CROSS-SECTION DEF

INTERACTION TAKES PLACE IN VOLUME V WITH $2E$ PARTICLES OF EACH KIND

$$\phi_A = N_A e^{i p_A x}$$

$$\int_V \rho \, d^3 r = \int 2E \, \phi_A^* \phi_A \, d^3 r = 2E N_A^* N_A V$$

SO THE NORMALISATION FACTOR IS

$$N_A = \frac{1}{\sqrt{V}}$$

- NUMBER OF TRANSITIONS IN UNIT TIME AND VOLUME

$$W_{fi} = T_{fi} T_{fi}^* \Big/ \text{UNIT TIME AND VOLUME}$$

CROSS SECTION

$$\sigma = W_{fi} \left(\frac{\text{NUMBER FINAL STATES}}{\text{INITIAL FLUX}} \right)$$

$$T_{fi} T_{fi}^{*} = \frac{e^4}{q^4} \iint \frac{1}{V^4} \left[(P_A + P_C)_\mu (P_B + P_D)^\mu \right]^2$$

$$e^{i(P_C + P_D - P_A - P_B)x} e^{-i(P_C + P_D - P_A - P_B)x'} d^4x \, d^4x'$$

$$= \frac{e^4}{q^4 V^4} \left[(P_A + P_C)_\mu (P_B + P_D)^\mu \right]^2 \delta^4 (P_C + P_D - P_A - P_B) \frac{(2\pi)^4 T}{\cancel{T} V}$$

NUMBER OF FINAL STATES
(PHASE-SPACE)

3-MOMENTUM

$$P_C \qquad P_C + d^3 P_C$$

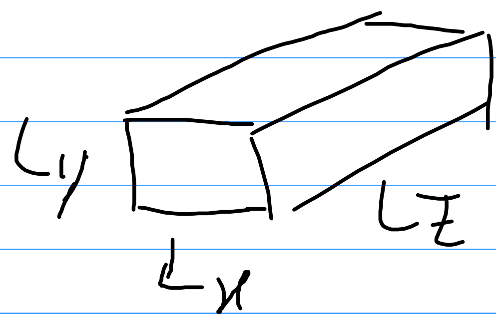
$$P_D \qquad P_D + d^3 P_D$$

ENERGY BETWEEN

E_c , $E_c + dE_c$

E_D , $E_D + dE_D$

INTERACTION VOLUME



PROPAGATOR ONLY EXISTS HERE

SIMILAR TO THE INFINITE POTENTIAL WELL

IN OUR CASE, MOMENTA HAVE TO OBEY THE RELATION

$$P_x L_x = 2\pi m_x$$

CALCULATE DIFFERENTIAL VARIATION

$$(P_x + \Delta P_x) L_x = 2\pi (m_x + 1)$$

SUBTRACTING PREVIOUS EXPRESSION

$$\Delta P_x = \frac{2\pi}{L_x}$$

$$N_p = \frac{\Delta P_x \, \Delta P_y \, \Delta P_z}{(2\pi)^3} L_x L_y L_z =$$

$$\frac{V}{(2\pi)^3} d^3 P$$

NORMALISING

$$N_p = \frac{V}{(2\pi)^3} \frac{d^3 P}{2E}$$

FOR 2 PARTICLES

$$N_p^{CD} = V d^3 P_C \, V d^3 P_D / 4 (2\pi)^6 E_C E_D$$

FLUX IS $\rho_A \rho_B v_{AB}$

$$v_{AB} = v_A - v_B$$

$$\text{FLUX} = \frac{2E_A}{V} \frac{2E_B}{V} \left(\frac{p_A}{E_A} - \frac{p_B}{E_B} \right)$$

$$= \frac{4 \cancel{E_A E_B}}{V^2} \left(\frac{p_A E_B - p_B E_A}{\cancel{E_A E_B}} \right)$$

IN CMS $p_A = -p_B$

$$\text{FLUX} = \frac{4}{V^2} p_A (E_A + E_B) =$$

$$= \frac{4}{V^2} p_A \sqrt{s}$$

SO, CROSS SECTION

$$d\sigma = \frac{e^4}{q^4}\frac{(2\pi)^4}{V^4}\left[(p_A+p_C)_\mu(p_B+p_D)^\mu\right]^2$$

$$\delta^4(p_C+p_D-p_A-p_B)$$

$$d\Omega\left\{\frac{V\,d^3p_C}{2E_C(2\pi)^3}\frac{V\,d^3p_D}{2E_D(2\pi)^3}\frac{V^2}{4p_A\sqrt{s}}\right.$$

LORENTZ-INVARIANT PHASE SPACE IN COM FRAME:

$$dQ = \cancel{(2\pi)^4}\,\delta\left(\sqrt{s} - (E_C + E_D)\right)$$

$$\delta^3\left(p_C + p_D - \cancel{p_A} - \cancel{p_B}\right)\frac{d^3 p_C}{2E_C}$$

$$\frac{d^3 p_D}{2E_D}\,\frac{1}{(2\pi)^{\cancel{6}\,2}}$$

INTEGRATE OVER p_D

(ASSUME DO NOT MEASURE THE ELECTRON)

$$dQ = \delta\left(\sqrt{s} - (E_C + E_D)\right)\frac{d^3 p_C}{2E_C}\,\frac{V^2}{2E_D(2\pi)^2}$$

IF WE DO NOT CARE ABOUT MUON DIRECTION

$\Rightarrow$ INTEGRATE OVER OUTGOING MUON ANGLES

$$\int d^3 p_c = \int 2\pi\, p_c^2 \sin\theta \, d\theta \, dp_c$$

$$= \int p_c^2 \, dp_c \left(2\pi \sin\theta \, d\theta\right) =$$

$$= \int p_c^2 \, dp_c \quad d\Omega$$

$$\sqrt{s} = E_c + E_D =$$

$$\sqrt{p_c^2 + m_c^2} + \sqrt{p_D^2 + m_D^2}$$

IN CMS $p_c = -p_D$

$$\sqrt{s} = \sqrt{p_c^2 + m_c^2} + \sqrt{p_c^2 + m_D^2}$$

$$d\sqrt{s} = \frac{\cancel{2} p_c \, dp_c}{\cancel{2}\sqrt{p_c^2 + m_c^2}} + \frac{\cancel{2} p_c \, dp_c}{\cancel{2}\sqrt{p_c^2 + m_D^2}}$$

$$= p_c \, dp_c \left(\frac{1}{\sqrt{p_c^2 + m_c^2}} + \frac{1}{\sqrt{p_c^2 + m_D^2}} \right)$$

$'' \; p_D^2$

$$d\sqrt{s} = p_c \, dp_c \left(\frac{1}{E_c} + \frac{1}{E_D} \right) =$$

$$= p_c \, dp_c \, \frac{E_c + E_D}{E_c E_D}$$

$$= p_c \frac{\sqrt{s}}{E_c E_D} \, dp_c$$

$$d\omega = \frac{V^2}{4\pi^2} \delta\left(\sqrt{s} - (E_c + E_D)\right) \frac{d\Omega}{4 E_c E_D} \, p_c^2 \, dp_c$$

$$dQ = \frac{V^2}{2\pi^2}\,\delta\left(\sqrt{s} - (E_c + E_D)\right)$$

$$\frac{d\Omega}{4}\,\frac{P_c}{\sqrt{s}}\,d\sqrt{s}$$

INTEGRATE OVER $d\sqrt{s}$

$$dQ = \frac{V^2}{16\pi^2}\,d\Omega\,\frac{P_c}{\sqrt{s}}$$

$$d\sigma = \frac{e^4}{q^4}\,\frac{1}{V^4}\left[(P_A + P_C)_\mu (P_B + P_D)^\mu\right]^2$$

$$dQ\;\frac{V^2}{2P_A\sqrt{s}}$$

$$= \frac{e^4}{q^4} \cancel{\frac{1}{\hbar^4}} \left[(p_A + p_C)(p_B + p_D) \right]^2$$

$$\frac{\cancel{V^2}}{4\omega_A \sqrt{s}} \frac{\cancel{V^2}}{16\pi^2} d\Omega \frac{p_C}{\sqrt{s}} =$$

$$= \frac{e^4}{q^4} \frac{1}{64\pi^2} \frac{\left[(p_A + p_C)(p_B + p_D) \right]^2 p_C}{p_A \, s}$$

FOR $E_i >> m_i$

$$q^2 = (p_A - p_C)^2 = \left(E_A - E_C, \underline{p}_A - \underline{p}_C\right)^2$$

IF $E >> m$, $E = |p|$

$$q^2 = (E_A - E_C)^2 - (p_A - p_C)^2 =$$

$$= \cancel{E_A^2} + \cancel{E_C^2} - 2E_A E_C - \cancel{E_A^2} + 2\underline{p}_A \cdot \underline{p}_C - \cancel{E_C^2}$$

$$= -2E_A E_C + 2|\underline{p}_A||\underline{p}_C| \cos\theta$$

$$= -2E_A E_C (1 - \cos\theta)$$

$$q^4 = 4 E_A^2 E_C^2 (1 - \cos\theta)^2$$

$$\left[(p_A + p_C)_\mu (p_B + p_D)^\mu\right]^2 =$$

$$= \left[p_A p_B + p_A p_D + p_B p_C + p_C p_D\right]^2 =$$

$$\begin{pmatrix} p_A = (|p|, \vec{p}) \\ p_B = (|p|, -\vec{p}) \\ p_C = (|p|, \vec{p}\,') \\ p_D = (|p|, -\vec{p}\,') \end{pmatrix}$$

$p_A p_B =$

$\cancel{p^2 = p_A p_B}$

$E_A E_B - \vec{p}_A \cdot \vec{p}_B$

$= 2p^2$

$p_A p_D =$

$p^2 + p^2 \cos\theta$

$$= \left[2p^2 + p^2 + p^2 \cos\theta + 2p^2 + p^2 \cos\theta + p^2\right]$$

$$\left(6p^2 + 2p^2\cos\theta\right)^2$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^4 s}\left(\frac{3+\cos\theta}{1-\cos\theta}\right)^2$$

DECAY CASE

$$A \to BC$$

NUMBER OF FINAL STATES SAME AS BEFORE

$$\frac{1}{16\pi^2}\,\frac{p_c V^2}{m_A}\,d\Omega$$

NORMALISATION EQUAL TO MASS OF REST PARTICLE

$$\gamma = \frac{2\,m_A}{V}$$

FLUX

$$\phi = \frac{1}{16\pi^2}\,\frac{P_c V^2}{m_A}\,d\Omega\,\frac{V}{2m_A}$$

$$= \frac{1}{32\pi^2}\,\frac{P_c}{m_A^2}\,V^3\,d\Omega$$

DECAY RATE

$$\frac{d\Gamma}{d\Omega} = |T_{fi}|^2 \frac{1}{32\pi^2} \frac{p_i}{m_A^2} V^3$$

FOR ISOTOPIC DECAY

INTEGRATE OVER $d\Omega$

FOR A GROUP OF N PARTICLES

$$\frac{dN}{dt} = -\Gamma N$$

$$N(t) = N_0 e^{-\Gamma t}$$

RELATIVISTIC SPIN $\frac{1}{2}$ PARTICLES

SPIN CONNECTED TO OPERATOR

$$S = \frac{\hbar \hat{\sigma}}{2}$$

σ = PAULI MATRICES

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = \underline{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$[S_x, S_y] = i\hbar S_z$

OR, IF $x=1$ $y=2$ $z=3$

$[\sigma_i, \sigma_j] = 2i\,\varepsilon_{ijk}\sigma_k$

$$\varepsilon_{ijk} = \begin{cases} 0 & i=j \text{ OR } k \text{ OR } j=k \\ 1 & \text{FOR } 123, 231, 312 \\ -1 & \text{FOR } 132, 321, 213 \end{cases}$$

FOR ANTI-COMMUT-

$\sigma_i\sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k$

FOR TWO VECTORS

$\underline{A}$, $\underline{B}$

$$(\sigma \cdot \underline{A})(\sigma \cdot \underline{B}) = \underline{A} \cdot \underline{B} +$$

$$+ i \varepsilon_{ijk} \sigma_k A_i B_j =$$

$$= \underline{A} \cdot \underline{B} + i \underline{\sigma} \cdot (\underline{A} \times \underline{B}) =$$

$$= \underline{A} \cdot \underline{B} + i \begin{vmatrix} \sigma_1 & \sigma_2 & \sigma_3 \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

WHEN $\underline{A} = \underline{B} = \underline{P}$

$$(\sigma \cdot \underline{P})(\sigma \cdot \underline{P}) = |P|^2$$

WE CAN REPLACE

$$E = \frac{p^2}{2m} + V$$

WITH

$$\underline{E} = \frac{(\sigma \cdot \underline{p})(\sigma \cdot \underline{p})}{2m} + V$$

DIRAC EQUATION

$$H\psi = (\bar{\alpha} \cdot \underline{p} + \beta m)\psi$$

$\bar{\alpha}$ AND β ARE 4X4 MATRICES

HAS TO BE CONSISTENT WITH

$$E^2\psi = (p^2 + m^2)\psi$$