Acoustic Detection of Cosmic Ray Neutrinos

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Abstract

A programme of work is proposed to develop hydrophones in water as a method of detecting the interactions of ultra high energy (UHE) cosmic ray neutrinos (≥ 10¹⁰ GeV), a region which is likely to prove difficult for conventional detectors. It is proposed to develop a test and calibration procedure for the detection of such events. A suitable test bed is the UK MoD noise range array situated in the waters off Rona in North West Scotland. This is a hydrophone array which is used to check the acoustic signatures of naval vessels. The purpose of the investigation is to prove the technique. It is also proposed to study the size and nature of an array necessary for full scale high energy neutrino astronomy. The advantages of allying the method to more conventional detectors will also be investigated.

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1 Executive Summary

We propose to develop the technique of acoustic detection of Ultra High Energy particles (UHE) (Energy $\gtrsim 10^{10}$ GeV) with a view to a future study of UHE cosmic ray neutrinos which have never before been detected. This is likely to be a difficult range for conventional optical and shower detectors. The observation of significant fluxes above those expected from secondary interactions of the known UHE cosmic ray primaries would be extremely interesting from a cosmological point of view. Measurements in new regimes, such as this, have often in the past led to new discoveries.

We seek joint MoD and PPARC funding via the Joint Grants Scheme to establish the following Research and Development programme.

- Develop a digital data acquisition system (DAQ) to read out hydrophones in the Rona array, with sufficient bandwidth to be able to detect pulses such as those produced by the interactions of UHE neutrinos.
- Develop the signal processing techniques to extract these bipolar pulses from the noise to as low an energy as possible;
- Develop a system to simulate and calibrate the acoustic pulses produced by the interaction of UHE neutrinos in water which will be detected by hydrophones;
- Prove the techniques by field studies of the noise from the existing MoD hydrophone array at Rona;
- Study the feasibility for UHE neutrino detection of either a stand alone acoustic array or an array working in conjunction with existing equipment such as an optical array.

The team of proposers has specialised expertise in acoustics (JA, CR), signal processing (RB, SD) as well as particle physicists (TS, DW) and particle astrophysicists (LT, SD). To undertake this R and D programme we request 278k pounds over a three year period, we propose that this amount be split 50-50 between PPARC and the MoD.

The proposal is summarised in the organigram shown in figure 1. The paper is arranged as follows. The first section describes the physics interest in detecting and measuring the fluxes of UHE neutrinos and a brief review of ongoing work. This is followed by sections (with details in appendices) on acoustic detection of particle showers. Sections then follow to describe our proposed programme of work and the apparatus and infrastructure which we will need to do it.

2 UHE Neutrino Physics

The origin of ultra high energy (UHE) cosmic rays remains one of the major unknown phenomena in modern day astrophysics. Furthermore, observations [1, 2] of energetic cosmic ray primaries with energies exceeding the GZK cut off [3] are a clear indication of new, unexplored
physics. This cut off is predicted to arise from pion photoproduction interactions of the pri-
maries with the cosmic background radiation. Neutrinos should not undergo such interactions.
Hence, if there are sources of cosmic radiation at such colossal energies in the Universe, neu-
trinos should be a good way to study them. The flux of UHE neutrinos has never before been
measured. UHE neutrinos lie outside the range of optical detectors which detect upward going
muons from neutrino interactions. UHE neutrinos can be observed with limited sensitivity in
shower detectors by measuring near horizontal deeply penetrating showers. However, to achieve
greater sensitivity and resolution for the detection of UHE neutrinos, new techniques will need
to be developed. We propose here to develop one such novel technique, namely acoustic detec-
tion.

Currently it is thought that UHE neutrinos are produced as a result of the interactions of
the primaries in the cosmos. The observation of fluxes above the levels predicted by such
models [4–6] would indicate either independent sources of neutrinos in the Universe or particle
physics beyond the standard model. Either of these would be significant discoveries.
2.1 Physics Interest in Detecting UHE Cosmic Neutrinos

The flux of UHE cosmic ray neutrinos has never been measured and the upper limits from previous lower energy searches are shown in figure 2 together with projected limits from future experiments. The current best estimate of the UHE neutrino flux in the standard model of particle physics is roughly represented by the Waxman-Bahcall (WB) flux limit of \( \sim 2 \times 10^{-8}/E^2 \) \((\text{GeV} \cdot \text{cm}^{-2} \cdot \text{sec}^{-1} \cdot \text{sr}^{-1})\) \([4, 5]\) which is \( \sim 1.5 \) orders of magnitude below the current measured limits \([7, 8]\) at lower energy (see fig. 2). This limit is derived assuming that neutrinos are produced from pion decays following the interactions of primary protons with photons or other material in a thin source. It is interesting to detect UHE neutrinos to see if the flux is at the level expected or higher than that from the standard model. Measurements at a higher level would indicate either the presence of unexpected sources of neutrinos or a neutrino interaction cross section above that expected in the standard model of particle physics. Either case would be a significant discovery in physics \([9]\).

An unexpected source of UHE cosmic ray neutrinos could be one in which a cosmic proton accelerator is situated in material of high density in contrast to the thin source assumption of the Waxman-Bahcall limit \([5]\). In such a source (“a beam dump”) the protons would lose all their energy to secondary mesons which eventually decay to neutrinos. This would lead to an enhanced neutrino flux in comparison to the flux of proton primaries. Such an object would be an unexpected astronomical cosmic ray source.

Other scenarios leading to higher than expected UHE neutrino fluxes have been proposed. In such so-called top down models there could be even larger fluxes of neutrinos than the observed primaries. One such mechanism is the Z burst model \([15]\) invoked to explain the possible observation of cosmic ray primaries of energy above the GZK limit. In this model an energetic (anti-neutrino) neutrino annihilates on a relic low energy cosmic (neutrino) anti-neutrino, which is assumed to have mass, to produce the super-GZK cosmic rays. The cross section for the process resonates at a centre of mass energy equal to the \(Z^0\) mass. The model requires a neutrino flux several orders of magnitude greater \([16]\) than the Waxman-Bahcall limit. Top-down mechanisms which would lead to higher than expected fluxes of UHE neutrinos are models in which heavy particles, which are assumed to constitute part of the dark matter in the Universe, decay to neutrinos. Several candidate particles for this have been proposed \([17, 18]\). A further mechanism is the presence of topological defects in the Universe \([19]\) which may lead to the production of UHE neutrinos.

The total interaction cross section for UHE neutrinos has been computed in the standard model \([20–22]\) and typical calculations are shown by the solid and dotted curves in figure 3. Some models predict much larger cross sections \([23, 24]\). An example of a calculation in the brane models labelled “typical p-brane model” is shown in figure 3. This curve is at the centre of the predictions of the model described in reference \([23]\). Such predictions vary by roughly 2.5 orders of magnitude about the curve shown depending on the chosen parameters. It can be seen that these cross sections are several orders of magnitude higher than the standard model prediction. This would lead to an enhanced rate of detection of UHE neutrinos.
Figure 2: Current and expected limits on the cosmic ray neutrino flux as a function of neutrino energy. 90% confidence level limits are quoted unless otherwise stated. The steeply falling curve at low energy represents the flux of atmospheric neutrinos. The curve labelled GZK represents a calculation by [6] and the limit labelled Waxman-Bahcall is from [5]. The best current limits on diffuse neutrino fluxes are given by the Baikal and Amanda experimental limits [7,8]. Also shown are the expected limits from the Antares experiment after 1 and 3 years of data taking [10] and the approximate expected limit after 3 years of data taking by a generic km$^3$ scale optical Cerenkov detector. The RICE curve indicates the approximate position of various model dependent 95% upper limits on neutrino fluxes that are obtained in a search for radio Cerenkov signals in South polar ice [11]. GLUE [12] and FORTE [13] experiments search for radio signals due to neutrino induced cascades in the lunar regolith and Greenland ice sheet respectively, setting the model independent upper limits shown. The Auger curves represent the expected 1 event/year/energy-decade counting rates for electron and muon neutrinos (upper curve) and tau neutrinos (lower band) [14].

2.2 Ongoing Searches for Cosmic Ray Neutrinos

Several classes of experiments have or are searching for cosmic ray neutrinos. One class uses optical techniques to detect the Cerenkov light from particles resulting from the interactions of the neutrinos in transparent media such as water or ice [7,8,10,25–27]. Current limits on the neutrino flux set by these techniques cover the energy range from atmospheric neutrinos up to $\sim 10^7$ GeV. These experiments detect the upward going muons from neutrino interactions using downward pointing phototubes immersed in water or ice. The Earth becomes opaque to neutrinos of energy more than $\sim 10^7$ GeV due to the increasing neutrino cross section with
energy and this limits the energy range of this technique. However, some sensitivity will be achieved by either detecting back scattered light or the hadronic decays of upward going tau leptons [28].

A second class of experiments detects cosmic ray showers. High energy neutrinos are detected in deeply penetrating, near horizontal air showers. This technique was pioneered by the Fly’s Eye collaboration some years ago [2] and it will be used at higher energy in the Pierre Auger apparatus [14]. The sensitivity of this method is limited by the allowed angular range of the technique. This class of experiment is mainly sensitive to the detection of the decay of the $\tau$ lepton produced by the charged current interaction of a $\nu_\tau$ and the hadronic part of the neutrino shower energy is not measured.

To achieve higher sensitivity new techniques will be necessary to detect and measure UHE neutrinos. The known possibilities are radio and acoustic techniques.

A programme of work began a few years ago on the detection of coherent radio Cerenkov emissions following a neutrino interaction [11–13, 29, 30]. This is a promising technique and
upper limits on the flux of UHE neutrinos have recently been published by a number of experiments (see figure 2). In certain energy regions the limits are within 2 orders of magnitude of the Waxman-Bahcall flux and at higher energies are beginning to constrain topological defect and Z-burst models. These limits will improve as more data is collected and next-generation experiments such as ANITA and SALSA come on-line. Note that RICE is mainly sensitive to electron neutrinos through CC interactions, but at higher energies there is sensitivity to all neutrino flavours through NC interactions.

Initial studies including those described here indicate a threshold for acoustic neutrino detection in the region of $10^{10} - 10^{11}\text{ GeV}$. Moreover, the sensitivity increases as the energy increases, at least up to $10^{13}\text{ GeV}$. Hence acoustic detection is complementary to Auger and the low energy radio Cerenkov experiments, with maximum sensitivity above the optimum energy ranges for these experiments. The ultimate sensitivity of radio detection at much higher energies is less well known, and the programme of work described here will help to establish the relative sensitivities of acoustic and radio detection methods. The discovery of a signal in any of the planned future experiments will require confirmation using all available detection techniques, and acoustic detection is the main alternative to radio at these ultra-high energies. Acoustic detection is also interesting in that the required detectors could be deployed alongside the antennae used for radio detection or PMT modules used for optical detection, possibilities that are receiving considerable attention worldwide. Building up expertise in acoustic detection will be an important route through which the UK can contribute to future developments in the field of ultra-high energy neutrino detection.

3 Acoustic Detection of UHE neutrinos

Acoustic detection of showers using hydrophones immersed in a large volume of water, although proposed many years ago [31], has not yet been implemented to detect cosmic rays. However, the technique has been proved on a smaller scale using accelerator beams [32]. Meanwhile hydrophone technology has advanced and arrays for investigating the acoustic signatures of naval vessels now exist. Hence, it seems timely to develop the technique on a large scale for UHE neutrino detection using the more advanced available technology. We wish to prove the technique to ascertain if UHE neutrino detection is feasible by this method. To this end a US collaboration has made a similar proposal [33] and we propose the present studies.

An important advantage of acoustic detectors in water is that the attenuation length of the sound from a cosmic ray shower is of order kilometers. Hence it may be possible to instrument large volumes of water quite cheaply. Since water is the target material there are fewer restrictions as to the location of a detector than for the radio or optical techniques.

A peculiar feature of the acoustic signal from a cascade is that the acoustic energy is concentrated in a small range of angles perpendicular to the direction of the shower (see figure 4 and appendix A). An array of hydrophones will be required to produce an omnidirectional detector capable of accurately reconstructing the shower location and direction, with the coincidence signal potentially enabling thresholds to be lowered significantly. The optimum design of such an array will be investigated as part of this R and D programme.
Further details are given below of the main properties of acoustic neutrino signals, an existing hydrophone array that we propose to exploit for R and D purposes, and the worldwide context of our activities.

### 3.1 Acoustic Signal Generation and Detection

Hadronic and electromagnetic cascades produced by the interactions of ultra-high energy cosmic ray neutrinos ($\gtrsim 10^{10}$ GeV, or 1 Joule) can deposit sufficient thermal energy in the surrounding medium to generate a measurable acoustic signal. The dominant and calculable coupling mechanism is the near-instantaneous expansion of the volume of material in which the thermal energy is deposited, although other coupling mechanisms such as micro-bubble production have also been proposed. The formalism for calculating the acoustic signal resulting from a given thermal energy deposition was developed in [34] and is explained in more detail in appendix A.2.

Hadronic showers at these energies deposit the majority of their energy within a narrow cylinder roughly 10 interaction lengths ($\approx 10$ m) long. Due to the Landau, Pomeranchuk, Migdal (LPM) effect, electromagnetic showers can be somewhat longer [35]. Since the speed of sound in water is so much less than the speed of the shower propagation, the energy deposition along the length of the shower can be considered to be quasi-instantaneous, giving rise to an acoustic line-source. At large distances from the shower and in the plane perpendicular to the shower axis, the acoustic emission is coherent, giving rise to large amplitude pressure pulses. The magnitude of the pressure pulse falls off very rapidly out of this plane. Figure 4 further illustrates the relevant geometry.

![Figure 4](image)

Figure 4: A schematic of the energy deposition model used in the simulation (see appendix A). The energy deposition is modelled by a cylinder of Gaussian cross-section. The mean radius, $R$, is a few centimetres and the length $L$ is fixed to 10 m. The signal detected at the distant hydrophone depends on the distance $r$ from the cascade and the angle $\theta$ out of the plane transverse to the cascade axis. The signal falls rapidly with angle out of the plane shown (see appendix A and figures 12 and 19).

The acoustic signal can therefore be visualised as a bipolar pressure pulse propagating approximately at right angles to the shower axis at the velocity of sound in water (1500 m s$^{-1}$).
Almost all the acoustic energy is contained within this thin “pancake”, although stochastic effects in shower development can presumably give rise occasionally to more spherical acoustic radiation patterns. A typical pressure pulse is shown in figure 5. The attenuation length of the pressure pulse falls from about 10 km for frequency components of 10 kHz to about 1 km for frequency components of 30 kHz. Most of the acoustic energy is contained within this frequency range [34], [33].

![Figure 5: The pressure pulse for a 1.2 × 10^{11} GeV shower in sea water at a distance of 1 km. The angle with the plane transverse to the shower direction is zero degrees.](image)

The pressure pulses generated at the ultra-high energies discussed here are well within the detection capabilities of modern hydrophones [36]. No information on the cascade direction can be gleaned from the signal in a single hydrophone, although coincidence signals in a multiple hydrophone array could be used to reconstruct a cascade direction as well as reduce the rate of fake hits from noise and background sources.

Further details, including preliminary studies of single hydrophone counting rates and thresholds, are given in appendix A.

### 3.2 The Rona Array

We will have access to selected hydrophone signals from the UK MoD Noise Range at Rona which is used to measure the acoustic characteristics of naval vessels. As such it is populated with measurement and tracking hydrophones with sufficient sensitivity and bandwidth to detect the signals described in this proposal. The array is situated in an acoustically quiet location in around 200m depth of water, and the ambient noise environment is well characterised. Average water temperature is around 10C through the year. The most suitable hydrophones to record
Data from are the measurement hydrophones located around the centre of the array. These are at midwater depth and are the best maintained sensors in the array, furthermore, these hydrophones are less likely to suffer from background due to reflections than ones deployed close to the sea bed or surface. Hydrophone data will be brought onshore via analogue cableways to a shore station.

The current data acquisition and recording systems are tailored to acoustic noise ranging of Royal Naval vessels, and do not have the required performance in terms of bandwidth and storage capacity for particle shower detection. Hence dedicated equipment will need to be purchased and installed for this project. Such equipment can be set up on a non-interfering basis, and left to run unattended for the duration of the experiment. It is planned to record data from up to 4 hydrophones for around one month (700hrs). The actual choice of hydrophones to be recorded will depend on the serviceability of the noise range at the time the experiment is conducted as well as allowing for any ongoing maintenance issues. A scalable recording system has been proposed that will allow the number of hydrophone-hours captured to be expanded as data storage prices continue to fall.

### 3.3 Other activities in Acoustic Detection

Three of the proposees recently attended a first mini-workshop on acoustic detection in Stanford, California which was attended by approximately 15 people. The existence of this workshop indicates that this field of research is indeed undergoing a renaissance. There are activities taking place in the US, Europe and Russia and opportunities for collaboration with some or all of these groups is certainly probable if this funding request is successful.

In the US a small group of scientists has successfully collected 230 live days worth of pre-filtered data from 7 hydrophones in the AUTEC array, owned by the US Navy and situated in the Bahamas. Analysis of these data is underway and there is agreement to share data taken with this array with the UK team via a visit to the Stanford group. The proposees are keen to reciprocate and also make data available to the US team. This AUTEC group uses imploding light bulbs as a calibration device which is non-optimal. This proposal’s aim to develop a calibrator is therefore complementary to the efforts in the US. Furthermore it is clear that a number of issues such as acoustic pulse shape studies, noise reduction studies, etc., ideally need to be carried out in a number of different water conditions in order to develop a full understanding of the mechanisms involved and the algorithms required for the reduction of noise. The fact that the UK has an existing hydrophone array (rather than one that requires development) clearly places the UK in an advantageous situation regarding such studies. In addition, there is much to be gained by studying data taken from the rather different sea environments of the AUTEC and Rona arrays.

In Europe there are studies taking place in three countries. In Germany (Erlangen ANTARES group) the work is concentrating on development of home-made hydrophones - whilst interesting the UK proposees do not see this to be a priority and so this work is largely complementary to the programme of work outlined here. Work in Italy (Catania/LNS ANTARES/NEMO group) is currently looking at developing a few hydrophones at a deep-sea site off the coast of Sicily with a view to starting noise studies. Finally, in France (CPPM, Marseilles ANTARES group)
work is taking place on simulating the electromagnetic and hadronic showers associated with the incident neutrino, this is non-trivial at such high energies. There is overlap here with the work programme discussed in this proposal; however, the UK and French teams have agreed to collaborate in this area to avoid duplication of effort.

There are two areas where the UK proposal certainly will bring considerable “added value” to the current global activities in this research field, namely in developing a robust, reproducible calibration device for simulating the initial energy deposit from the neutrino - something that is of interest to other groups in Europe and the US. Secondly, our intention to record a significant amount of unfiltered data from an existing hydrophone array will enable ourselves and any colleagues wishing to share our data to study noise and sea state conditions with a view to developing sophisticated algorithms for signal processing. In this respect the dataset that we propose to take at Rona will be unique.

4 Programme Goals and Methodology

4.1 Data Taking

The ability to detect neutrinos from their acoustic signatures depends critically on the ambient noise environment. Underwater ambient noise is generated by many mechanisms such as shipping, bio noise (e.g. from whales, dolphins, snapping shrimps etc) as well as wind noise transmitted from the surface. A quantitative study of the properties of the neutrino signal and of the noise are discussed in appendix B. Appendix C describes a method proposed to extract the signal from the noise and figure 23 illustrates the expected theoretical effectiveness of this method.

The purpose of data taking will be to ascertain whether or not the method proposed to extract the neutrino shower signal will be effective and if not the data will be used to develop an appropriate method. The shower signals will be generated artificially (see section 4.2). We also wish to ascertain whether it will be possible to trigger on these signals. In appendix C.4 it is shown that, in theory, the apparatus can be triggered by a neutrino signal which is 2.85 times the root mean square noise level assuming that the noise level is purely Gaussian.

Such a trigger level should give us a false trigger rate of 1 per ten years, a figure that is used later in appendix B. It should be stressed that this figure assumes a purely Gaussian noise level such as that that might be expected purely from sea noise and does not include any non-Gaussian components from, e.g., bio noise. The false trigger rate quoted is therefore unrealistic. A better estimate however will only be possible after having studied the noise environment at Rona in some detail - studies which form part of this proposal. The bipolar signals expected from a neutrino shower (see appendix A) will be replicated by the calibration apparatus and the proposed signal generators and superimposed on the noise. The data will then be analysed using methods such as that outlined in Appendix C.

The equipment at the Rona array is not optimised for UHE neutrino detection and we therefore need to purchase and install additional items. In particular it will be necessary to construct a dedicated, unattended, wideband, multichannel digital recorder that can record acoustic data
from a number of hydrophones in the array over an extended period of time. An additional requirement is the need to be able to project representative synthetic UHE neutrino signals into the water around the array. A possible way of doing this has been outlined in Section 4.2. Both the recorder and projectors are specific to this project and require some development. They are not items that can be furnished through a single purchase of existing equipment.

We foresee two phases of data taking. The first phase will be to prove the calibration, pulse generating and data acquisition systems in a quiet pool such as the one at Kelk Lake [37] (see section 5). Then we would propose to test the procedures in the more realistic environment at Rona.

4.2 Calibration and Test Procedure

It is proposed to build a signal test and calibration system. The purpose is to generate pulse forms of the shape and amplitude corresponding to those created by high energy neutrino interactions with water. A number of methods are being investigated to effect this simulation and calibration. The most promising one involves the use of a laser or collimated flash tube as a light source.

4.2.1 Use of Collimated Light Sources

Assuming that the dominant mechanism for the dissipation of the energy from a neutrino interaction is thermal deposition [34], we propose an optical calibration method using either a pulsed laser or a collimated flash gun, working in the wavelength region between 550 and 600nm where the absorption coefficient is typically in the region of 0.1 m\(^{-1}\) (see figure 6). In this region, in the absence of significant suspended matter, the beam attenuation coefficient is dominated by absorption with scattering being of minor importance. The beam intensity therefore can be simply calculated using

\[ I = I_0 \exp(-\alpha x) \]  

(1)

where \( I \) is the intensity at distance \( x \) m and \( \alpha \) (m\(^{-1}\)) is the absorption coefficient. The energy loss \( dI \) within an infinitesimal segment \( dx \) is then simply given by

\[ \frac{dI}{dx} = -I_0\alpha \exp(-\alpha x). \]  

(2)

This energy loss will appear largely in the form of heat.

We propose to simulate the energy absorption from an UHE neutrino shower by shining a collimated pulsed light beam through a 10m column of water and reflecting it back to the source. The wavelength is chosen so that a significant fraction of the light energy is absorbed in the water. This is not a perfect simulation of a UHE cosmic ray neutrino shower since the source by its nature is exponentially decaying in intensity. However, the angular spread of the sound, particularly in far field, will be very similar to that from such a shower. This is shown in figure 7.

The costs of a system to make this calibration source are estimated in section 5. In addition to this method of calibration other ideas are being considered.
Figure 6: The absorption coefficient of light in pure sea water as a function of the wavelength of the light.

Figure 7: Far field angular distribution of acoustic energy from the collimated light source. The angle in degrees is measured from an axis in the plane at right angles to the shower axis (see also appendices A and C.1, figure 19).

4.2.2 Artificial Sound Sources

- A simple omnidirectional projector will be used to assess the sensor performance, recording system and data processing techniques. It will be straightforward to project pulse
shapes at a controlled level using an omnidirectional projector. This will give confidence that the expected signals can be detected, recorded and processed using Rona, or any other collection array.

- A parametric system; one of the characteristic features of the neutrino signal is the very narrow beam pattern generated by the cascade. In order to replicate this and easily investigate its effect, it should be possible to use a parametric projector that can put an appropriate signal in the water in a very narrow beam pattern. The technique involves simultaneously driving a transducer with two slightly differing frequencies. Whether it can be made to work with a broadband pulse needs to be investigated. Alternatively, systematic adjustment of the level of the signal generated by the omnidirectional projector can be used to investigate beam pattern effects. Both the omnidirectional projector and parametric technique will involve generating a signal with a PC, driving a card and a hydrophone through a poweramp.

The basic equipment required for this approach is depicted schematically in figure 8. All these techniques will depend upon being able to use a small craft to deploy safely a sound source in the vicinity of the test hydrophone.

![Figure 8: Schematic of the parametric calibration system.](image)

### 4.3 Hydrophone Array Studies

In addition to the basic research on acoustic detection outlined above, we will undertake studies aimed at assessing the optimal design and expected performance of a future dedicated hy-
drophone array.

4.3.1 Experimental Studies

Multiple hydrophones (up to three) will be read out simultaneously during the deployment of a calibration device and/or acoustic signal projector as described above. The feasibility of combining signals from multiple hydrophones to extract position and direction information will be investigated. Other issues that can be addressed in such a study include the degree of noise correlation between closely spaced hydrophones, and the impact of relative hydrophone position uncertainties on shower reconstruction.

4.3.2 Simulation Studies - Future Array Topologies

Existing hydrophone arrays are unlikely to be arranged in an optimal way for the detection and reconstruction of UHE neutrino induced showers. The narrow acoustic radiation pattern from such showers may make it difficult to ensure that signals are recorded in multiple hydrophones, as is required for accurate position and direction reconstruction. Using a Monte Carlo simulation we will investigate the effective detection volume and reconstruction resolutions of various hydrophone arrangements. The goal of these studies will be to arrive at an optimal hydrophone array design, and in addition to estimate the number of elements required in order to have appreciable sensitivity to given incident neutrino flux levels (e.g. the Waxman-Bahcall flux, as indicated on figure 2). Practical considerations such as how such an array might be constructed and read-out, or integrated with an optical detector such as ANTARES and cubic-kilometre scale Cerenkov detectors currently being planned, also need to be fed into this exercise as constraints.

5 Funding Requested

5.1 Equipment

As discussed above, there are two main hardware objectives to the proposed programme of work, namely the upgrade of the Rona array to take several hundred hours worth of data (which it is not capable of doing at the moment) and to develop a suitable calibrator to simulate acoustic signals in water. For the Rona upgrade a PC needs to be furnished with the appropriate high bandwidth ADCs (available as plug-in PCI cards) and a high capacity RAID disk array for remote data recording as the site has no WAN infrastructure. Such a system has already been costed by the DSTL, MoD via a recognised contractor for such work, i.e. Kaon (Real Time Systems and Processing) Ltd. Their estimate for a system incorporating a 16-channel, 16-bit PCI ADC with anti-aliasing filtering (included in consumables as less than £3k), 4Tb RAID array plus interfacing to Rona hydrophones, GUI development costs and on-site deployment and testing of the upgrade is summarised in the table 1.
In the case of a calibrator a number of different approaches will be explored (as outlined in Section 4). A reduced version of the Rona DAQ (PC plus ADC card without RAID array) will need to be purchased, along with 3 hydrophones (thus facilitating some directionality studies), to support these efforts. The estimated costs are shown in table 1.

### 5.2 Consumables

This section lists the consumables shown in table 2 that will be required to support the hardware activities.

### 5.3 Staff Costs

The request for staff costs comprises three distinct elements:

1. A research associate, to be based at Sheffield, to work full time on hardware and DAQ development, and to co-ordinate with the other groups represented in the proposal. This researcher would be employed on the Research and Analagous staff RA1A pay scale, starting at spine point 4 (£21125 per year).

2. Technical support from the MoD and the Universities. For the MoD component, it is assumed that the upgrade work at Rona, in particular, will require the expertise of the technical staff who are responsible for the existing array. A total of 80 man hours (2 man weeks) of effort is foreseen, this corresponds, following MoD advice, to £4,000. From the University side, it is assumed that throughout the project 30% of a University technician will be needed, this has been costed at an appropriate point (Grade D, point 1) on the University technical staff pay scale.

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Table 1: Equipment costs.

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<th>Description</th>
<th>Unit Cost</th>
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<td><strong>Rona array DAQ upgrade</strong></td>
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<td>Bespoke system from Kaon, discussed above</td>
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<td>comprising:</td>
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<td>£66479</td>
</tr>
</tbody>
</table>

---

15
<table>
<thead>
<tr>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dell Latitude C840 laptop PC% for portable DAQ</td>
<td>£2167</td>
</tr>
<tr>
<td>General Standards Inc. ADC* for portable DAQ</td>
<td>£2738</td>
</tr>
<tr>
<td>Optical test bench for laser based calibrator</td>
<td>£1440</td>
</tr>
<tr>
<td>Light sources and detectors, laser diodes</td>
<td>£1080</td>
</tr>
<tr>
<td>Design and construction of mechanical mountings for prototype calibrators</td>
<td>£1850</td>
</tr>
<tr>
<td>Short duration flash lamps, replacement bulbs</td>
<td>£2280</td>
</tr>
<tr>
<td>Protective clothing and security items for deployment operations on water</td>
<td>£1650</td>
</tr>
<tr>
<td>Benthos spheres, pressure and water resistant glass spheres</td>
<td>£1280</td>
</tr>
<tr>
<td>Miscellaneous electrical components</td>
<td>£1020</td>
</tr>
<tr>
<td>PCB prototyping for control electronics</td>
<td>£1540</td>
</tr>
<tr>
<td>Pressure and water resistant electrical cabling for deployment</td>
<td>£1300</td>
</tr>
<tr>
<td>Steel cabling for support of calibrators during deployment</td>
<td>£1500</td>
</tr>
<tr>
<td>GPS units for spatial synchronisation with underwater Rona array</td>
<td>£900</td>
</tr>
<tr>
<td>2 PCs for acoustic laboratories</td>
<td>£2460</td>
</tr>
<tr>
<td>Total</td>
<td>£23205</td>
</tr>
</tbody>
</table>

Table 2: Consumables costs.

% Full specification: Dell Latitude C840 PIV 2.2GHz, 60Gb HDD, 512Mb memory, 2nd battery
* Model number: PCI-16SDI-HS-4-DB50

3. It is usual, in the case of proposals submitted under the PPARC-MoD Joint Grants Scheme (JGS) the the costs of the MoD personnel on the project are included in the funding requested. As a consequence, costs for Dr. Chris Rhodes for a total of 300 hours on project are included here. The quoted cost of £15,000 are as advised by the MoD.

The total staffing costs are summarised in table 3.

<table>
<thead>
<tr>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Research Associate, 3 years incl. USS,NI and overheads</td>
<td>£120618</td>
</tr>
<tr>
<td>University Technician, 3 years at 30% effort incl. USS,NI and overheads</td>
<td>£25325</td>
</tr>
<tr>
<td>Ranges technical support</td>
<td>£4000</td>
</tr>
<tr>
<td>Dr. Chris Rhodes, DSTL, MoD, project support costs</td>
<td>£10000</td>
</tr>
<tr>
<td>Total</td>
<td>£159493</td>
</tr>
</tbody>
</table>

Table 3: Staff costs.

5.4 Operations

Both the development of the calibrator and the upgrade of the Rona array will involve some degree of operations at a deep water site or in the sea itself. Initial tests of prototype calibrator devices will be performed in suitable local water volumes, e.g. university swimming pool,
possibly Kilder Water reservoir outside Newcastle, etc. Following this however, further develop-
ment must take place at a dedicated facility with suitable infrastructure. Such a facility exists
at Kelk Lake, a 60000m² fresh water lake in East Yorkshire. Kelk Lake is owned by Neptune
Sonar Ltd., a company specialising in acoustic transducers for commerical and military appli-
cations, their estimated costs are summarised in table 4. For calibration of the Rona array a
number of deployments of a calibrator must take place from the sea above the Rona array. This
will necessitate hiring a boat. A suitable vessel, a 9.5 m RIB, is owned by the Seafari Adven-
tures company on the Isle of Skye, their quoted hire costs of this vessel are listed in table 4.
These costs are incorporated into the Consumables heading in the Je-SRP1.

<table>
<thead>
<tr>
<th>Description</th>
<th>Unit Cost</th>
<th>Amount</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neptune Sonar Calibration Facility, Kelk RIB hire</td>
<td>£470/day</td>
<td>15 days</td>
<td>£7050</td>
</tr>
<tr>
<td></td>
<td>£470/day</td>
<td>15 days</td>
<td>£7050</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td><strong>£14100</strong></td>
</tr>
</tbody>
</table>

Table 4: Operations costs.

5.5 Travel

As discussed above, after initial hardware development at the represented Universities and labs,
the calibrator will be developed at a facility such as that owned at Kelk Lake in East Yorkshire
before being deployed at Rona. In parallel with this the Rona upgrade will take place which
will also involve travel by the collaboration members to the Rona site. Our best estimate of
expected travel costs over the three year duration of the project are summarised in table 5.

<table>
<thead>
<tr>
<th>Description</th>
<th>Unit Cost</th>
<th>Amount</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train travel to collaboration meetings (4/year x 5 people)</td>
<td>£45/trip</td>
<td>4x4x3</td>
<td>£2700</td>
</tr>
<tr>
<td>Car Hire (incl. insurance, petrol) 20 days/year</td>
<td>£50/day</td>
<td>60 days</td>
<td>£3000</td>
</tr>
<tr>
<td>Rona: Accommodation, subsistence (10 man days/year)</td>
<td>£80/day</td>
<td>30 days</td>
<td>£2400</td>
</tr>
<tr>
<td>10 day trip to Stanford to work on AUTEC data</td>
<td>£1200</td>
<td>1</td>
<td>£1200</td>
</tr>
<tr>
<td>Incidental travel between institutes</td>
<td>£300/year</td>
<td>3 years</td>
<td>£900</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td><strong>£10200</strong></td>
</tr>
</tbody>
</table>

Table 5: Travel costs.

5.6 Summary

A summary of the funding requested is given in table 6. Through the PPARC-MoD JGS it is
intended that the total costs are split 50-50 between PPARC and the MoD.

6 Milestones, Time-Lines and Deliverables

The programme of work envisaged is summarised in figure 9.
Table 6: Summary of costs.

<table>
<thead>
<tr>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equipment</td>
<td>£71384</td>
</tr>
<tr>
<td>Consumables</td>
<td>£18300</td>
</tr>
<tr>
<td>Staff Costs</td>
<td>£159493</td>
</tr>
<tr>
<td>Operations</td>
<td>£14100</td>
</tr>
<tr>
<td>Travel</td>
<td>£10200</td>
</tr>
<tr>
<td><strong>Grand Total</strong></td>
<td><strong>£273477</strong></td>
</tr>
</tbody>
</table>

Figure 9: Gantt chart outlining time planning, milestones and deliverables.

7 Conclusions

An R and D programme is proposed to prove the acoustic detection of very high energy showers in water which should allow the detection of UHE cosmic ray neutrinos. The resources needed and the goals and milestones for the project are described in some detail. This work would take place in close collaboration with other groups in Europe and the US currently exploring this emergent field of particle astrophysics. It is expected that during or following this pilot programme the proposees will be formally collaborating in a larger initiative.
Appendices

A Acoustic Neutrino Detection: A Simulation

In this appendix the simulation of the acoustic pulse due to UHE neutrino induced cascades is described. The expected signal counting rate in a single hydrophone is estimated.

A.1 Total Interaction Rates

The following flux of ultra-high energy neutrinos\(^1\) is assumed, corresponding to an upper limit on the neutrino flux from photo-pion production in astrophysical sources that are relatively transparent to the primary protons \([5]\):

\[
E^{2}_\nu \frac{d\Phi}{dE_\nu} = 2 \times 10^{-8} \text{ GeV cm}^{-2} \text{s}^{-1} \text{sr}^{-1}
\]  

(3)

There are no direct measurements from neutrino scattering or DIS experiments of the neutrino nucleon cross section at a centre of mass energy corresponding to incident neutrinos above \(\approx 10 \text{ TeV}\). We take the cross section from a recent analysis that uses structure function measurements from HERA and elsewhere to extrapolate to the energy range of interest \([20]\). Then the rate of interactions per \(\text{cm}^3\) of water is given by:

\[
R = 4\pi N \int_{E_{\nu,\text{min}}}^{E_{\nu,\text{max}}} \frac{d\Phi}{dE_\nu} \sigma_{\nu N}(E) \, dE_\nu ,
\]  

(4)

where full solid angle coverage is assumed and \(N\) is the number of nucleons per \(\text{cm}^3\) of water. This integral for \(10^{10} \text{ GeV} < E_\nu < 10^{13} \text{ GeV}\), evaluated numerically, gives:

\[
R = 7.13 \times 10^{-25} \text{ cm}^{-3} \text{s}^{-1} .
\]  

(5)

For a sphere of radius 10 km, this translates into a yearly rate:

\[
R \approx 100 \text{ yr}^{-1} .
\]  

(6)

Upward going neutrinos at these energies are largely attenuated by the earth. As a result, a rough estimate is 50 events/yr in the energy range \(10^{10} \text{ GeV}\) to \(10^{13} \text{ GeV}\), for a detector that performs perfectly out to a range of 10 km. All further rate estimates are simply based on multiplying this number by the fraction of simulated events in the same energy range and volume that pass the given detection requirements.

---

\(^1\)The flux is for \(\nu_\mu + \bar{\nu}_\mu\) at source. Neutrino oscillation should equalise the flux of the different flavours at the Earth, but acoustic detection is sensitive to both hadronic and electromagnetic cascades, with comparable sensitivity to all incident neutrino flavours. There are considerable uncertainties in this flux from the different cosmological assumptions.
A.2 Single Event Simulation

The simulation of the acoustic signal produced by a neutrino induced cascade in water follows the approach described in [33], which in turn relies on the formalism developed in detail in [34]. The pressure measured at hydrophone location \( \mathbf{r} \) is computed by evaluating the following integral over the volume \( V \) of the cascade energy deposition:

\[
p(\mathbf{r}, t) = \int_V \rho_E(\mathbf{r}') \, G(|\mathbf{r} - \mathbf{r}'|, t) \, d^3 \mathbf{r}' .
\]

(7)

\( \rho_E \) is the spatial energy density of the cascade and \( G \), the pressure pulse resulting from a point-like thermal energy deposition, is given by:

\[
G(\mathbf{r}, t) = -\frac{\beta}{4\pi C_P} \frac{(t - r/c)}{r \sqrt{2\pi \tau^3}} \exp\left(-\frac{(t - r/c)^2}{2\tau^2}\right) .
\]

(8)

\( \tau = \sqrt{r/(\omega_0 c)} \) and the numerical values for the following constants take the values specified in [33]:

- \( \beta \): coefficient of thermal expansion \( \approx 2.0 \times 10^{-4} \) K\(^{-1}\),
- \( C_P \): specific heat capacity of water \( \approx 3.8 \times 10^3 \) J kg\(^{-1}\) K\(^{-1}\),
- \( c \): speed of sound in water \( \approx 1500 \) m/s,
- \( \omega_0 \): attenuation frequency \( \approx 2.5 \times 10^{10} \) s\(^{-1}\).

We have used a simple cylindrical model for the energy deposition by ultra-high energy neutrino induced cascades, as indicated in figure 4. A fixed cascade length of 10 m and a Gaussian cross section with a mean radius of a few centimetres roughly corresponds to, and reproduces the waveform amplitudes resulting from, the hadronic cascades described in [33]. Future studies will require detailed shower simulations of both hadronic and electromagnetic cascades in water.

A.3 Event Ensemble

100,000 events are generated randomly following a \( 1/E^2 \) distribution in energy in the range \( 10^{10} < E < 10^{13} \) GeV according to the integrand of equation 4. The events are generated with uniform density in a 10 km-radius sphere and with random spatial orientations. For each event, the angle between the plane transverse to the shower direction and the hydrophone at the coordinate origin is calculated (see figure 4). This angle, together with the distance from the hydrophone, form the input to the single event simulation. For reasons of CPU efficiency, events for which the hydrophone is more than 5 degrees out of the transverse plane are not fully simulated, since the fall of peak pressure with angle, plotted in figure 12, is extremely steep. The assumption that events beyond 5 degrees contribute negligibly to the overall rate is

\[^2\text{The coefficient of thermal expansion has been calculated for the temperature and salinity of RONA water.}\]
shown to be safe in section A.4. Events less than 500 metres from the hydrophone are simulated regardless of the detection angle.

As an example, the pressure pulse at a hydrophone from an event with energy $1.2 \times 10^{11}$ GeV, computed according to equations 7 and 8 at a distance of 1 km and at an angle of zero degrees with the plane transverse to the shower direction, is shown in figure 5. The peak pressure for the event is 0.08 Pa. The distribution of peak pressures for the ensemble of events is shown in figure 10.

![Figure 10: The distribution of peak pressure for the ensemble of events described in the text.](image)

A.4 Results of the Simulation

We assume that a $10^{11}$ GeV event at 1 km and zero degrees represents the limit of detection. Events of this energy and range are discussed in [33], although the analysis described therein is preliminary and is currently being revised. This corresponds to a peak pressure of 0.067 Pa. Clearly detection at this level has still has to be firmly established but is a suitable metric on which to base this study. A more sophisticated analysis of hydrophone sensitivities and signal extraction in the presence of noise may enable this threshold to be lowered. Figure 11 shows various distributions for the whole ensemble of events, and those passing the cut on the peak pressure. A number of features are noteworthy:

- The distribution of the radius of detected events is fairly flat. Reducing the threshold to improve the statistics results in a distribution peaking at several km. There may be some small additional sensitivity beyond 10 km, although detection at such large distances faces other challenges such as significant refraction of the acoustic pulse due to the velocity-depth profile of the ocean.
Figure 11: The distributions on the left are for all 100,000 events in the ensemble. The corresponding distributions on the right are for those events that pass a threshold cut of 0.067 Pa on the peak pressure detected at a single hydrophone.
The energy distribution of detected events indicates that there may be some additional sensitivity beyond $10^{13}$ GeV, despite the rapidly falling flux. The interplay between flux, cross section, signal strength and effective volume appear to result in very little sensitivity below $10^{11}$ GeV.

The very strongly peaked angle distribution of detected events indicates that the cut in the simulation of 5 degrees is safe.

The number of events passing the simple threshold cut of 0.067 Pa is 50, resulting in a naive rate estimate of 0.03 events/year. However a number of factors probably act to reduce this number significantly, as discussed in the following sub-sections.

### A.4.1 Angular Dependence

Figure 12 compares the peak pressure as a function of angle out of the transverse plane for this simulation and the results of [33]. The pressure falls more steeply with angle for the latter analysis, presumably due to the more realistic shower model used. Also shown in the figure is the result of scaling the pressures predicted in this simulation by an ad-hoc factor of $1.8^{-\theta}$, to bring the angular distribution more into line with the prediction of [33]. Applying this factor to all events reduces the number events above threshold by almost a factor of two, to 31.

![Figure 12: The peak pressure as a function of detection angle out of the transverse plane perpendicular to the shower direction. The solid line with solid circle markers is the prediction of the simple simulation used here. The solid line with open circle markers is the prediction of [33], normalised to the same peak pressure and shifted by $-0.6^\circ$ such that the pressure maximum is at zero degrees. The dashed line is the result of scaling the first curve by an ad-hoc angle-dependent factor, to bring the predictions into better agreement. See text for more details.](image-url)
A.4.2 Energy Fraction

The simple simulation used in this analysis assumes that the entire energy of the incident neutrino is deposited as thermal energy in the resulting cascade. This cannot be the case, since at least some fraction of the incident neutrino energy (for example, the energy carried away by the neutrino in neutral current interactions, the muon in charged current interactions and, at UHE, electrons via the LPM effect) does not contribute to the thermo-acoustic coupling of the cascade. Note that the simulation used here should reproduce the fraction of thermal energy that is converted into acoustic energy, since the equations used to generate the pressure pulse are those derived in [34] from first principles. Moreover, the dimensions of the simple cylindrical energy deposition model have been tuned to roughly reproduce the peak amplitudes presented in [33], so any additional factors should not be large. If it is assumed that only 50% of the initial neutrino energy is converted into thermal energy, this corresponds to multiplying the peak pressures by 0.5. Applying this factor results in a reduction of the number of detected events to just 11.

A.5 Conclusions

The final estimate based on the simulation described here is that the rate of detectable events in the energy range $10^{10} - 10^{13}$ GeV by a single hydrophone in a 10 km-radius sphere of water is approximately $5.5 \times 10^{-3}$ per year, assuming a Waxman-Bahcall incident flux. There may be some sensitivity outside this energy range and detection volume, but we do not expect a large factor. The error on this estimate resulting from uncertainties in the signal modelling is large - probably an order of magnitude. A detailed particle level shower simulation is required in order to reduce these uncertainties, work which is currently underway (see figures 13 and 14).

A single hydrophone count rate of $\mathcal{O}(10^{-2} - 10^{-3})$ /yr indicates the likely number of hydrophones that would ultimately be required in an array in order to achieve a flux sensitivity comparable to the Waxman-Bahcall flux. Coincidence signals will certainly enable thresholds to be lowered and reduce noise, but there will probably be a significant geometric penalty due to the shape of the acoustic radiation pattern.

It must be stressed that the efficiencies and fake rates of the signal extraction algorithms that will need to be employed are not yet known, and will probably only be known after extensive analysis of RONA hydrophone data.

B Acoustic Modelling for High Energy Neutrino Detection

B.1 Introduction

The interaction of a high-energy neutrino in seawater has been addressed by Lehtinen et al [33] and in Appendix A for a $1.2 \times 10^{11}$ GeV event. The results of these calculations indicate that, given certain assumptions about energy deposition, a characteristic bipolar pressure pulse of approximately 100 $\mu$s in duration is generated. Figure 15 shows the form of the pressure pulse,
Figure 13: The thermal energy deposition due to an electromagnetic cascade in water, calculated using GEANT4 [38]. A 1 TeV electron is incident at the point indicated by the arrow. Positive and negative azimuthal angles map onto positive and negative radii in the \( \{r, z\} \)–plane. The relative energy density is indicated by the colour scale on the right hand side.

at a distance 1km and on a plane perpendicular to the shower, and its associated power spectrum for a \( 1.2 \times 10^{13} \) GeV event. It is assumed that the signal is sampled at a rate of 200kHz.

The purpose of this Appendix is to present basic sonar modelling calculations on the effects of propagation and noise on the detectability of such a pulse. Through the use of Receiver Operation Curves (ROCs) it is possible to investigate the interplay of effective detector volume with false alarm rates. The procedure used is the standard approach to modelling underwater acoustics and detection, and is presented in Refs [39, 40].

B.2 Calculating the Source Level of the Pulse

The pulse shown here is calculated at 1000m range from a highly directional cylindrical source. The frequency distribution of the pulse shows a broadband signature, with most of the energy
Figure 14: The pressure pulse at 1 km due to a 1 TeV electromagnetic shower, calculated using equation 7 but with the energy density taken from a detailed shower simulation. The pressure pulse is similar to that derived from a toy simulation described in the text.

Figure 15: The waveform and its frequency components.

around 10kHz. Most of the acoustic energy concentrated is in a very narrow lobe in a plane
perpendicular to the cylinder. It is desirable to be able to estimate an approximate value for the source level of the pulse for use in subsequent performance assessment calculations. It is noted at this stage that whilst it is normally difficult to estimate the pressure spectrum directly, as it contains both amplitude and phase information it is trivial to calculate the intensity spectrum (which is phase independent), hence we proceed by calculating the intensity spectrum and re-convert to a pressure spectrum. The root mean square (rms) pulse level over the period from \(-50\mu s\) to \(50\mu s\) can be calculated using

\[ P_{rms} = \sqrt{\frac{1}{21} \sum_{-10}^{10} x_n^2} = 0.052 \text{ Pa} \]  

(9)

where \(x\) is the signal and \(n\) the sample. Note also with the 200 kHz sampling rate the pulse is approximately 21 samples in length.

The relationship between intensity, \(I\), and pressure \(P\) is given by

\[ I_{rms} = \frac{P_{rms}^2}{Z_0} \]

where \(Z_0\) is the characteristic acoustic impedance of water \((Z_0 = \rho c = 1.5 \times 10^6 \text{ kg m}^{-2} \text{ s}^{-1})\), yielding \(I_{rms} = 1.8 \times 10^{-9} \text{ W m}^{-2}\). Dividing by the signal bandwidth (taken to be 20kHz) yields an intensity level per unit bandwidth (Hz) of \(9 \times 10^{-14} \text{ W m}^{-2} \text{ Hz}^{-1}\). Converting this figure to \(\text{dB} \) to give the intensity spectrum level

\[ I_l = 10\log(I/B) = -130dB_W \]  

(10)

(Note \(dB_W\) is relative to \(1\text{ W m}^{-2}\) in a 1Hz band.)

It is straightforward to convert this value to a pressure spectrum level by the addition of +182\text{dB}, giving 51\text{dB} (re 1\text{\mu Pa Hz}^{-1/2})^3.

### B.3 Calculating the Pulse Level as a function of distance

As stated above, these results are for the pulse as measured at 1000m. To refer back to a source level at 1m, we assume a \(20\log(1000)\) dB geometric spreading, and take an absorption value at 10kHz of around 1dB/km. This gives a total transmission loss of 60 (geometric loss) + 1 (absorption) = 61dB.

The overall source level \((SL)\) is therefore given by summing the 51dB from Section B.2 with this value, i.e.

\[ 51 + 61 = 112dB \]  

(re 1 \(\mu\text{ Pa per Hz}^{1/2}\) at 1m around 20kHz).

As a cross-check, taking the \(SL\) to be 112 dB re 1\(\mu\text{ Pa per Hz}^{1/2}\) at 1m, it is straightforward to calculate the received level as a function of range. This is given by the equation:

\[ SL = 112 - 20\log(R) - 1 \times 10^{-3}R \text{ dB} \]  

(12)

where \(R\) is the distance from the source. We note that in the absence of attenuation this yields a pulse pressure level which falls off linearly with distance as shown in figure 16.\(^{3}\)

\(^{3}\text{1Wm}^{-2} = 1.225 \times 10^6 \mu\text{ Pa}\)
B.4 Directionality of the Pulse

The source level in a direction away from perpendicular will be modulated by the beam pattern corresponding to a line source of this size at approximately 20kHz. The radiation pattern for a coherent thin line source corresponding to the dimensions of a $10^{11}$ GeV shower can be easily calculated in the far field using the Fraunhofer approximation and is of the form:

$$I = I_0 \left( \frac{\sin \beta}{\beta} \right)^2$$

where $\beta = \pi L/\lambda$ and $L$ is the length of the source ($\sim 10$m) Figure 17 gives the intensity level versus bearing from broadside for the 10kHz component of the signal. It is clear from this plot that the maximum 112dB of source level is only available in a narrow sector perpendicular to the interaction volume, and over most aspects the source level is attenuated by 30-50dB with respect to this peak level.

B.5 Calculating the Energy in the Pulse

For a given source level $SL$, the power in the pulse is given by

$$SL = 171 + 10 \log P + DF$$

(14)
where $DF$ is the directionality factor. This is zero for an omni directional source (spherical source). For our geometry this can be approximated assuming the energy is radiated in an annulus of width $\lambda/L$ and can be calculated using

$$DF = 10 \log \left( \frac{4\pi}{2\pi \lambda/L} \right) = 10 \log \left( \frac{2L}{\lambda} \right) = 21 \text{ dB}$$

for $L=10$ m and $\lambda=15$ cm (10 kHz). This gives a power level of $1 \times 10^{-8}$ W. Taking the pulse duration to be around $100 \mu s$, this gives a pulse energy of $1 \times 10^{-12}$ J.

The incoming particle has energy $1.2 \times 10^{11}$ GeV = 3.2 J. The efficiency of the conversion of particle energy to acoustic energy is therefore of order $10^{-13}$.

### B.6 Noise, Propagation Loss and Detection

In the above we have estimated a source level $SL$ of 112 dB (re. $1 \mu Pa$ per Hz$^{1/2}$ at 1m) for the pulse waveform at around 10kHz. We now wish to calculate the effects of losses due to propagation through the water and the effect of noise. Both of these have a consequential effect on the detectability of the signal.

The propagation loss, $PL$, for this pulse is taken to be a combination of geometric spreading and absorption i.e. $PL = 20 \log R + \alpha R$, where $\alpha$ is taken to be the absorption co-efficient for a 10kHz source (i.e. 1dB/km).
The noise level in any detection system will be $N$ taken from Knudsen-Wenz curves [39] or locally measured curves. Here we take the ambient noise level $N$ to be no greater than 45dB at 10kHz, from historic Rona data. This corresponds to around Sea State 3 for this site.

For broadband passive detection:

\[ SL = PL + N - DI + DT \]  \hspace{1cm} (16)

Where $N$ is the noise, $DI$ is the sensor directivity index and $DT$ is the detection threshold (discussed below).

Assuming an omni-directional hydrophone detector,

\[ DI = 0, \text{ so } SL = PL + N + DT. \]  \hspace{1cm} (17)

Assuming a correlator (matched filter) detector [39],

\[ DT = 5 \log d - 5 \log BT \]  \hspace{1cm} (18)

where $d$ is the detection index. Thus

\[ SL = 20 \log R + \alpha R + N + 5 \log d - 5 \log(BT) \]  \hspace{1cm} (19)

From the above discussions,

- source level, $SL = 112$dB re 1 $\mu$Pa at 1m;
- absorption coefficient, $\alpha = 1$dB/km;
- noise level, $N = 45$dB;
- bandwidth, $B = 50$kHz;
- pulse duration, $T = 100 \mu$s.

To determine an appropriate value for the detection index, $d$, we assume a Probability of Detection (PoD) = 0.5 and a Probability of False Alarm (PFA) = $10^{-13}$. Therefore, $5 \log d = 13.5$dB, $5 \log BT = 3.5$ so $DT = 13.5 - 3.5 = 10$dB. (For the purposes of this calculation it is appropriate to use Receiver Operating Characteristic curves relating to active detection to determine $d$. The ROC curves characterise the thresholding that is central to the decision process when making a detection. For a given signal type in noise it is possible to systematically vary the threshold and infer the probability that an event will be correctly identified [39, 40].)

PoD corresponds to - if a pulse is generated in the water what is the probability that a SIGNAL PRESENT event will be called, and PFA corresponds to - the probability that SIGNAL PRESENT will be called in the absence of a real signal in the water. Even with matched filtering, to get the probability of detection reasonably high and a false alarm rate that is low will require several dB of signal/noise. Quite how the false alarm rate is set will determine the effective detection range and hence detector volume.
Substituting all these values in the above equation, we are left with an expression to be solved for range, \( R \).

\[
20 \log R + 1 \times 10^{-3}R = 112 - 45 - 10 = 57 \text{dB.}
\] (20)

\( R \) will therefore be approximately 800m.

This corresponds to the maximum range at which it is possible to make detections with the stated PoD and PFA. (No sonar degradation factors have been included here). This detection range corresponds to a detector size of \( 200 \cdot \pi \cdot 800^2 \approx 4 \times 10^8 \) m\(^3\) for each individual hydrophone at Rona. However, if we are off-axis (i.e. off the beam) the source level will be down by 40-60dB which means that it will not possible to make a detection at this range.

In the paper by Butkevitch [41] there is a \( 10^7 \text{GeV} \) pulse calculation, with a bipolar pulse peaking at 60\( \mu \text{Pa} \). This corresponds to a \( SL \) of around -18dB re 1\( \mu \text{Pa} \) per Hz\(^{1/2}\) at around 10kHz. Allowing for propagation, \( 20 \log(400) = 52 \text{dB} \) so this gives a source level \( SL \) of -18+55db = 37dB re 1\( \mu \text{Pa} \) per Hz\(^{1/2}\) at 1m. This would not be detectable at Rona. Interpolating between the source levels inferred from the Butkevitch figure and the Lehtinen paper, we get \( SL = 20 \log_{10}(E) - 274 \text{dB} \). We can now solve \( 20 \log R + \alpha R = SL - 55 \) for the detection distance or range, \( R \). The numerical solution is shown in Figure 18.

![Figure 18: Detection distance (range) vs energy.](image)

The curve in figure 18 can be interpreted as follows. As the neutrino interaction energy increases the signal to noise ratio increases because the ambient noise background remains the same. This gives an ever-increasing signal excess that results in longer detection range. Hence the distance at which events can be reliably detected will increase with energy.
C. A Digital Signal Processing Perspective

C.1 Angular distribution of the pulse

We note that in far field ($R$ is over approximately 500m from the shower in our case) for a single frequency the pressure may be calculated using the spatial Power Spectral Density function:

$$I(\theta) = \frac{P^2}{Z_0 R^2} \left[ \int_{d/2}^{d/2} f(x) \exp(-i\omega_{sp} x) dx \right] \left[ \int_{d/2}^{d/2} f(x) \exp(-i\omega_{sp} x) dx \right]^*$$

(21)

where $P$ is the pressure at source, $Z_0$ the characteristic acoustic impedance of the water, $d$ the length of the source, $f(x)$ the aperture function, which will be a constant in this case, and $\omega_{sp} = k \sin \theta$ is the spatial frequency ($k = 2\pi/\lambda$) and the * denotes the complex conjugate. This integration yields an acoustic intensity:

$$I(\theta) = \left( \frac{P^2 d^2}{Z_0 R^2} \right) \left( \frac{\sin^2 \beta}{\beta^2} \right)$$

(22)

where $\beta = \omega_{sp} d/2$. For small angles ($\sin \theta \sim \theta$) the half width of the central lobe works out simply as $\lambda/d$.

Here we consider the intensity profile from a coherent source radiating over a range of frequencies from 0 up to $f_c$, the cut off frequency. The pressure profile is thus given by integrating (summing) the various $(\sin \beta/\beta)^2$ curves produced by each of the individual frequencies. Expressed simply each individual frequency will produce a $(\sin \beta/\beta)^2$ curve with a central lobe half width of $\lambda/d$. This width depends on $1/f$, therefore the high frequency component will be much more directional than the low frequency component.

We note also that a mathematically equivalent way of producing this set of $(\sin \beta/\beta)^2$ curves is to keep the frequency constant but to vary the width of the aperture function, rather than varying $\lambda$ we vary $d$. (Note the height of the aperture function also needs to be varied to keep the energy per unit bandwidth constant).

Furthermore because the Fourier transform is linear this addition can be done before the Fourier transform is taken. The angular distribution of the pulse has been calculated using the pulse spectrum at 1000m (illustrated in figure 15) to weight each frequency component. Figure 19 has been integrated numerically using a 1024 point Fast Fourier Transform.

C.2 Matched Filter Recovery Simulation

C.2.1 Introduction

A comprehensive description of matched filtering is given in [42] however it is couched in language unfamiliar to a larger part of the Astroparticle Physics community. We attempt here a brief explanation of matched filtering. A standard filter will pass certain frequencies and not pass others. It is trivial to demonstrate that with a matched filter the frequency response of the
filter has to be optimised to that of the signal. Indeed if there is no noise present, or if the noise is flat ("white noise") the frequency response of the matched filter will be identical to that of the signal. Should the noise have a non-flat frequency response, the filter needs to be optimised such that it will maximise the response to the signal, whilst simultaneously minimising the response to the noise. Matched filters also vary from standard frequency filters in that they also match the phase of the signal. Indeed they are designed so that the filter produces an output which is proportional to the energy of the pulse at time $t$ where the signal completely overlaps the impulse response of the filter. The basic procedure is illustrated in figure 20.

Let us now look at a few examples.

We assume we are using a non recursive digital filter and assume $x[n]$ is the input to the filter and $y[n]$ the output. Here $x[n]$ represents the $n^{th}$ reading. Then we can design a filter such that

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \ldots + b_k x[n-k]$$

(23)

The output is a weighted sum of the current input, the previous input, the next previous input and so on. The output $y$ is the convolution sum of the filter coefficients and the input sequence:

$$y[n] = b[n] \ast x[n] = \sum_{k=-\infty}^{\infty} b[k] x[n-k]$$

(24)

where $b[i]$ represents the fraction of the signal in the $i^{th}$ reading. Note in these examples only readings are taken before the event. However, it is possible in the off-line analysis to use the data before and after the event.

**Example 1**

Input sequence 1,0,0,0,0,0
Figure 20: A block diagram showing the proposed signal extraction procedure.

\[ b_0 = 1, b_1 \text{ to } b_k = 0 \]

then \( y[n] = x[n] \). Note that if all the energy of the signal is deposited in within one sampling period, the detection is already optimal. Note that the filter will also pass noise. If the Noise level is \( N \), at the input to the filter, it will also be \( N \) at the output of this filter. Or \( Y = 1.X + N \).

**Example 2**

Input sequence \( 1/\sqrt{2}, 1/\sqrt{2}, 0, 0, 0 \). Here the energy is split between two bins and we keep the total amplitude as 1.

\[ b_0 = 1/\sqrt{2}, b_1 = 1/\sqrt{2} b_2 \text{ to } b_k = 0 \]

\[ y[n] = 1/2, 1, 1/2, 0, 0, 0 \] and the output at time \( t \) is proportional to the energy in the sequence.

Note that the filter will also pass noise. We assume the Noise level in \( N \), at the input to the filter and that the noise is normally distributed, totally uncorrelated between one sample and the next (white noise). We assume an orthogonal basis set, (simply a Cartesian \( x \) and \( y \) axis in this case) with one axis aligned \( x[n] \) and the other with \( x[n+1] \). The output of the filter is a linear combination of these two coordinates and can be expressed in vector format as \( Y = B.X + B.N \) or the dot product between the filter and the signal plus noise. As the noise has no preferred direction it will appear to occupy a circle in this vector space. Furthermore \( |B| = 1 \). Irrespective of the orientation of \( B \): \( Y = B.X + N \)
Also as B is aligned in the direction of X (at when there is total overlap between the filter and signal) we get $Y = |1.X| + N$ which is identical to example 1.

**Example 3**

$x = 0.2673, 0.5345, -0.8018$. Here the energy of the signal is divided unequally between three bins

$b_0 = -0.8018, b_1 = 0.5345, b_2 = 0.2673$

$y = -0.2143, -0.2857, 1.0000, -0.2857, -0.2143$

Note that using the same argument as in Example 2 and again white noise, we can choose a three dimensional vector space. The noise will now be distributed in a sphere of radius N and have no preferred direction. Indeed in general the noise will form a hyper sphere of radius $|N|$. Indeed again we will get an output identical to example 1.

We note that in general the output of a non-recursive digital filter is the convolution sum of the input sequence $x$ with the filter coefficients $b$. It is worth noting also that in all three cases the detection level is the same. Assuming a flat noise distribution a matched filter is simply a time reversed copy of the input signal and the output of the filter is the autocorrelation function of the signal. If the noise level is not flat (not white) then a pre-whitening filter can be used in order to make the noise flat.

### C.3 A matched filter for the Rona Hydrophone array

Figure 21 gives the Noise level as measured at Rona. We have used the smoothed curve illustrated in the figure. We wish to model this assuming a white noise source passed through a digital filter. Note that it is not necessary to model the filter over the entire frequency range but within the bandwidth of the signal, which we have chosen to be between 500Hz and 50kHz. We assume the filter can be modelled by a 3rd order continuous transfer function $H(s)$. This transfer function is the Laplace transform of the output over the Laplace transform of the input and is a solution to the 3rd order differential equation:

$$a_n \frac{d^n y}{dt^n} + \ldots + a_1 \frac{dy}{dt} + a_0 y = b_n \frac{d^n u}{dt^n} + \ldots + b_1 \frac{du}{dt} + b_0 u$$  \hspace{1cm} (25)

where $a_i$ and $b_i$ are constants, $u$ the input to the system and $y$ the output from the system and $n = 3$ in this case. This system has a corresponding transfer function:

$$H(s) = \frac{b_n s^n + \ldots + b_1 s + b_0}{a_n s^n + \ldots + a_1 s + a_0}$$  \hspace{1cm} (26)

This function was then optimised using Nelder Mead Simplex optimisation yielding the transfer function constrained such that the transfer function (and its inverse are stable). This optimisation yielded the following transfer function:

$$H(s) = \frac{1.414 \times 10^{-13} s^3 + 3.079 \times 10^{-8} s^2 + 0.3161 \times 10^{-3} s + 0.09933}{4.969 \times 10^{-13} s^3 + 5.07 \times 10^{-8} s^2 + 0.3391 \times 10^{-3} s + 1}$$  \hspace{1cm} (27)
Figure 21: Typical ambient noise level at Rona and the Knudsen curves. The curves labelled SS0-3 represent estimates of the noise in different sea states (SS). The blue curve shows the measurements at Rona at a typical wind speed of 11 knots i.e. somewhere between SS 1 and 3 and the red curve a parameterisation of these data. The Rona data show more noise than normal in the low frequency range 1-100 Hz which is outside our range of interest.

Figure 22 illustrates the fit

We note that the fit is within 1/2 dB for the region specified and there is no need to attempt a higher order transfer function.

Although such a filter is highly useful it is computationally difficult to implement. It is computationally much easier to solve difference equations of the form:

\[ a_k y[n-k] + \ldots + a_1 y[n-1] + a_0 y[n] = b_k x[n-k] + b + \ldots + a_1 x[n-1] + b_0 x[n] \]  

(28)

Indeed as we will be implementing the filter on a sampled sequence at Rona this format is more useful. Just as differential equations can be converted into transfer functions using the Laplace transform, difference equations can be converted by using the z transform:

\[ Z(x) = \sum_{n=-\infty}^{\infty} x[n] z^n \]  

(29)

where \( x_n \) is the sequence and \( z \) is a complex variable. A transfer function can now be written in terms of the z transform of the output over the z transform of the input

\[ H(z) = \frac{a_n z^n + \ldots + a_1 z + a_0}{b_n z^n + \ldots + b_1 z + b_0} \]  

(30)
Figure 22: The observed spectrum is the blue curve and the fit described in the text is given by the green curve.

Converting from a transfer function $H(s)$ to $H(z)$ is a standard procedure, see for example [43]. This filter was converted into the digital domain using the Bilinear Transform with a 200kHz sampling rate (to match the sampling frequency it is intended to use at Rona) and prewarping at 10kHz. This ensures a perfect match between $H(s)$ and $H(z)$ at 10kHz. Yielding:

$$h(z) = \frac{0.3528z^3 - 0.7982z^2 + 0.5509z - 0.1054}{z^3 - 2.579z^2 + 2.172z - 0.5926} \quad (31)$$

This is a recursive digital filter of the form:

$$y[n] = 0.3528x[n] - 0.7982x[n - 1] + 0.5509x[n - 2] - 0.1054x[n - 3] + 2.579y[n - 1] - 2.172y[n - 2] + 0.5926.$$

Thus the output at time $t$, depends not only on current and previous input but also on previous outputs. Care must be taken to ensure the filter is stable. The inverse filter $h^{-1}(z)$ is simply given by

$$h^{-1}(z) = \frac{z^3 - 2.579z^2 + 2.172z - 0.5926}{0.3528z^3 - 0.7982z^2 + 0.5509z - 0.1054} \quad (33)$$

Figure 23 illustrates the process. A Gaussian pseudo-random number generator is used to simulate white noise. This noise is filtered to match the spectrum at Rona using digital filter
\[ h(z) \] above. The signal is then added. The signal and noise are inverse filtered to ensure the noise is white. (This is also known as maximum entropy).

The process modifies the shape of the pulse, (however in this case as the variation is only about 6dB across the pulse bandwidth the change is slight) and can be seen in Fig 23a. The matched filter will be a time-reversed copy of this (inversely) filtered pulse. The process is illustrated in figure 23 showing the noise and signal at the input to the matched filter and Figure 23c the output.

We note that the enhancement factor for the matched filter is 2.65 or 8.5dB as opposed to a discriminator based upon the original signal.

### C.4 Setting the Threshold level

Given the very low signal rate we need to ensure the probability of noise triggering the electronics is low. We limit our rate to one event every 10 years assuming a random noise distribution. As we are sampling at 200kHz we would take the equivalent of \(6.3 \times 10^{13}\) measurements in this period, hence we need a probability of triggering which is less than about \(1/6.3 \times 10^{13}\).
Assuming the noise obeys Gaussian statistics we can determine the probability of such an event by integrating the Gaussian curve from the threshold to infinity (figure 24). This yields a value of approximately 7.5 sigma. With the matched filter in place this reduces to $7.5/2.65 = 2.85$ sigma. Hence the pressure threshold is such that the peak signal must be 2.85 times the root mean square of the noise signal. One of the purposes of the project is to prove the feasibility of this computation, i.e. to check for the presence of non-Gaussian tails which would invalidate the calculation.

Figure 24: The probability of an event occurring above threshold as a function of the threshold level.

C.5 Determination of Detection Range

The overall noise level (assuming an anti-aliasing filter) can be calculated from the measured noise spectrum at Rona, as illustrated in Figure 21, by normalising to a particular frequency and integrating the area under the curve. It must be remembered in this methodology that the noise is pre-whitened before being applied to the matched filter. It is advantageous therefore to have most of the noise out of the bandwidth of the signal (500Hz-20kHz). Furthermore the noise will be dominated by the frequency region from 20kHz to 100kHz as this contains 80% of the bandwidth. In our case unfortunately the noise level is highest within the bandwidth of the signal, thus the apparent noise will be increased. Performing the integration and dividing by a flat distribution normalised to 10kHz yields an increase of noise level of 1.54 or 3.75dB. The noise level at Rona is of the order of 45dB Hz$^{1/2}$ at 10kHz, the pre whitening will increase this to a 48.72 dB Hz$^{1/2}$ over the 0-100kHz bandwidth. The effective threshold level will be: 

$$48.72 + 20 \log(2.85) = 57.8 \text{ db.}$$

(The factor of 2.85 is taken from section C.4). It is noteworthy that the figure determined in Appendix B is 57dB and hence the two values are in very close agreement, giving confidence that the threshold level estimation is fairly robust. The effective radius of the detector at various energy levels is thus in agreement with that of appendix B.
References


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