

$$p_A^{\mu} p_{B\mu} = \frac{s - m_A^2 - m_B^2}{2}$$

$$E_A^{CMS} = \frac{2 m_A^2 + s - m_A^2 - m_B^2}{2\sqrt{s}}$$

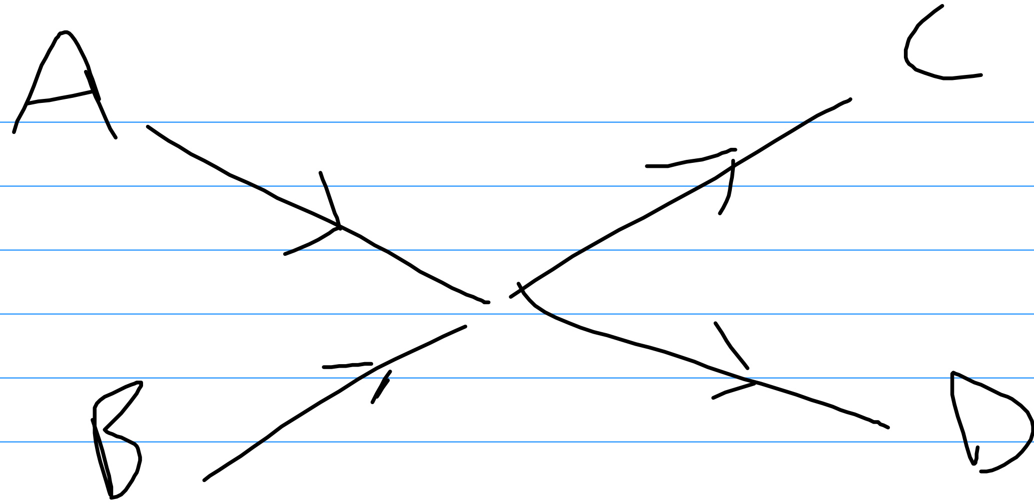
$$= \frac{s + m_A^2 - m_B^2}{2\sqrt{s}}$$

$$\left(p_A^{CMS}\right)^2 = \left(E_A^{CMS}\right)^2 - m_A^2 =$$

$$= \left(\frac{s + m_A^2 - m_B^2}{2\sqrt{s}}\right)^2 - m_A^2 =$$

$$= \frac{s^2 + (m_A^2 - m_B^2)^2 + 2s m_A^2 - 2s m_B^2 - 4s m_A^2}{4s}$$

$$= \frac{\left[s - (m_A + m_B)^2\right]\left[s - (m_A - m_B)^2\right]}{4s}$$



MANDELSTAN VARS

$$s = (p_A + p_B)^2$$

$$t = (p_A - p_C)^2 = (p_B - p_D)^2 =$$
$$= q^2 = -Q^2$$

$$u = (p_A - p_D)^2 = (p_C - p_B)^2$$

$$s+t+u = (p_A + p_B)^2 + (p_A - p_C)^2 +$$
$$+ (p_A - p_D)^2 = p_A^2 + p_B^2 + 2p_A p_B + p_A^2 + p_C^2$$
$$- 2 p_A p_C + p_A^2 + p_D^2 - 2 p_A p_D$$

$$3m_A^2 + m_B^2 + m_C^2 + m_D^2$$
$$+2p_A(p_B - p_C - p_D)$$

$$p_A + p_B = p_C + p_D$$

$$p_B - p_C - p_D = -p_A$$

so

$$s+t+u = \cancel{3}m_A^2 + m_B^2 + m_C^2 + m_D^2$$
$$-\cancel{2m_A^2}$$