Structure of Matter

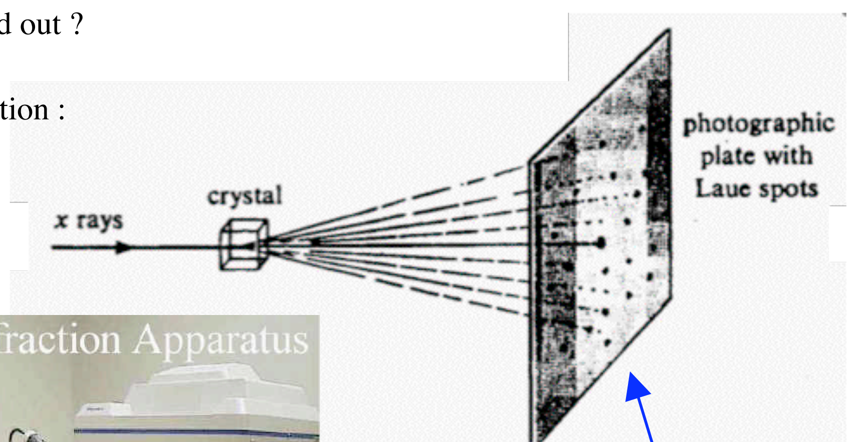
<u>Outline :</u>

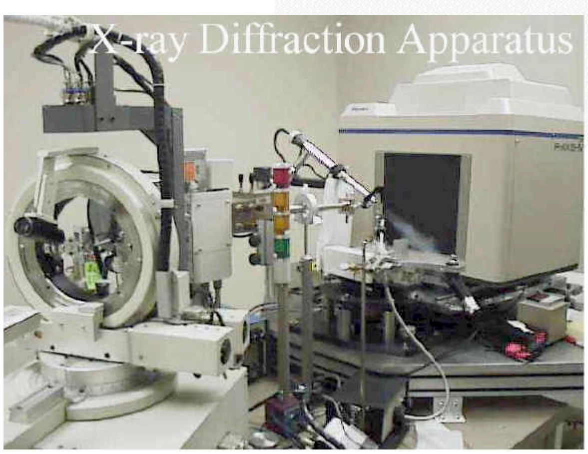
- **How big are atoms ?**
- **How big are nuclei ?**
- **How big are quarks ?**

How Big Are Atoms ?

- How do we find out ?

1) X-ray diffraction :





Spacing of spots gives spacing of atoms in sample.

How Big Are Atoms ?

2) From densities of solids and liquids.

$N_{AVO} = 6\times10^{23}$ **atoms per gramme-atom**

A grammes of an element of atomic number A

Number of atoms/cm^3 = $N_{AVO} \times (\rho/A)$; ρ = density in g/cm^3

Therefore the volume occupied by a single atom, $V_A = 1 / [N_{AVO} \times (\rho/A)]$

Therefore the spacing between atoms is given by :

$$\approx \sqrt[3]{V_A} = \sqrt[3]{\frac{1}{N_{AVO} \times (\rho / A)}}$$

How Big Are Atoms ?

Atomic Numer	Element	A	Density ρ (g/cm^3)	ρ / A	$\sqrt[3]{\rho / A}$ ($\propto$ "number density")
1	Hydrogen	~1	(0.071)	0.071	0.41
2	He	~4	(0.125)	0.031	0.31
4	Be	~9	1.848	0.2	0.58
6	C	~12	2.265	0.19	0.57
10	Ne	~20	(2.7)	0.06	0.39
13	Al	~27	2.7	0.1	0.46
26	Fe	~56	7.9	0.14	0.52
56	Xe	~131	(2.95)	0.02	0.27
78	Pt	~195	21.45	0.11	0.48
82	Pb	~207	11.35	0.055	0.38

1 to 82

1 to 207

0.27-0.58 much smaller range

$\Rightarrow$ All atoms are ***roughly*** the same size

How Big is That ?

$$\text{Size} \approx \sqrt[3]{\frac{1}{N_{\text{AVO}} \times (\rho / A)}} = \sqrt[3]{\frac{1}{N_{\text{AVO}}}} \Big/ \sqrt[3]{\frac{\rho}{A}} = \sqrt[3]{\frac{1}{6 \times 10^{23}}} \Big/ 0.4 \approx 3 \times 10^{-8} \text{ cm}$$

In fact, nearly all are (2 to 4) × 10^{-10} metres.

This scale is set by the Bohr radius of hydrogen :

quantum effects; allowed energy levels

$$a_{\text{Bohr}} = \frac{4\pi\varepsilon_0}{e^2} \times \frac{\hbar^2}{m_e} = 0.53 \times 10^{-10} \text{ metres}$$

constants of nature **dynamics of motion**

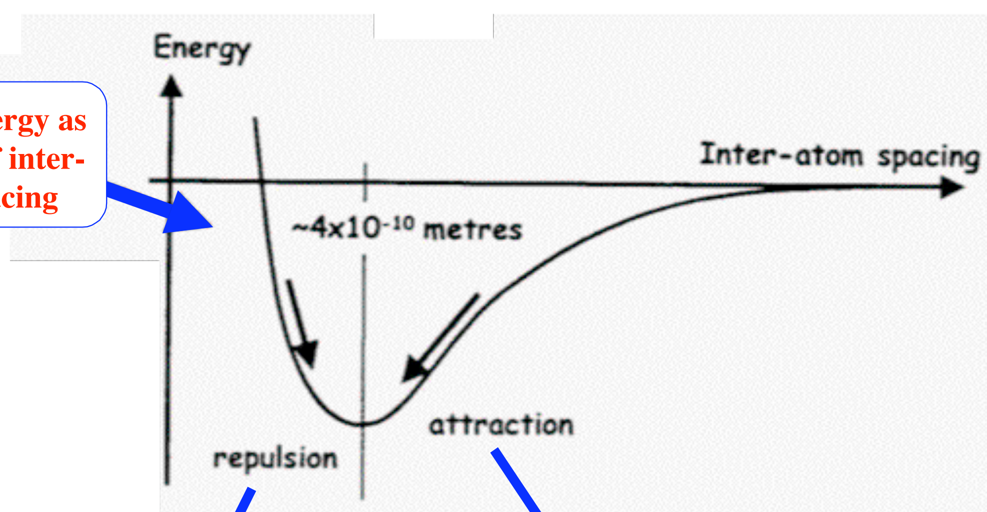
[Derived by solving the Schrödinger equation for a Coulomb potential]

What Forces are at Work ?

The atoms are bound differently for different elements (crystal structures, molecules)

Consider the “noble gases” (He, Ne, Xe) :

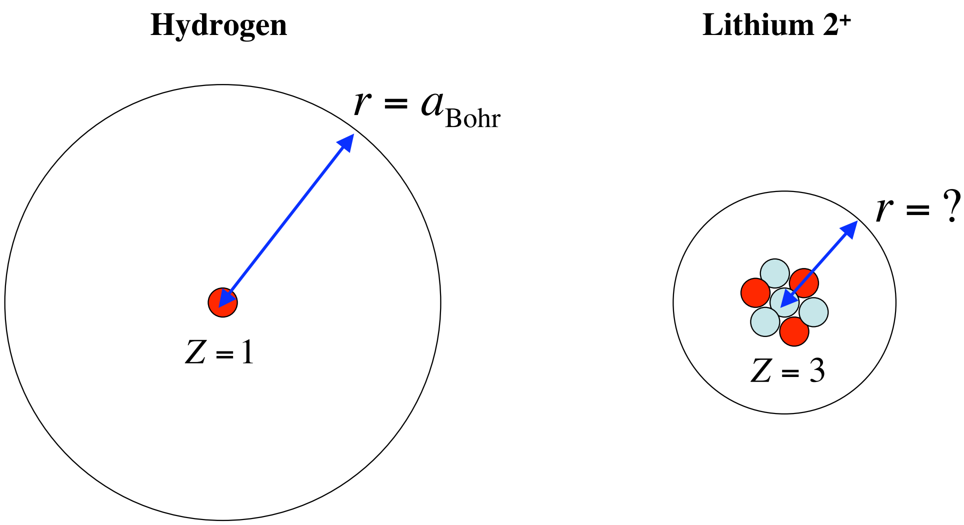
Potential energy as a function of inter-atomic spacing



Caused by ? **Caused by ?**

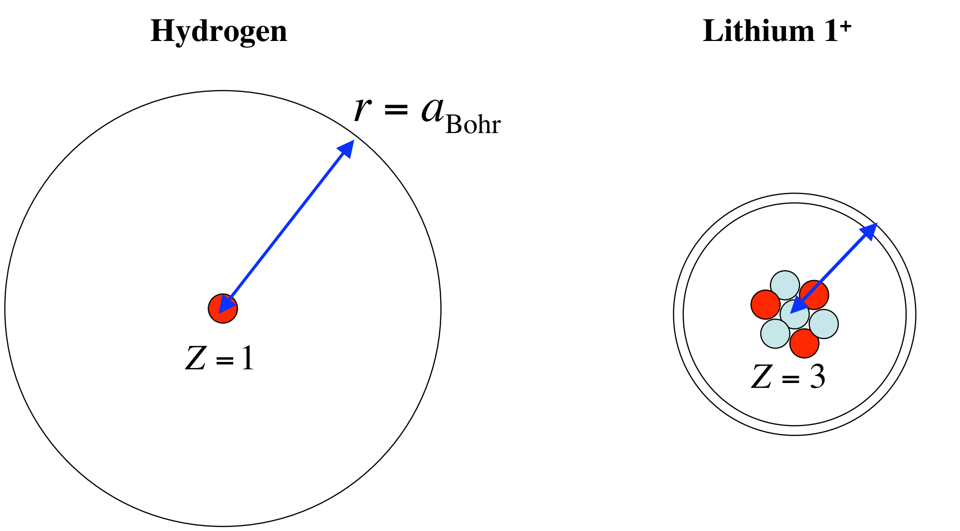
All Atoms are *Roughly* the Same Size

Why ?



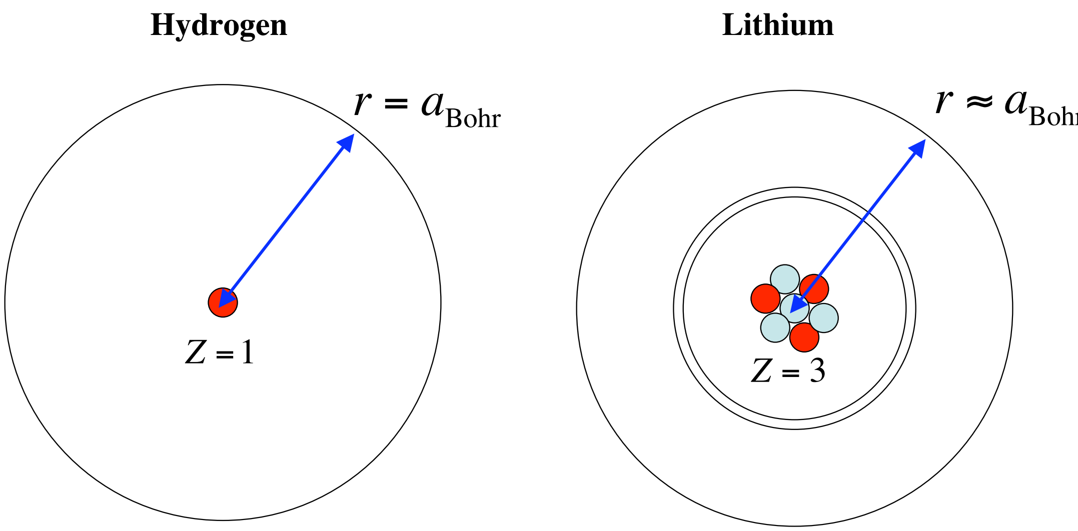
All Atoms are *Roughly* the Same Size

Add one more electron to the lithium ion :



All Atoms are *Roughly* the Same Size

Neutralise the lithium ion :



All Atoms are *Roughly* the Same Size

1) Why don't all electrons fall into the lowest energy level, closest to the nucleus ?

<u>Pauli Exclusion Principle :</u>

"Once lower energy levels are full, electrons have to go into higher levels"

or

"No two fermions can occupy the same quantum state"

Electrons are **fermions** because they have intrinsic angular momentum = $\hbar/2$

(or "spin 1/2". All half-integer spin particles are fermions).

Examples of **bosons** ?

All Atoms are *Roughly* the Same Size

1) Why is the outermost electron orbit in any atom roughly the same size as that for an electron in a hydrogen atom ?

How Big Are Nuclei ?

- How do we find out ?

Diffraction again.

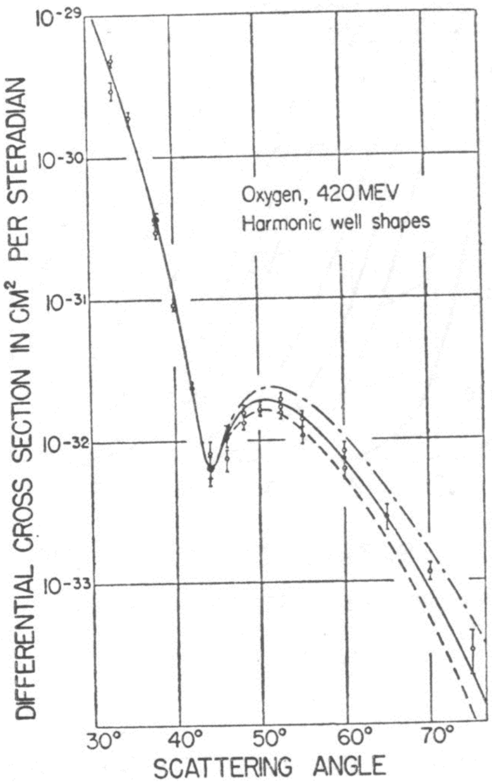
Need smaller wavelengths, hence larger momenta :

$$\text{De Broglie}: \; p = h/\lambda$$

Use beams of particles (electrons or sometimes nucleons).

CEBAF multi-GeV electron accelerator

How Big Are Nuclei ?



De Broglie wavelength of the electrons :

$$E = p \times c = 420 \text{ MeV}$$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{2\pi \times (\hbar c)}{pc}$$

$$\approx \frac{2\pi \times (197 \text{ MeV fm})}{420 \text{ MeV}}$$

$$\approx 3 \text{ fm}$$

1 fm = 10^{-15} m

First minimum for a simple diffraction grating :

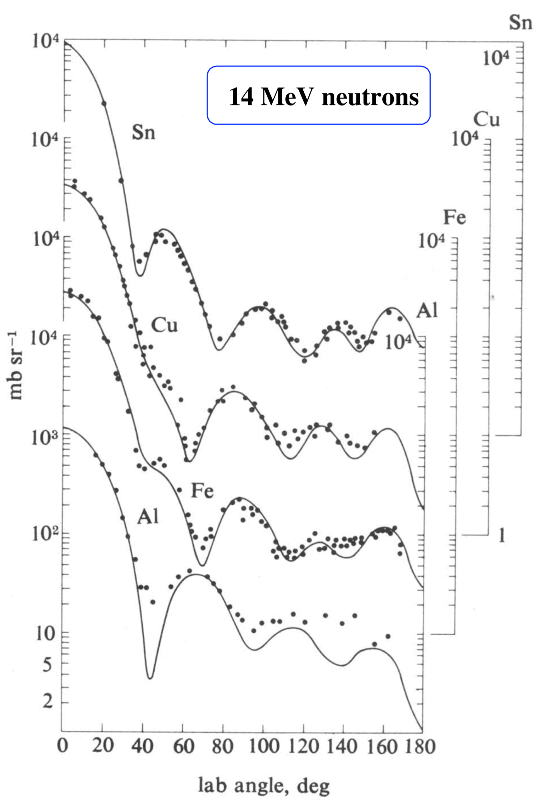
$$d = 0.5 \times \lambda / \sin\vartheta$$

For a sphere it's only slightly different :

$$R = 0.6 \times (\lambda / \sin\vartheta)$$

$\vartheta_1 = 44^o$: ***R* (Oxygen) = 2.5 fm**

How Big Are Nuclei ?



De Broglie wavelength of the neutrons :

$$E = \frac{p^2}{2m}$$

$$pc = \sqrt{2mc^2 E} = \sqrt{2 \times 940 \times 14} \approx 162 \text{ MeV}$$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{2\pi \times (\hbar c)}{pc}$$

$$\approx \frac{2\pi \times (197 \text{ MeV fm})}{162 \text{ MeV}}$$

$$\approx 7.6 \text{ fm}$$

Again taking position of first minimum :

$$R = 0.6 \times (\lambda / \sin\vartheta)$$

$\vartheta_1 = 45^o$: ***R* (Al) = 6.4 fm**

How Big Are Nuclei ?

Measuring nuclei with a wide range of mass number A :

$$R_A = 1.2\ A^{1/3} \times 10^{-15}\ \text{metres}$$

Unlike atoms, nuclei get consistently bigger as they get heavier.

What does $R_A \propto A^{1/3}$ imply for the ***density*** of nuclear matter ?

What does this imply for the range of the force between two nucleons (protons or neutrons ?)

The Constituents of Nuclei

Protons : spin 1/2, charge **+e**, mass $\mathbf{m_p \sim 2000 \times m_e}$

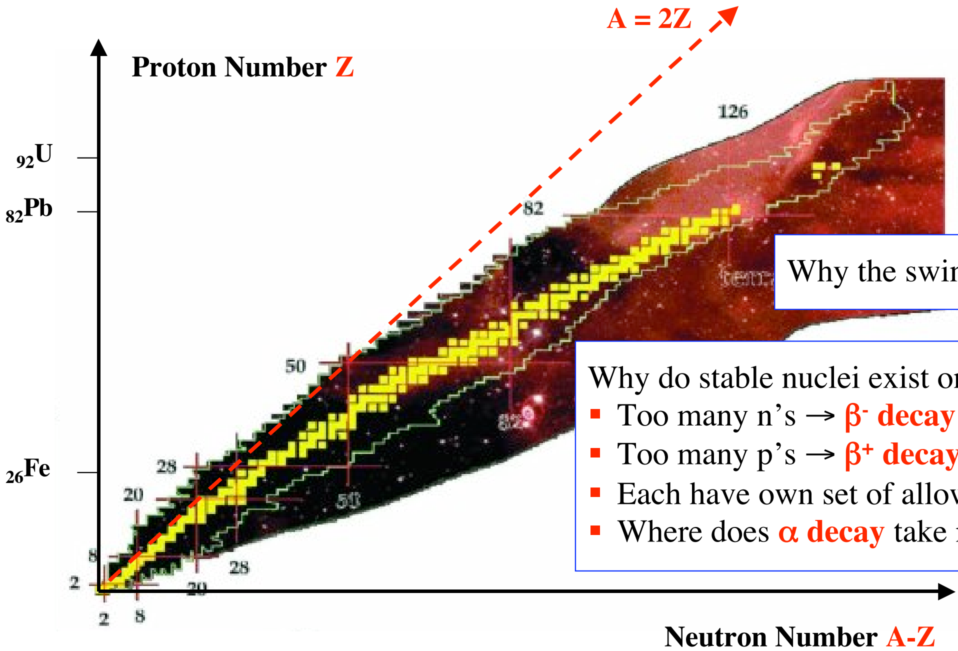
Neutrons : spin 1/2, charge **0**, mass $\mathbf{m_n = m_p + 2.5 \times m_e}$

Free neutrons decay : $n \to p e^- \bar{\nu}$

Why don't neutrons bound inside nuclei decay ?

- Sometimes they do : β decay.
- Usually neutrons are protected from decaying because replacing a neutron with a proton would put the nucleus in a higher energy state. Binding energies of protons and neutrons inside nuclei are $\sim 10 \times m_e$
- Sometimes the protons decay to a lower energy neutron : $p \to n e^+ \nu$ This is called β^+ decay. We'll discuss positrons in the next lecture.

Which Nuclei Exist ?



Why the swing to the right ?

Why do stable nuclei exist only on this band ?

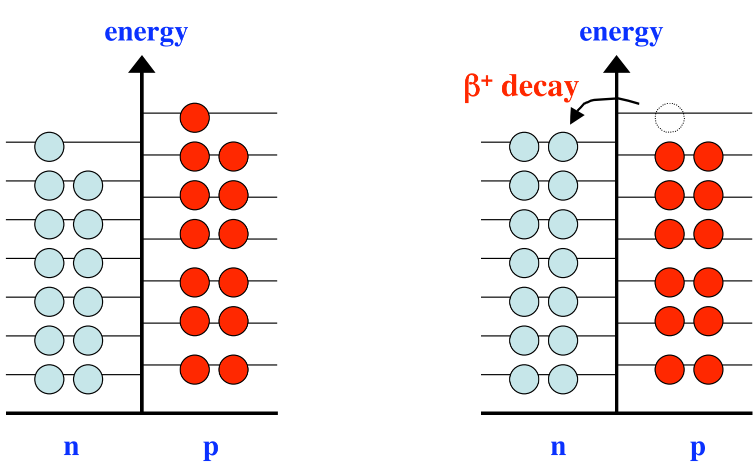
- Too many n's → β^- **decay**
- Too many p's → β^+ **decay**
- Each have own set of allowed energy levels
- Where does α **decay** take nuclei ?

Which Nuclei Exist ?

Why the swing to more neutrons ?

Neutrons don't have electrostatic repulsion - their energy levels are slightly lower than proton energy levels. A lower overall energy state can therefore be reached by turning some protons into extra neutrons.

At some point even the additional neutrons cannot protect the nucleus from the electrostatic repulsion between the protons - what happens then ?



The Force Between Nucleons

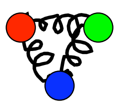
The strong nuclear force

It's short range : nuclear scattering experiments reveal that neutrons and protons have to pass within a few femto-metres of a nucleus to feel the effect of the strong nuclear force.

It saturates : nucleons are only feeling the force from their neighbours, not from nucleons far away. This gives rise to ~ constant nuclear density.

What does this resemble ? The van der Waal's force between molecules has very similar properties - for example that which binds atoms together in liquid Xe.

In the case of atoms or molecules the van der Waal's attraction is a residual electrostatic interaction. In the case of nucleons bound together inside a nucleus it is a residue of which interaction ?



Red, green and blue quarks are bound inside protons and neutrons through the exchange of "gluons". A little bit of the left over interaction between quarks generates a van der Waal's force between nucleons.

More in the particle physics lecture!

How Big Are Quarks ?

The smallest De Broglie wavelengths are achieved by going to the highest possible energies.

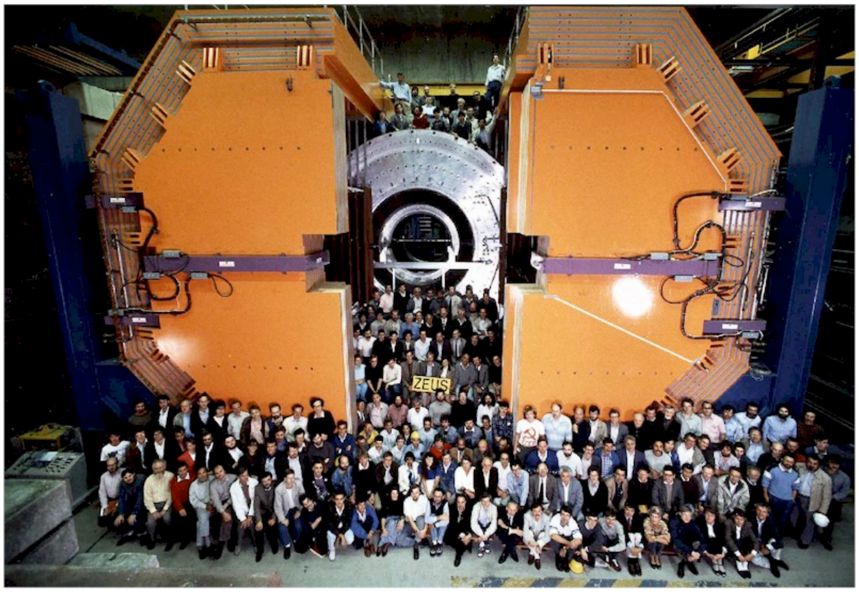
For example the HERA accelerator (shown left) collides :

e (27.5 GeV) → ← (820 GeV) **p**

Corresponding De Broglie wavelength of the electrons is :

~ 10^{-18} metres

How Big Are Quarks ?

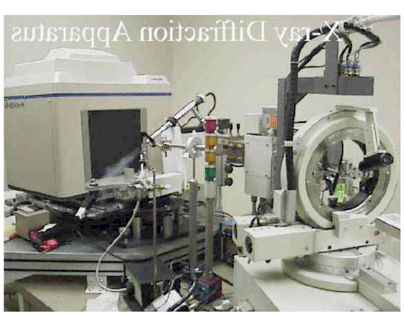


Giant detectors such as these search for any sign of the constituent quarks in the proton having a finite size. So far no evidence has been found :

$$R_q < 10^{-18} \text{ metres}$$

Summary

- **Atoms**
  - Typical size : few × 10^{-10} metres.
  - Small variation in size across a wide range of elements : the last electron in any atom always feels a net charge of +1.
  - Liquid Xe : atoms bound together by van der Waal's force, a residual electrostatic interaction.
- **Nuclei**
  - Typical size : few × 10^{-15} metres.
  - Small variation in density across a wide range of elements : "liquid-drop"
  - Nucleons are bound by a residual strong interaction between constituent quarks.
- **Quarks**
  - All we know is that they are smaller than ~ 10^{-18} metres.



The smaller the structure being probed the larger the apparatus required.