Brunel University
Queen Mary, University of London
Royal Holloway, University of London
University College London

Intercollegiate post-graduate course in High Energy Physics

Paper 1 : The Standard Model

Tuesday, 5 February 2013

Time allowed for Examination : 3 hours

Answer 6 from 8 questions

Books and notes may be consulted

Question 1 (20 marks)

(a) The highest energy cosmic rays have $E \sim 10^{20}$ eV. Assuming the cosmic ray collides with a proton in the atmosphere, determine the ratio of the centre-of-mass energy in such a collision to the centre-of-mass energy for proton-proton collisions at the LHC. **[5]**

(b) Particle C of mass m_C decays into two particles, A and B, with masses m_A and m_B. Determine the energies of the decay products in the rest frame of the parent particle in terms of the masses of A, B and C. **[5]**

(c) Also for $C \to A+B$, show that in the rest frame of the decaying particle, C, the outgoing momentum can be written in terms of the masses of A, B and C as

$$|\mathbf{p}| = \left[m_A^4 + m_B^4 + m_C^4 - 2m_A^2 m_B^2 - 2m_B^2 m_C^2 - 2m_A^2 m_C^2\right]^{1/2} / 2m_C \,,$$

where $\mathbf{p}$ is the outgoing momentum of either A or B. **[5]**

(d) Show that the flux factor for the collinear collision of A and B is given by

$$E_A E_B |\mathbf{v}| = \left[(p_A \cdot p_B)^2 - m_A^2 m_B^2\right]^{1/2}$$

where $\mathbf{v}$ is the velocity of incident A (energy, E_A, four-momentum, p_A, and mass, m_A) in the rest frame of target B (energy, E_B, four-momentum, p_B, and mass, m_B) . **[5]**

Question 2 (20 marks)

(a) The α_i and β matrices of the Dirac equation, have the following identities :

$$\begin{aligned} \alpha_i \alpha_j + \alpha_j \alpha_i &= 2\delta_{ij} \\ \alpha_i \beta + \beta \alpha_i &= 0 \\ \alpha_i^2 = \beta^2 &= I \,. \end{aligned}$$

From these, derive the anti-commutation relationship for γ^μ matrices,

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \,.$$

[8]

(b) Prove the identity,

$$\not{a}\,\not{a} = a^2 \,.$$

[3]

(c) Show that

$$\mathrm{Tr}\,[\beta] = 0 \,.$$

[4]

(d) From its definition, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, show that

$$\mathrm{Tr}\,[\gamma^5] = 0 \,.$$

[5]

Question 3 (20 marks)

(a) Show that $\gamma^0\gamma^{\mu\dagger}\gamma^0 = \gamma^\mu$. **[5]**

(b) For Coulomb scattering, $e^-(k) \to e^-(k')$, the lepton tensor can be written as

$$L^{\mu\nu} = \frac{1}{2}\text{Tr}\left[(\not{k}' + m)\gamma^\mu(\not{k} + m)\gamma^\nu\right]$$

Show that this can be evaluated as

$$L^{\mu\nu} = 2\left[k'^\mu k^\nu + k'^\nu k^\mu - (k' \cdot k)g^{\mu\nu}\right] + 2m^2 g^{\mu\nu}$$

You do not have to prove but should quote any trace theorems used. **[6]**

(c) Evaluate the lepton tenser, L^{00}, giving your answer in terms of the incident energy, E, velocity, v, and the scattering angle, θ. **[9]**

Question 4 (20 marks)

(a) Contrast the advantages and disadvantages of e^+e^- and pp colliders and give an example result from each to illustrate the differences. **[7]**

(b) Describe the features of Fig. 1, the deep inelastic scattering cross section, and how this relates to the structure of the proton. **[7]**

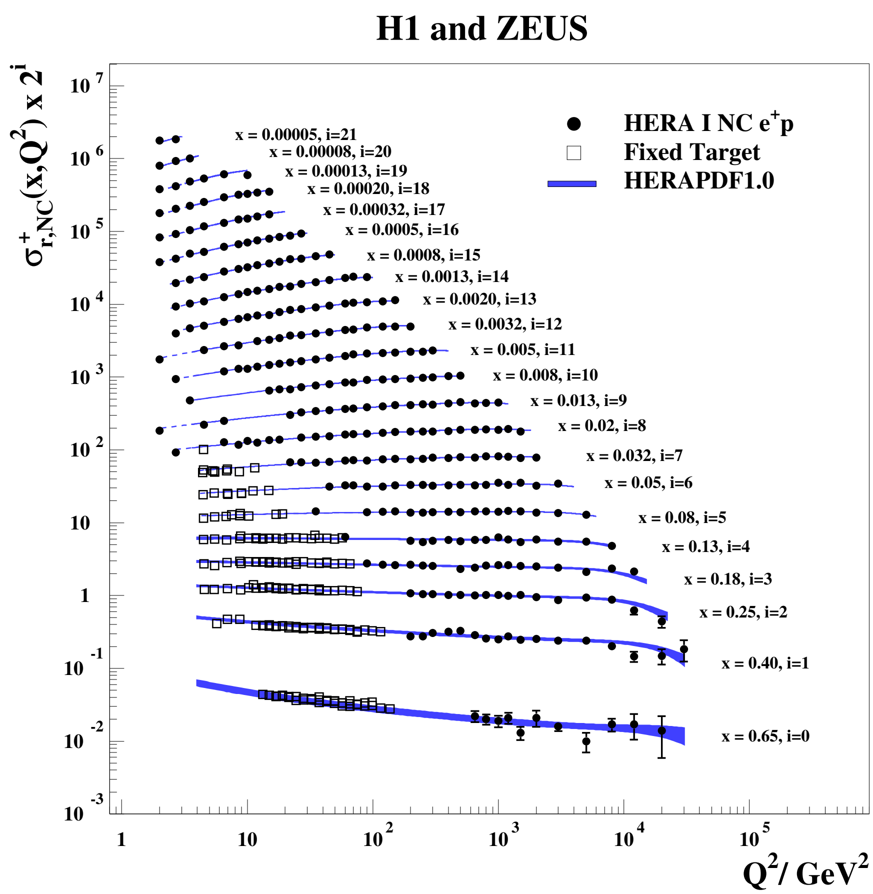

Figure 1: Cross section for neutral current deep inelastic scattering.

[Continued over]

(c) Describe features of Fig. 2, showing the total cross section, $e^+e^- \rightarrow$ hadrons, and ratio, $e^+e^- \rightarrow \text{hadrons}/e^+e^- \rightarrow \mu^+\mu^-$. **[6]**

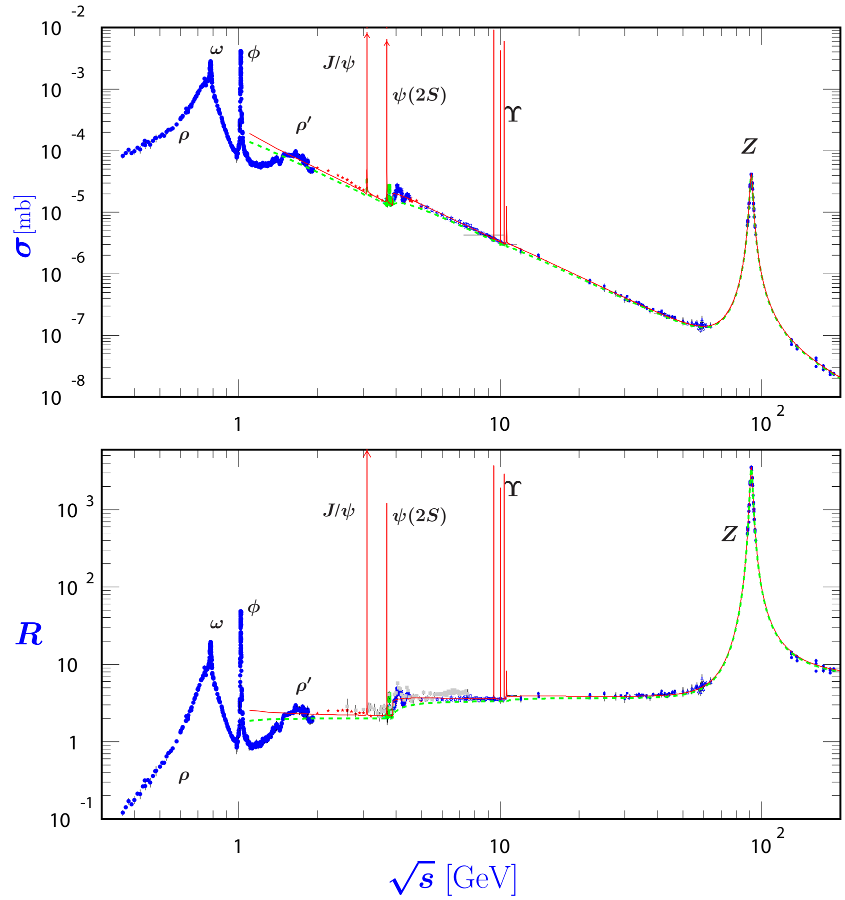

Figure 2: Measurement of the total cross section, $e^+e^- \rightarrow$ hadrons, and ratio, $e^+e^- \rightarrow \text{hadrons}/e^+e^- \rightarrow \mu^+\mu^-$.

Question 5 (20 marks)

(a) State what are meant by global and local gauge transformations. **[2]**

(b) Explain the four terms in the Lagrangian of QED :

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi + e\bar{\psi}\gamma_\mu A^\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

[4]

(c) Using the Euler-Lagrange equation,

$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\left(\partial_\mu\phi\right)}\right) - \frac{\partial\mathcal{L}}{\partial\phi} = 0\,,$$

i. substitute the Lagrangian,

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right) - \frac{1}{2}m^2\phi^2\,,$$

and show that this gives the Klein-Gordon equation. **[5]**

ii. substitute the Lagrangian,

$$\mathcal{L} = i\overline{\psi}\gamma_\mu\partial^\mu\psi - m\overline{\psi}\psi$$

and show that this gives the Dirac equation. **[3]**

(d) From the Lagrangian

$$\frac{1}{8}\left[g_W^2(v+h)^2(W_\mu^1 - iW_\mu^2)(W_\mu^1 + iW_\mu^2) - (v+h)^2(g'B_\mu - g_W W_\mu^3)(g'B^\mu - g_W W_3^\mu)\right]$$

derive the ZZH and $ZZHH$ couplings. (Simplify your answer to give results in terms of g_W and θ_W and remove dependencies on both v and g'.) **[6]**

Question 6 (20 marks)

(a) The amplitude for the decay $\pi^-(q) \to \mu^-(p) + \bar{\nu_\mu}(k)$ is given by:

$$|T_{\text{fi}}|^2 = \frac{G_F^2}{2} f_\pi^2 \cos^2\theta_c m_\mu^2 \text{Tr}\left[(\not{p} + m_\mu)(1-\gamma^5)\not{k}(1+\gamma^5)\right]$$

Use Trace theorems to show this simplifies to

$$|T_{\text{fi}}|^2 = 4G_F^2 f_\pi^2 \cos^2\theta_c m_\mu^2 (p \cdot k)$$

[5]

(b) The total decay width is given by

$$\Gamma = \frac{1}{8\pi m_\pi^2} |T_{\text{fi}}|^2 |\mathbf{p}|$$

where m_π is the pion mass and $\mathbf{p}$ the momentum of the muon in the centre-of-mass frame. Hence show that

$$\Gamma = \frac{1}{8\pi m_\pi^3} G_F^2 f_\pi^2 \cos^2\theta_C m_\mu^2 (m_\pi^2 - m_\mu^2)^2$$

[9]

(c) From this, derive the ratio of decay rates:

$$R = \frac{\Gamma(K^- \to e^- + \bar{\nu_e})}{\Gamma(K^- \to \mu^- + \bar{\nu_\mu})}$$

in terms of the particle masses. Use this relation to give the value to 3 significant figures showing that the rate is close to that measured from experiment, $\sim 2.44 \times 10^{-5}$. **[3]**

(d) Explain the above difference in decay rates to $e^-\bar{\nu}_e$ or $\mu^-\bar{\nu}_\mu$. **[3]**

Question 7 (20 marks)

(a) At lowest order, the transition amplitude for the Higgs decay, $H \to f(p_1)\bar{f}(p_2)$, to two fermions of mass, m_f, is

$$T_{\text{fi}} = -i\frac{g_W m_f}{2M_W}\bar{u}(p_1)v(p_2)$$

where M_W is the W mass. Show that

$$|T_{\text{fi}}|^2 = \frac{g_W^2 m_F^2}{4M_W^2}(2M_H^2 - 8m_f^2)$$

[7]

(b) The total decay width is given by

$$\Gamma = \frac{1}{8\pi M_H^2}|T_{\text{fi}}|^2|\mathbf{p}|$$

where M_H is the Higgs mass and $\mathbf{p}$ the momentum of the fermion in the centre-of-mass frame. Hence show that

$$\Gamma(H \to f\bar{f}) = \frac{G_F\, m_f^2\, M_H}{4\sqrt{2}\pi}\left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

[7]

(c) In fact the above expression is valid for decays to leptons; how would it be modified for decays to quarks ? **[1]**

(d) Determine the widths for decays to muons and to bottom quarks. Which is the dominant rate of the two modes for the Higgs-like Boson discovered at the LHC ? **[5]**

Question 8 (20 marks)

(a) Draw the leading order Feynman diagram for the scatter of a neutrino of four-momentum, k, off an anti-quark of four-momentum, p. **[2]**

(b) The cross section for $\nu\bar{q}$ scattering is given by

$$\frac{d\sigma}{d\Omega}(\nu\bar{q}) = \frac{G_F^2}{4\pi^2}\,\frac{u^2}{s}\,.$$

First convert this into an expression for $\frac{d\sigma}{dt}(\nu\bar{q})$ and then find an expression relating $y = \frac{p{\cdot}q}{p{\cdot}k}$ (q is the exchanged four-momentum) and u and s to express $\frac{d\sigma}{dt}(\nu\bar{q})$ in terms of y. **[11]**

(c) The neutrino DIS cross section can be written in terms of the kinematic variables, x and y, and structure functions, $F_2^{(\nu)}$ and $F_3^{(\nu)}$, as

$$\frac{d^2\sigma^{(\nu)}}{dx\,dy} = \frac{G_F^2}{2\,\pi} s F_2^{(\nu)} \left(\frac{1+(1-y)^2}{2} + \frac{xF_3^{(\nu)}}{F_2^{(\nu)}} \frac{1-(1-y)^2}{2} \right)\,.$$

Use the parton model prediction

$$\frac{d^2\sigma^{(\nu)}}{dx\,dy} = \frac{G_F^2}{\pi} sx \left[q(x) + \bar{q}(x)(1-y)^2 \right]$$

in order to derive the dependence of $F_2^{(\nu)}$ and $\frac{xF_3^{(\nu)}}{F_2^{(\nu)}}$ on the quark, $q(x)$, and anti-quark, $\bar{q}(x)$, densities. **[7]**

[Total Marks = 120]

END OF PAPER