Brunel University
Queen Mary, University of London
Royal Holloway, University of London
University College London

Intercollegiate post-graduate course in High Energy Physics

Paper 1: The Standard Model and Beyond part 1

Monday, 2 February 2015

Time allowed for Examination: 3 hours

Answer four questions out of six (80 marks)

Books and notes may be consulted

The Standard Model

Question 1 (20 marks)

Write down all possible quark-antiquark combinations for the first family only (up and down quarks only) in SU(2), all combinations for up, down and strange quarks in SU(3), and their interpretation in terms of mesons. **[5]**

Write down Schroedinger's equation for a harmonic oscillator, and explain the difference between Schroedinger's and Heisenberg's approaches to time evolution for operator and quantum states. **[5]**

Explain why the operators a and $a^\dagger$ in the standard description of a harmonic oscillator are called destruction and creation operators, respectively. **[3]**

Define and explain the number operator for a harmonic oscillator.

[2]

Consider an anharmonic oscillator with an additional term of the form λx^5 in addition to the harmonic potential, and write the bra-ket expression for the first-order corrections to the energy states. **[5]**

CONTINUED

Question 2 (20 marks)
Explain the condition necessary to allow an event happening at coordinates described by the four-vector (x_1, y_1, z_1, t_1) to influence another happening at coordinates (x_2, y_2, z_2, t_2).

[4]

In a collision between two particles A and B, producing two other particles C and D, write the momentum of particle A in the centre of mass frame as a function of the masses of the four particles, as well as the Mandelstam variables u and t.

[6]

Write down the Klein-Gordon equation, and show that it leads to a conserved current.

[5]

Explain what is a Green's function in position space between an initial and final state for a scattering amplitude, and write its value in momentum space for the case of scattering between scalar planar waves. **[5]**

CONTINUED

Question 3 (20 marks)

Draw the Feynman diagram and write down the transition amplitude for electron-muon scattering in the case when spin is neglected. **[4]**

In the above case, derive the current j_μ^{fi} between the initial and final state by integrating by parts the transition amplitude, and apply the result obtained to the case of planar waves.

[6]

Using Maxwell's equations, find a suitable expression for the four-momentum A^μ, and derive the transition amplitude using that expression.

[5]

Indicate, and briefly describe, all terms by which the transitional amplitude has to be multiplied to obtain a full cross-section calculation.

[5]

CONTINUED

Question 4 (20 marks)

Write down the Pauli matrices, their commutation and anti-commutation relations, and their relations with Dirac's α_i and β matrices.

[5]

Demonstrate how the combination of the covariant and contravariant form of the Dirac equation leads to a conserved current.

[5]

Derive the free particle solutions of Dirac's equation.

[5]

Define the helicity operator, and derive its eigenvalues. **[5]**

CONTINUED

Question 5 (20 marks)
Write down Proca's equation, and derive the propagator for a massive spin-1 particle with current j. **[5]**

Write the transitional amplitude between initial and final state for Compton scattering between two vertices denominated as positions 1 and 2. **[4]**

Derive the expression for this transitional amplitude as a function of the Mandelstam variables, after averaging over initial and final spin states, and neglecting particle masses. **[6]**

Discuss the relation between Compton scattering and electron-positron annihilation, using Feynman diagrams and the Mandelstam variables. **[5]**

CONTINUED

Question 6 (20 marks)
State the main differences between the treatment of electron-muon scattering without and with spin. **[4]**

Write down the transitional amplitude for electron-muon scattering in the case with spin, using the expression for the Dirac potential

$$V_{DIRAC} = e(\alpha_k A^k - VI).$$

[6]

Denoting the electron mass by m and the muon mass by M, and considering that

$$|T_{fi}|^2 = \frac{e^4}{4q^4} Tr\Big[\left(\not{k}' + m\right)\gamma_\mu \left(\not{k} + m\right)\gamma_\nu\Big] Tr\Big[\left(\not{p}' + M\right)\gamma^\mu \left(\not{p} + M\right)\gamma^\nu\Big]$$

Show how this expression can be simplified in the relativistic limit, considering that products involving an odd number of γ matrices are zero. **[6]**

Rewrite the simplified expression for $|T_{fi}|^2$ using the Mandelstam variables. **[4]**

END OF PAPER