Anomalous Couplings

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- Reminder of method (from Benjamin & Errede).
- Testing the method using $W\gamma$ pseudo−experiments.
- Results of dummy pseudo−experiments.
- Feldman−Cousins confidence intervals.
- Application of method to dummy pseudo−experiments.
- Summary.

Outline of Method

- Directly following Run I method of Benjamin & Errede.
- The total cross section depends on anomalous couplings as :

$$\sigma_{TOT}(W\gamma) = \sigma_{SM} + (\Delta\kappa)\sigma_1 + (\Delta\kappa)^2\sigma_2 + (\lambda)\sigma_3 + (\lambda)^2\sigma_4 + (\Delta\kappa\lambda)\sigma_5$$

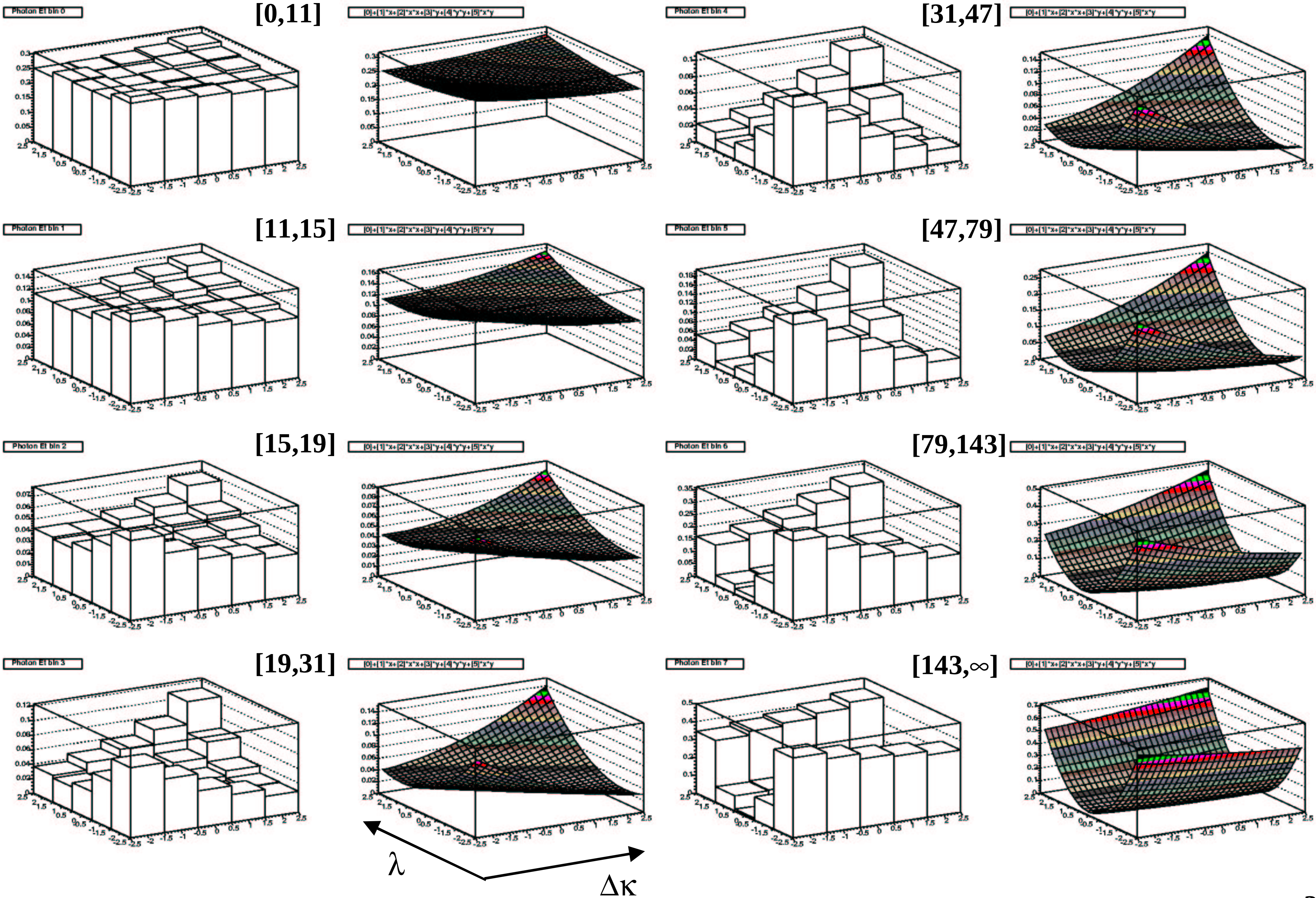
- Differential cross sections also behave in this way (1) :

$$\frac{\mathrm{d}\sigma(W\gamma)}{\mathrm{d}X} = \frac{\mathrm{d}\sigma_{SM}}{\mathrm{d}X} + (\Delta\kappa)\frac{\mathrm{d}\sigma_1}{\mathrm{d}X} + (\Delta\kappa)^2\frac{\mathrm{d}\sigma_2}{\mathrm{d}X} + (\lambda)\frac{\mathrm{d}\sigma_3}{\mathrm{d}X} + (\lambda)^2\frac{\mathrm{d}\sigma_4}{\mathrm{d}X} + (\Delta\kappa\lambda)\frac{\mathrm{d}\sigma_5}{\mathrm{d}X}$$

- After applying cuts and integrating over bin *i* :

$$\sigma^i_{TOT}(W\gamma) = \sigma^i_{SM} + (\Delta\kappa)\sigma^i_1 + (\Delta\kappa)^2\sigma^i_2 + (\lambda)\sigma^i_3 + (\lambda)^2\sigma^i_4 + (\Delta\kappa\lambda)\sigma^i_5$$

- Fit the parameters σ^i_j using Monte Carlo.
- [Note : the Monte Carlo could be adapted to generate event samples corresponding to each differential cross section in (1) above, which can then be combined with relative weights according to the anomalous coupling combination in question. No fitting required.]


[0,11]
[11,15]
[15,19]
[19,31]
[31,47]
[47,79]
[79,143]
[143,∞]
λ
Δκ

Outline of Method

- With these parameterisations of the cross section in each bin, the expectation for a given value of anomalous couplings can be computed.
- Then compute the log−likelihood in the normal way :

$$L = \prod_i \frac{e^{-\mu_i} \mu_i^{n_i}}{n_i!} \qquad \mu_i = \mu_i(\Delta\kappa, \lambda)$$

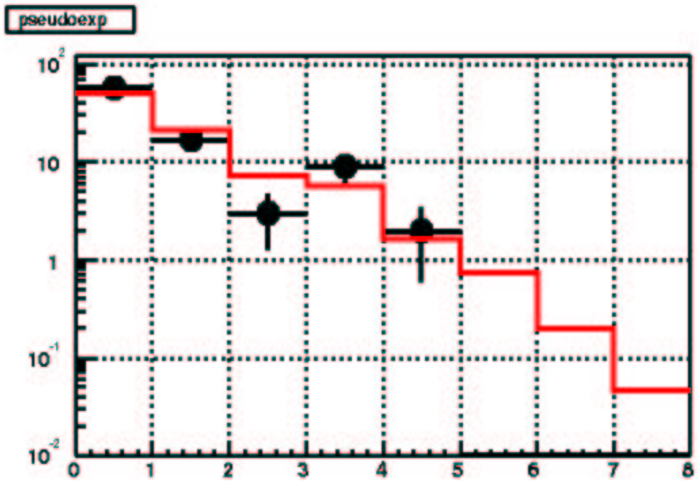
$$\log(L) = \sum_i [-\mu_i + n_i \log(\mu_i)] + C$$

- 1−sigma and 2−sigma confidence intervals are formed in the standard way :

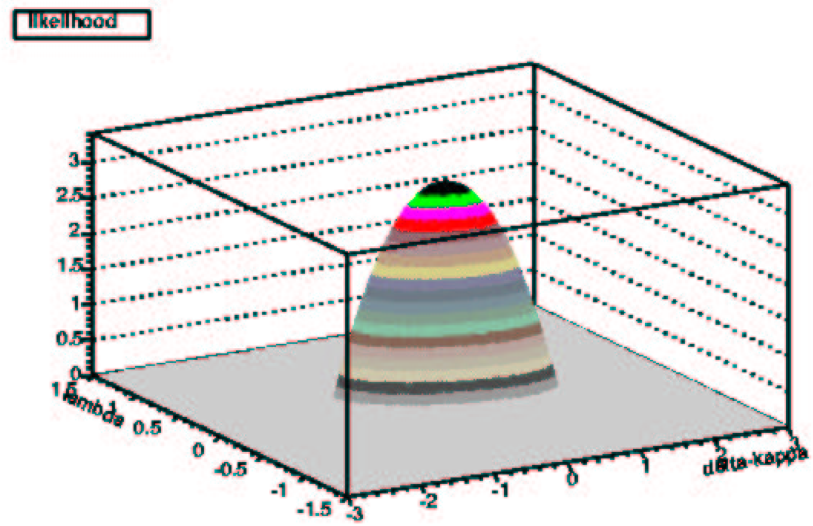
$$\Delta[\log(L)] = 1.15 \qquad (1\sigma)$$
$$\Delta[\log(L)] = 3.09 \qquad (2\sigma)$$

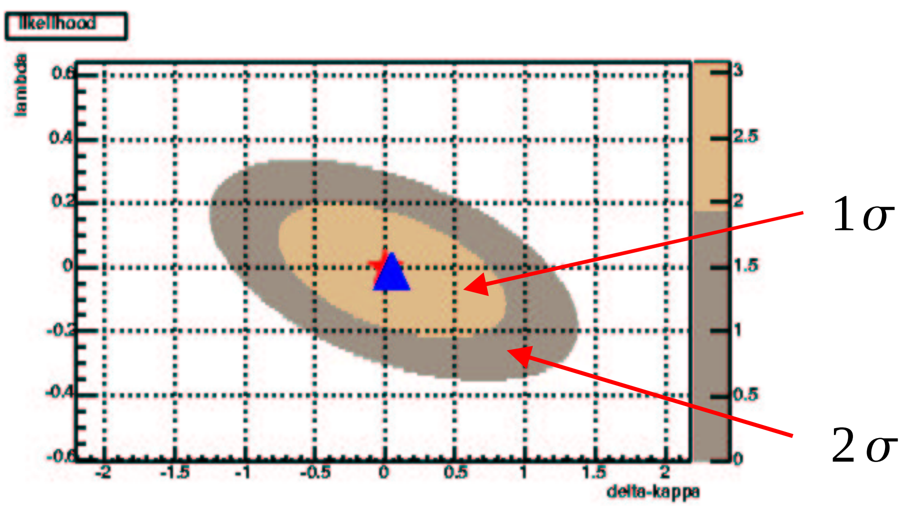
Wγ Pseudo−Experiments

<u>example ~50pb^{-1} Monte Carlo experiment :</u>



photon E_T bin





<u>Run IA :</u>

- $|\lambda| \approx 1$
- $|\Delta\kappa| \approx 3$

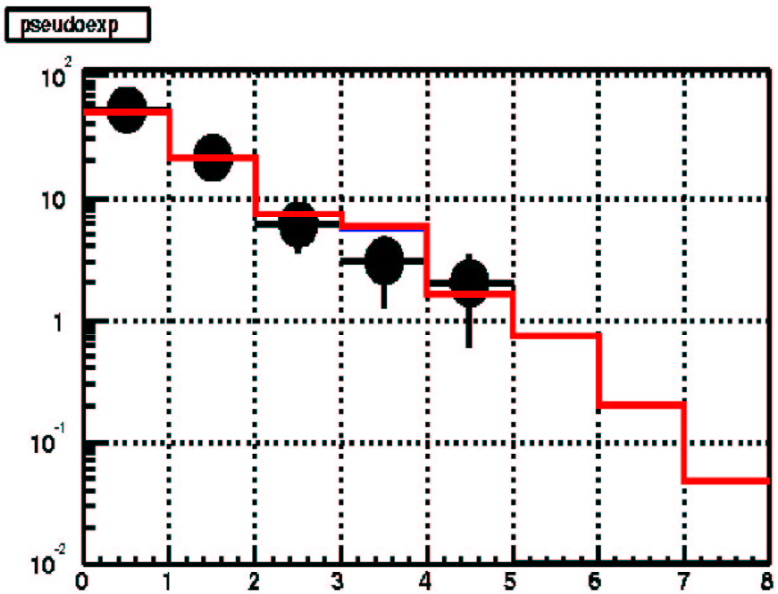
<u>TDR for 2 fb^{-1} :</u>

- $|\lambda| \approx 0.1$
- $|\Delta\kappa| \approx 0.3$

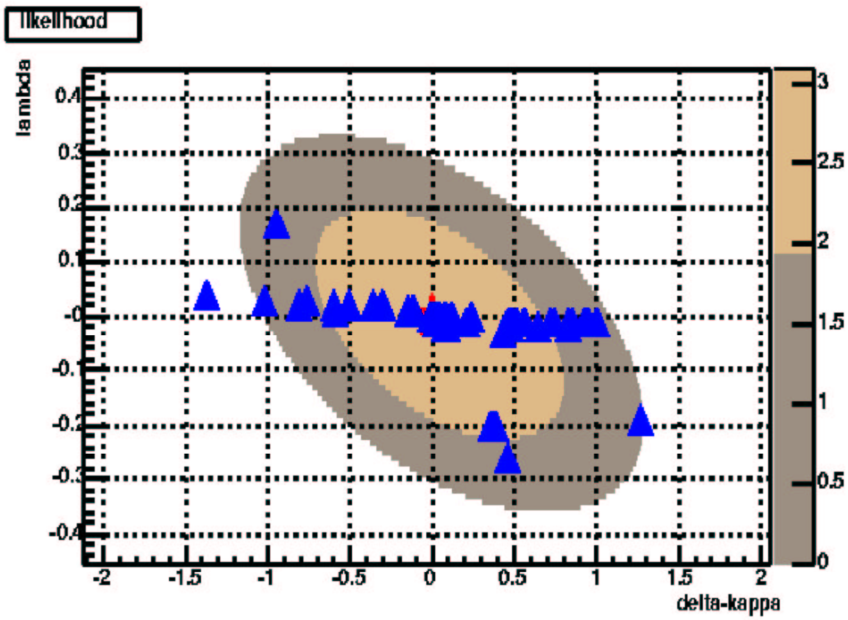
} extent of ellipses

Wγ Pseudo−Experiment Ensembles

<u>example experiment :</u>



<u>fit results for 100 experiments :</u>



- Fit results don't seem to be distributed as you would expect based on the log−likelihood contours for a single pseduo−experiment.
- The frequentist coverage is also not right : all pseudo−experiments are generated at the Standard Model point ($\Delta\kappa=\lambda=0$), yet $\ll$32% of 1σ contours exclude this point.

Dummy Pseudo−Experiments

- To see if this is a bug or feature of the Wγ distributions in particular, we have looked at a simpler set of dummy pseudo−experiments.
- Pseudo−experiments are generated according to the following distributions, also used as the fitting templates :

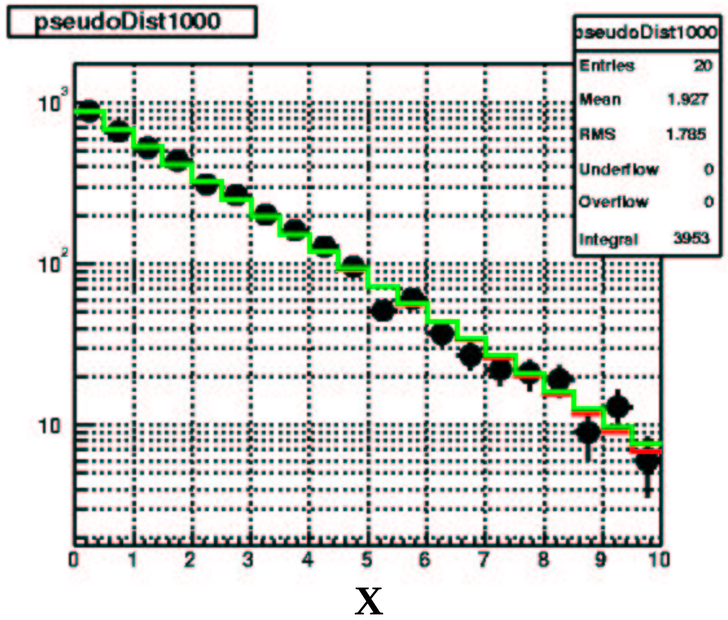
$$\frac{\mathrm{d}N}{\mathrm{d}x} \propto Ae^{-Bx} + Kx$$ **LINEAR**

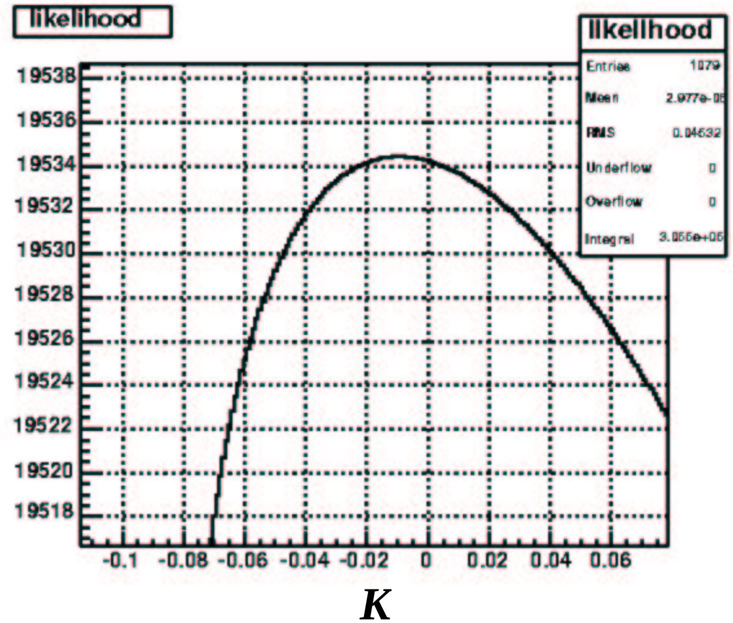
$$\frac{\mathrm{d}N}{\mathrm{d}x} \propto Ae^{-Bx} + (Kx)^2$$ **QUADRATIC**

➡ The coverage problem is reproduced only for the quadratic case (plots ⇒)

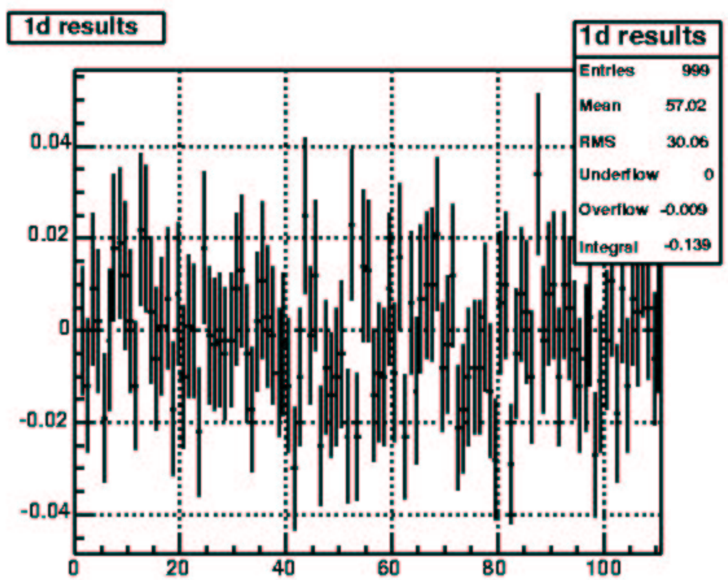
LINEAR

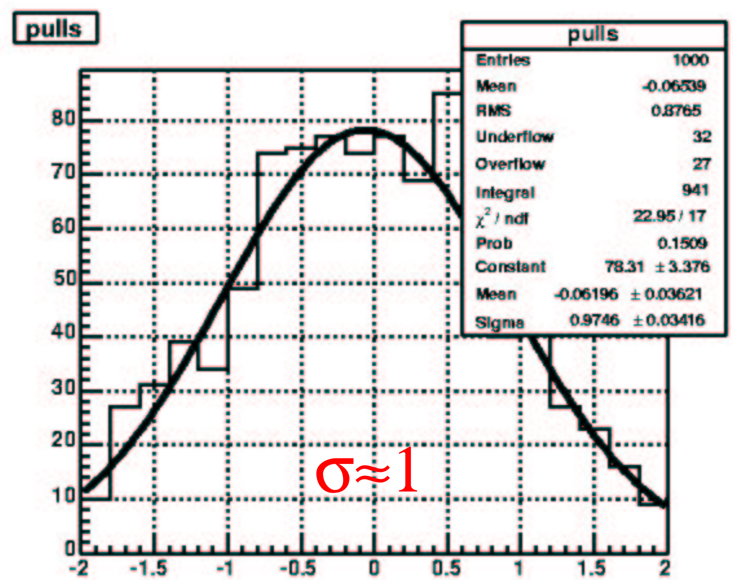
example experiment





ensemble results

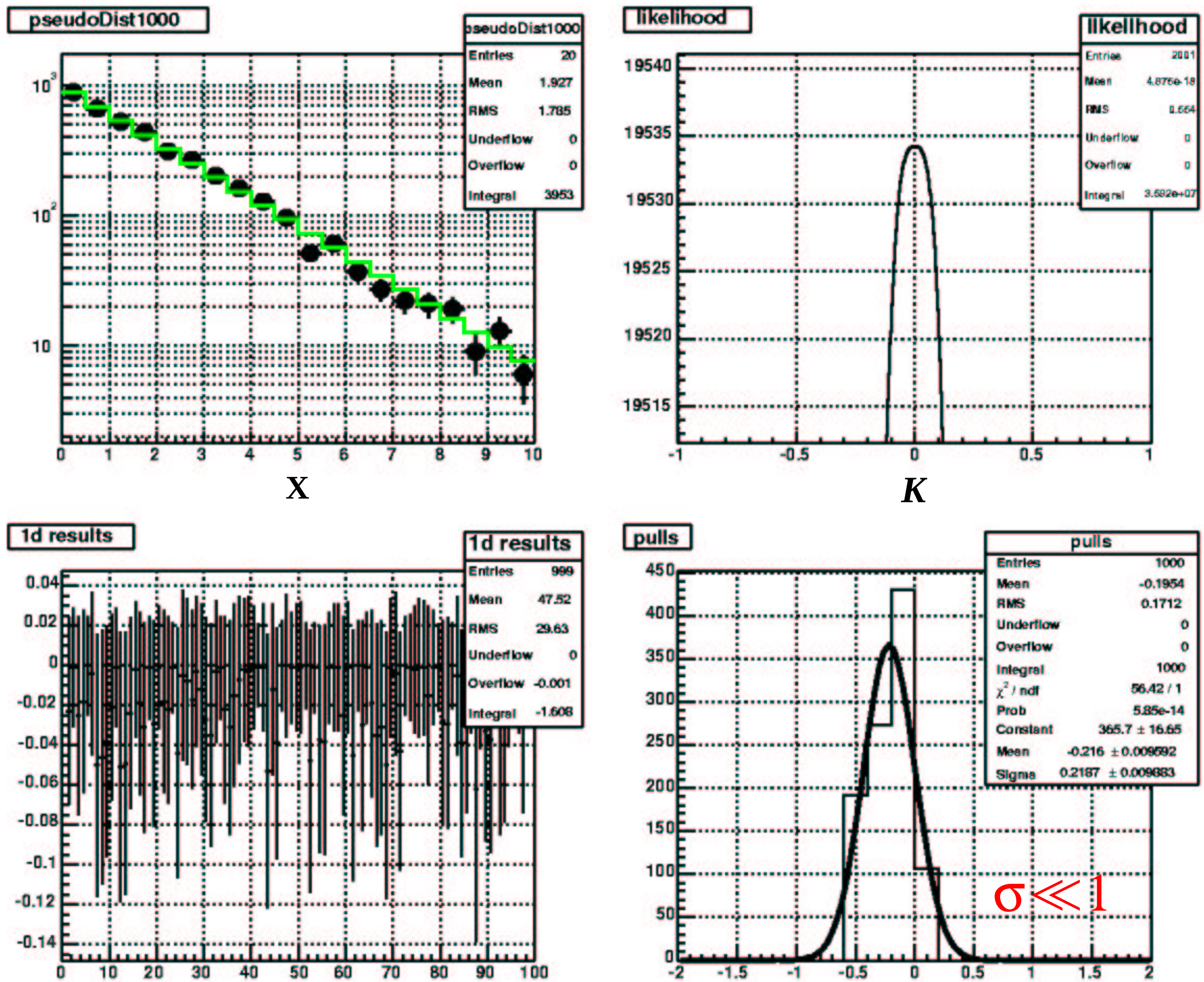




QUADRATIC

example experiment

ensemble results



Feldman−Cousins Confidence Intervals

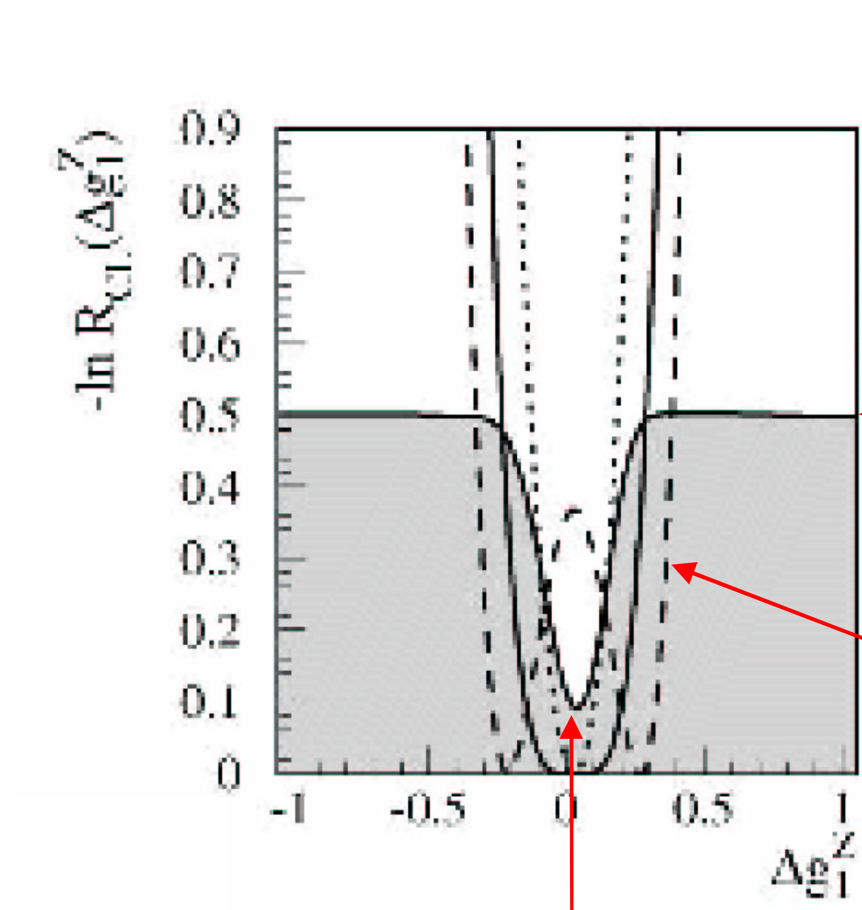
- Feldman−Cousins provide a prescription for the construction of frequentist confidence intervals based on a likelihood ratio method.
- The operational prescription :
  - For each point in parameter space, run a large number of pseudo−experiments.
  - For each pseudo−experiment, perform a log−likelihood fit (the parameters being allowed to vary).
  - Compute the log of the likelihood ratio *(R)* between the best fit value of the parameters and the true value of the parameters for that pseudo−experiment :

$$R = \sum_i \left[\mu_i - \mu_i^{best} + n_i \log\left(\frac{\mu_i^{best}}{\mu_i} \right) \right]$$

  - Make a histogram of this ratio for all pseudo−experiments and find the cut value such that 68.27% of pseudo−experiments have $R < R_{cut}$.
  - For the real experiment, compare the likelihood ratio at each value of the parameter with R_{cut} at that value. Practically, this means comparing the log−likelihood curve with the R_{cut} distribution.

Feldman−Cousins Confidence Intervals

- LEP have applied this method to the determination of confidence intervals in the measurement of TGC’s :



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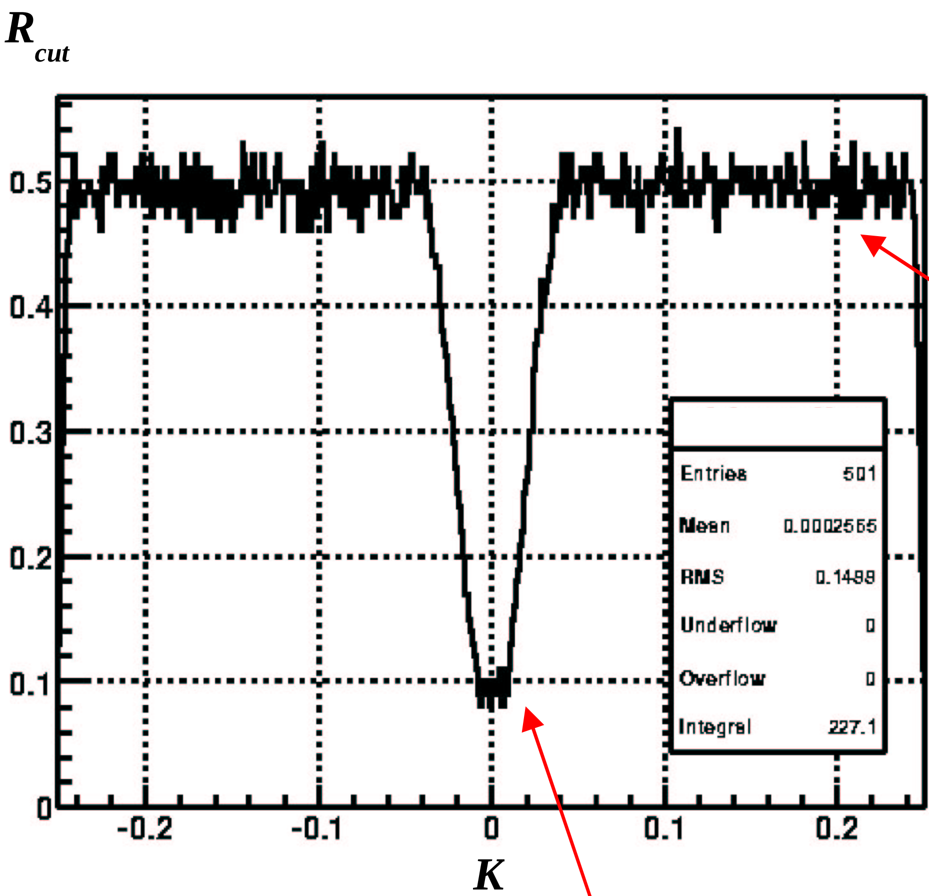
A cut value of 0.5 would be the "normal" way of determining the confidence interval

Curves represent the inverse log−likelihood curves from particular experiments

The Feldman−Cousins cuts value differs significantly from the usual value close to boundaries

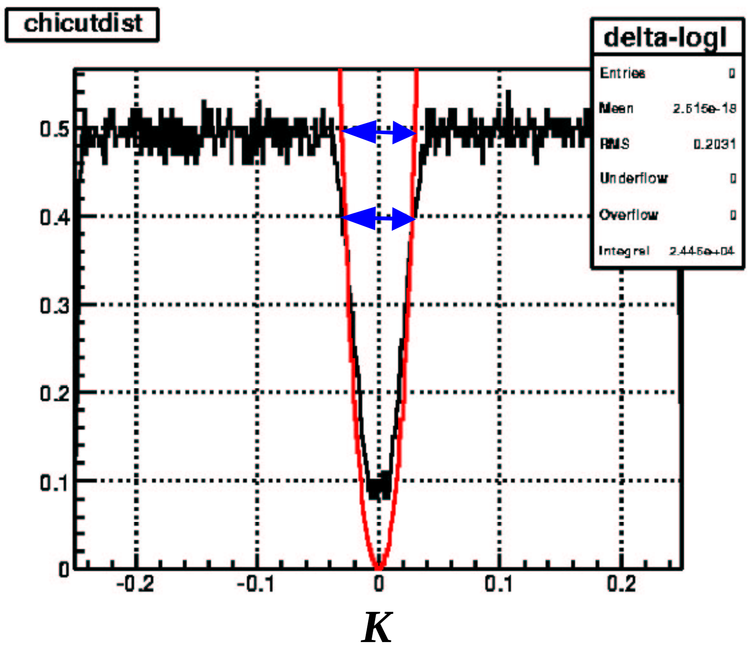
Feldman−Cousins Confidence Intervals

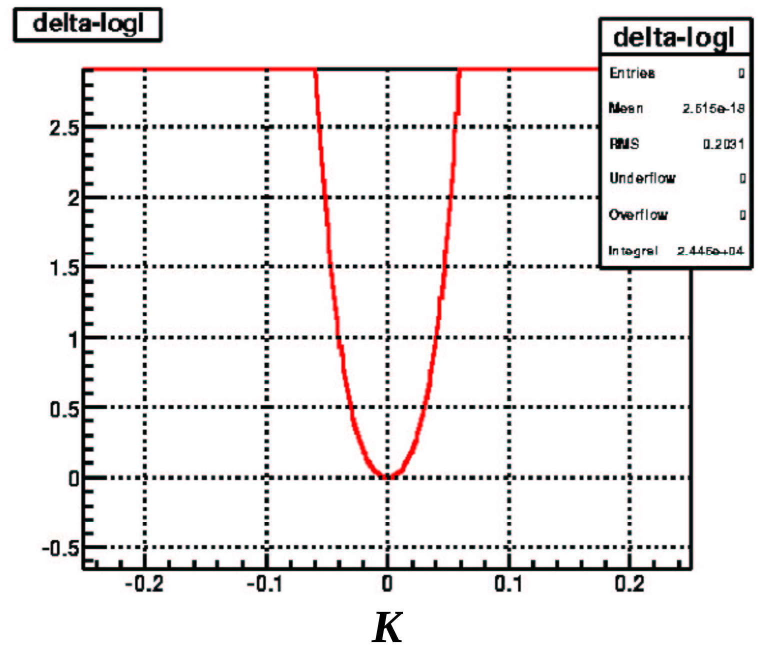
- Applying the method to the quadratic dummy experiment :

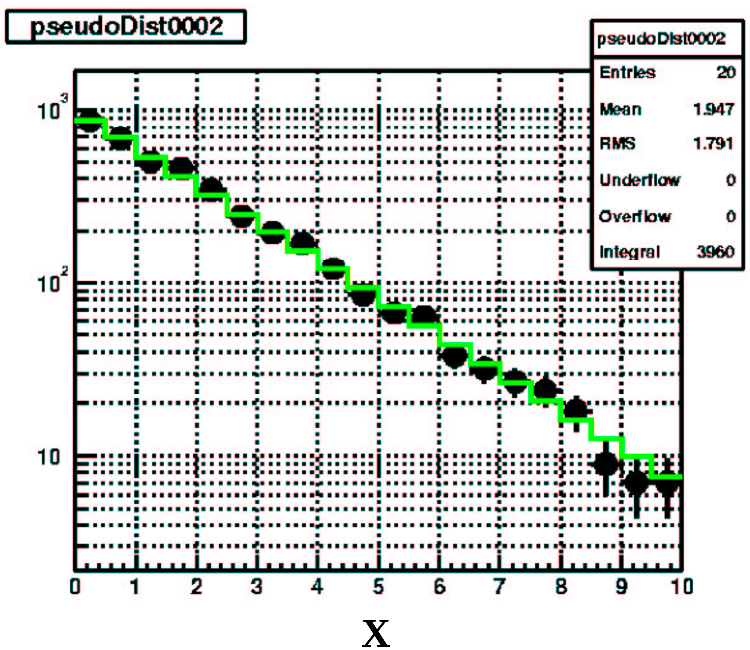


Each bin in the histogram corresponds to the results of running 4000 pseudo−experiments and fitting each one − **very time consuming** !

Overall structure is very similar to LEP TGC curve. There is a dip near the "boundary" of K=0. The "boundary" is due to all variations of K about this point giving rise to *positive cross section increases only*.

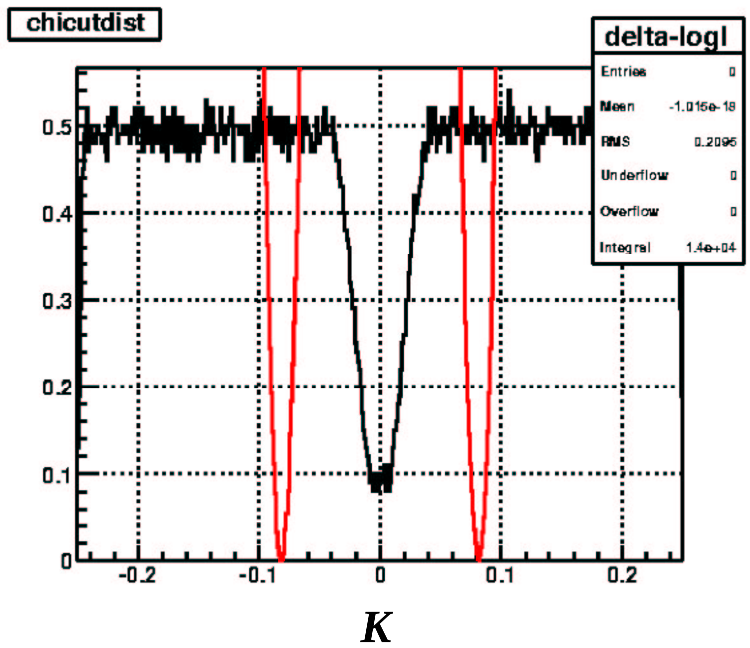


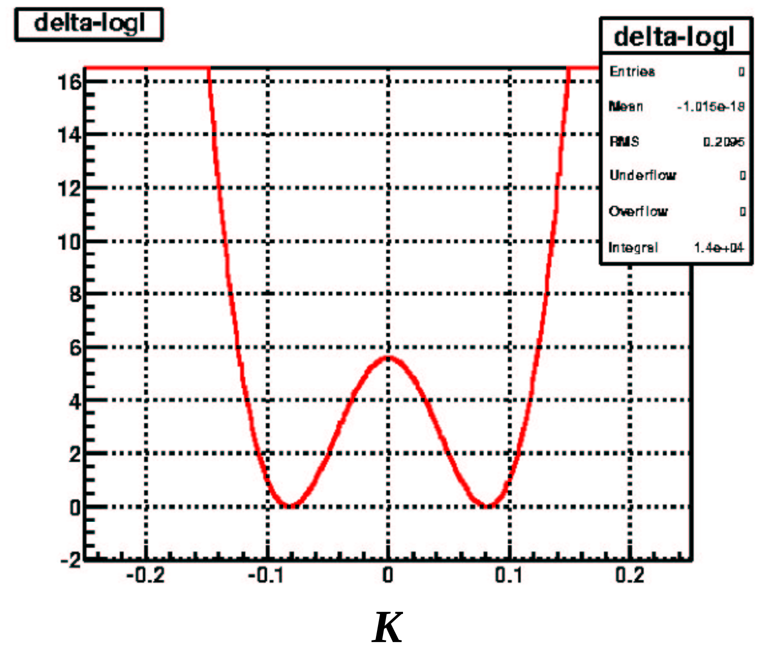


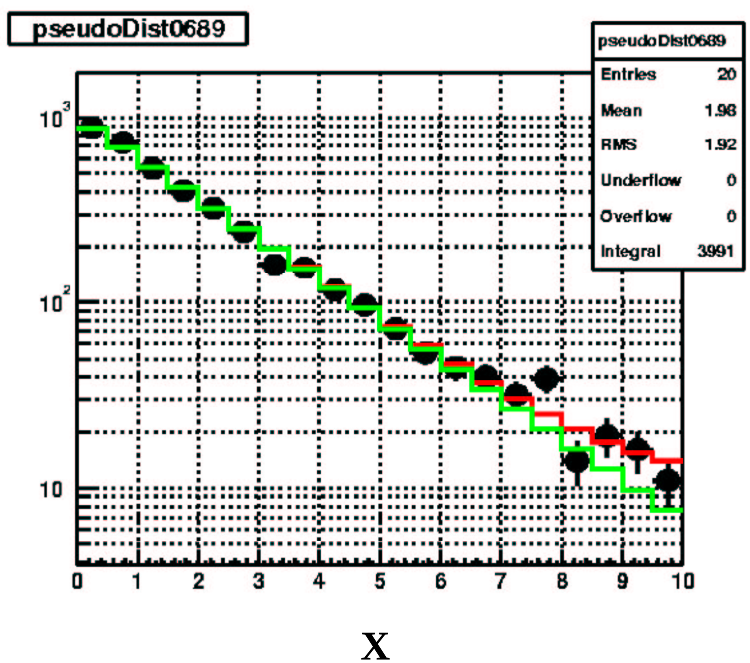


<u>Example 1:</u>

"Normal" and Feldman−Cousins confidence intervals would be very similar with Feldman−Cousins slightly narrower.

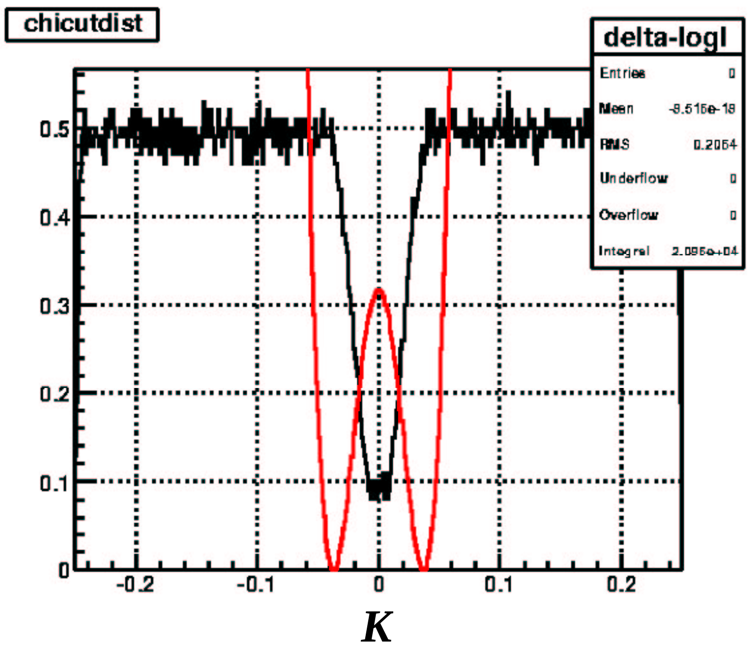


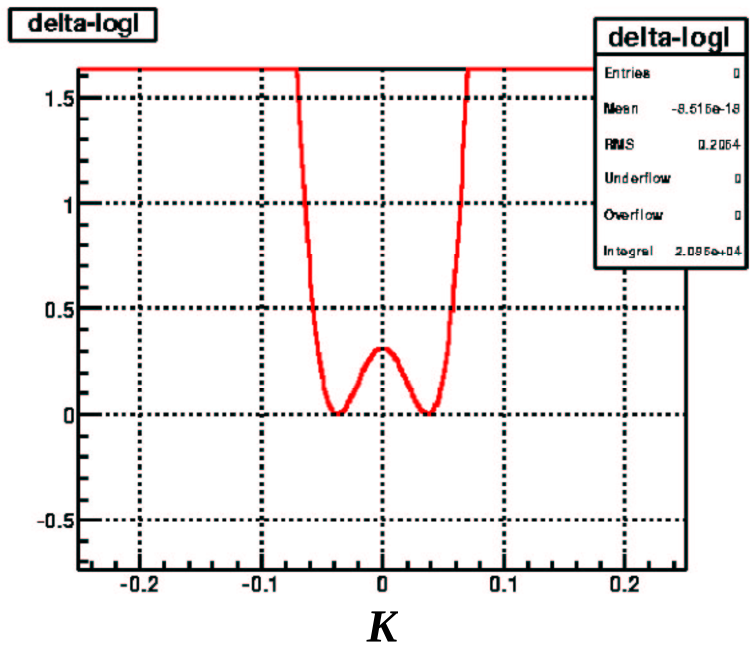


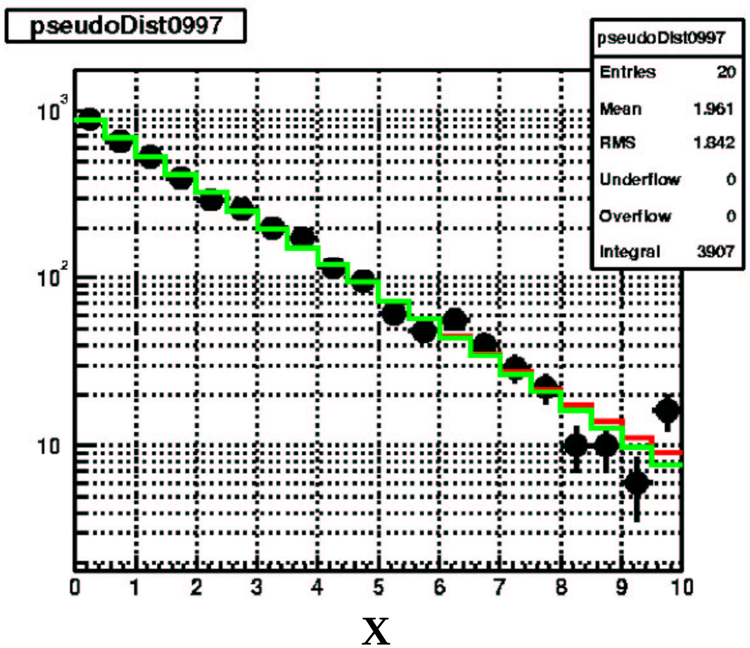


<u>Example 2:</u>

Both "normal" and Feldman−Cousins confidence interval determination would give the same results : disconnected confidence interval.







Example 3:

"Normal" confidence interval would be single interval [−0.6,0.6]. Feldman−Cousins interval would be two disconnected regions with K=0 excluded at 68% CL.

Feldman−Cousins intervals have the right coverage. The input value of K lies outside the CI in 32% of experiments (has been checked explicitly for this case).

Summary

- Run I log−likelihood method seems to work well and is very convenient but there seem to be some issues regarding frequentist coverage of the resulting intervals.
- Fitting for anomalous couplings *is* a problem with boundaries. The boundaries are due to the quadratic dependence of the cross section with couplings : there is a cross section *minimum* and so not all downward fluctuations can be fitted with an anomalous coupling.
- LEP have hit the same problem and have used the Feldman−Cousins technique for constructing confidence intervals with the right frequentist coverage.
- We have showed that this method works well for a dummy experiment in which we fit for a quadratic term superimposed on a falling exponential.
- However :
  - The Feldman−Cousins method is extremely time consuming.
  - It’s not clear that it will change the "size" of our error ellipses very much. I suspect not.
- We should probably use it at least as a cross−check of confidence intervals obtained using the standard log−likelihood technique.