Symmetries & Conservations Laws – Exam Question – 3 Feb 2012

SJH

Answer <u>all</u> the questions. Time 1.5 hours. Total 20 marks.
Hint: Roughly every key step or point corresponds to 0.5 marks.

1) Group Theory in Modular Arithmetic [6 marks]

Consider the following sets and operators:

a) {0,1,2,3,4} and ⊕, where ⊕ is addition modulo 5. Eg. 3 ⊕ 4 = 2.
b) {1,2,3,4} and ⊗, where ⊗ is multiplication modulo 5. Eg. 3 ⊗ 4 = 2.

Determine whether these are groups.
(*When considering associativity, you will need to do a bit more than just saying it is the same as for normal arithmetic*.)

2) Generators for SU(n) [5 marks]

Find the properties of the generator X for a Unitary transformation $U = \exp(i\alpha X)$, $\alpha \in \Re$, corresponding to the group SU(n).
(*You should not assume the result for det(1+ε), but sketch the proof.*)

Explain what the group U(1) corresponds to.
Is there a group SU(1) ? If so identify the elements.

3) Structure Constants in SU(3) [4 marks]

Find the non-zero structure constants associated with

a) $[X_1,X_2]$
b) $[X_1,X_4]$

Generators for SU(3):

$$X_1 = \tfrac{1}{2}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\quad X_2 = \tfrac{1}{2}\begin{pmatrix}0 & -i & 0\\ i & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\quad X_3 = \tfrac{1}{2}\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$X_4 = \tfrac{1}{2}\begin{pmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ 1 & 0 & 0\end{pmatrix}\quad X_5 = \tfrac{1}{2}\begin{pmatrix}0 & 0 & -i\\ 0 & 0 & 0\\ i & 0 & 0\end{pmatrix}$$

$$X_6 = \tfrac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}\quad X_7 = \tfrac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & -i\\ 0 & i & 0\end{pmatrix}\quad X_8 = \tfrac{1}{2\sqrt{3}}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{pmatrix}$$

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4) SU(4)$_{\text{flavour}}$ [5 marks]

Consider $q\bar{q}$ mesons in SU(4)$_{\text{flavour}}$, ie. $q \in \{u,d,s,c\}$.
Write the Young Tableaux representing the combinations of a quark and antiquark in SU(4), indicating their multiplicities.

The Weight Diagrams in SU(3) for the mesons are derived by combining an upside-down triangle (3) and a rightside-up triangle ($\bar{3}$), resulting in a hexagon (8) and a singlet (1).
Show how this might look for SU(4), labelling the nodes with quark content.
(*If you don't think your drawing is up to doing this in 3D, do it in layers* ☺)

Assuming perfect SU(4) symmetry and building on the wavefunctions found in SU(3), suggest suitable wavefunctions for the "neutral" states (ie. those containing $u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}$).
(*Ensure your wavefunctions are orthogonal and remember the form for singlets*.)