

Symmetries & Conservations Laws – Exam Question – Jan 2008

SJH

Answer all the questions. Time 1 hour. Total 20 marks.

1) Exponentiation of operators [5 marks]

$[A,B]=1$. Consider the expression $P = \exp(A)B \exp(-A)$. By expanding the first exponential as a power series (to all orders), simplify the expression.

By differentiating $P(\alpha) = \exp(\alpha A)B \exp(-\alpha A)$ wrt α , simplifying the expression, integrating and then evaluating $P(\alpha = 1)$, verify the expression you found for P .

2) A simple group [5 marks]

Consider a set of elements $S = \{e = 1, a = -1, b = i, c = -i\}$. Show that the combination of the set S and the operation “multiply” is a group.

Which group is this ?

Identify a group formed from geometrical transformations which is isomorphic to this.

Identify all subgroups.

3) Descriptions of charmed baryons [5 marks]

Charmed baryons with two light quarks $q = \{u, d, s\}$ can be written qqc , where the light quarks are described by $SU(3)_{\text{flavour}}$.

Use Young Tableaux to identify the multiplicities for the baryons and to describe the symmetries of the wave-functions. Write suitably symmetrised wave-functions for all the possible states. (*Only the light quarks should be considered under the $SU(3)_{\text{flavour}}$ symmetry; the heavy quark should simply be “added” to the light quark combinations.*)

Identify the isospin singlet ($I = 0$) and triplet ($I = 1$) states.

Very simply, why would you not expect to find a state $q\bar{q}c$?

4) Terms in the Standard Model Lagrangian [5 marks]

The SM Lagrangian contains a term proportional to $\bar{L}_L H l_R$ where L_L is the left-handed lepton doublet and l_R is the right-handed charged-lepton singlet. Assuming this contribution to the Lagrangian is invariant under all the gauge symmetries of the SM, what can you deduce about the quantum numbers of the Higgs field H , including the charges of the component fields?

You should use the relationship $Q = T_3 + \frac{1}{2}Y$ to deduce hypercharge.