Symmetries & Conservations Laws – Exam Question – Jan 2009

SJH

Answer all the questions. Time 1 hour. Total 20 marks.

Formulae: $\exp(i\alpha\mathbf{n}\bullet\boldsymbol{\sigma}) = \cos\alpha + i\sin\alpha\,\mathbf{n}\bullet\boldsymbol{\sigma}$ $\quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

1) Colour states in SU(3) generators [4 marks]

Consider the combination of a single-particle colour state and with a single-particle anticolour state. Use Young Tableaux to identify the possible states (in terms of YT; no need to consider wavefunctions) and their multiplicities.

For the above, which states are a valid description of a) gluons and b) mesons ?

Consider the combination of three single-particle colour states. Use Young Tableaux to identify the possible states and their multiplicities.

For the above, which states are a valid description of baryons ?

2) Handling SU(2) generators [7 marks]

By replacing the exponentials in the expression $Q = \exp(i\alpha\sigma_x)\exp(i\beta\sigma_x)$ with trig functions, derive a simplified form for Q, expressing this as an exponential.

Why is this expression what you would expect ?

Now consider a product $P = \exp(i\alpha\sigma_x)\exp(i\beta\sigma_y)$ – where the second transformation is now wrt the y-axis. Evaluate this in terms of trig functions. Can it be represented as a simple exponential ?

Expand $\exp(i\alpha\sigma_x)\exp(i\beta\sigma_y)$ and $\exp(i\alpha\sigma_x + i\beta\sigma_y)$ to second order in α and β to confirm your answer to the last question.

3) Reduced SU(2) group [3 marks]

Using the results of Question 2, demonstrate that the reduced SU(2) group made up of the set of operators $\{\exp(i\alpha\sigma_x)\,;\,\alpha \in \Re\}$ and the operation "follows" (or matrix multiplication) is a group.

To which of the following groups is this reduced group isomorphic ? Explain briefly.
a) SO(2), b) U(1), c) Real numbers under addition.

PTO

4) SU(2) transformations of a J=1 state [6 marks]

Consider a general SU(2) transformation $\exp(i\alpha\mathbf{n}\bullet\boldsymbol{\sigma})$ acting on a two-quark state uu – quark flavour (but could also be spin). $\boldsymbol{\sigma} = \boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2$ – the σ operators act on the individual quarks and hence are independent (commuting), i.e. σ_1 acts on the first quark etc.. Express the resultant state in terms of both quark flavour $\{\mathrm{uu,ud,du,dd}\}$ and isospin $\{\mathrm{I=1,\ I=0}\}$ states.

What is the projection on to the I=0 state ? Explain.

Hint: Express quark states as base-vectors: $u \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad d \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

This file is in *H:/Teaching/lectures/RH_Group_theory*