Relativity

Outline :

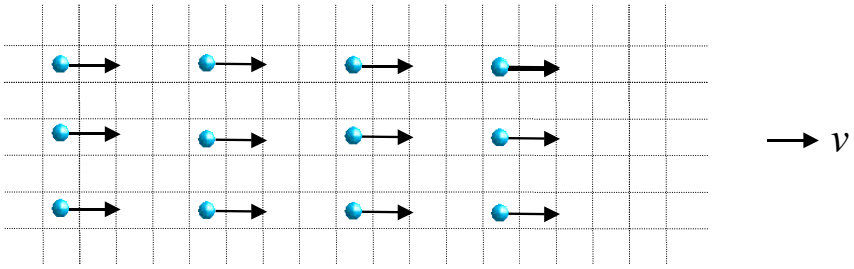
- **The Principle of Relativity**
- **Galilean Relativity**
- **Electromagnetism & Relativity**
- **Time Dilation**
- **Length Contraction**
- **Mapping Space-Time**
- **$E=mc^2$**
- **Antimatter**

Principle of Relativity

- Symmetries are fundamental to physics.
- For example, the results of an experiment do not depend on the spatial orientation of the laboratory : **isotropy**.
- The principle of relativity refers to the symmetry between observers in co-moving inertial reference frames.

Inertial reference frame :

- A frame in which test particles with no external forces acting upon them move at a constant speed in a straight line.



- The frame in which the imaginary array of particles is at rest is also an inertial frame, with a velocity relative to frame S of ***v***. All inertial frames are related to each other in this way.

Principle of Relativity

- With this definition, the principle of relativity takes the following simple form :

> The laws of physics are the same in all inertial reference frames

Weak :
Only the ***form*** of the physical law is invariant.

Strong :
Not only the ***form*** but the ***constants*** appearing in our physical laws are invariant.

↓

This is what we currently assume.

Galilean Relativity

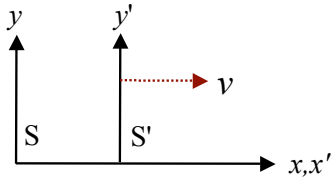
Newton's Laws :

(1) Objects remain at rest or in a state of uniform motion unless acted upon by an external force.
(2) Force = mass × acceleration
(3) Action and reaction are equal and opposite.

Galilean Relativity :

- We know the force law in frame **S** :

$$F = ma = m \times \frac{d^2x}{dt^2}$$



- For an observer in a relatively moving frame **S'**:

$$x' = x - vt \quad \Rightarrow \quad \frac{dx'}{dt} = \frac{dx}{dt} - v \quad \Rightarrow \quad \frac{d^2x'}{dt^2} = \frac{d^2x}{dt^2}$$

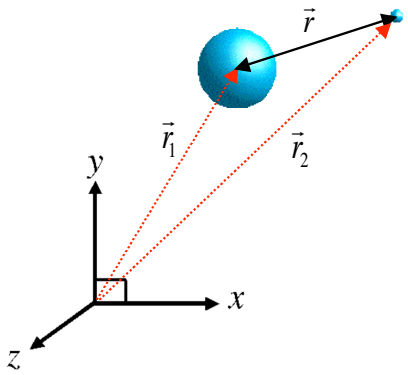
- Assuming the same force in both frames :

$$F' = F = m \times \frac{d^2x}{dt^2} = m \times \frac{d^2x'}{dt^2} = ma' \Rightarrow$$

Force law is the same in all inertial frames.

Galilean Relativity

- It also seemed natural that forces were invariant between different inertial frames :



$$F_{\text{GRAVITY}} = \frac{GM_1M_2}{r^2}$$

(directed along the line between the two bodies)

But under Galilean transformations :

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$

the separation is invariant :

$$r'^2 = r^2$$

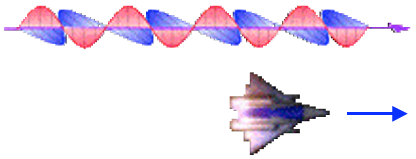
hence the force is the same in all inertial frames.
This is a consequence of the force **NOT** depending on velocity.

- In Newtonian mechanics, Galilean relativity is regarded as a ***consequence*** of the underlying theory. The modern view is that invariance under transformations between inertial reference frames is a ***pre-requisite*** of any theory.

Electromagnetism

- As soon as Maxwell's equations of electromagnetism were formulated, it was clear that they were not invariant under Galilean transformations.

They predict a constant light speed in vacuum, ***c***. But according to the Galilean transformations, I ought to be able to "chase a light beam" and so end up in a frame in which the light travels at a speed different to ***c***.



Speed = ***c***

Speed = ***v***

➢In pilot's restframe, light would have Speed = ***c -v***

The Lorentz force law :

$$\vec{F}_{\text{ELECTROMAGNETIC}} = q\left(\vec{E} + \vec{v} \wedge \vec{B}\right)$$

depends on the velocity of the moving charge, $\vec{v}$.

➢ The proof of Galilean invariance that we saw for Newton's gravity will necessarily fail.

Electromagnetism

- In the face of the non-invariance of Maxwell's equations under Galilean transformations, we have two choices :

Abandon Relativity

- There is a preferred rest-frame in the universe, in which the speed of light is ***c*** and is isotropic. This is the rest frame of the "luminiferous ether" which carries electromagnetic waves.
- Maxwell's equations are true only in this special frame.
- Search for the ether.

Impose Relativity

- Find different transformations between inertial reference frames that preserve Maxwell's equations and the speed of light.
- These transformations, the **Lorentz transformations**, were known at the same time as Maxwell's equations, but their interpretation was unclear.
- Einstein in 1905 provided the first clear space-time interpretation.

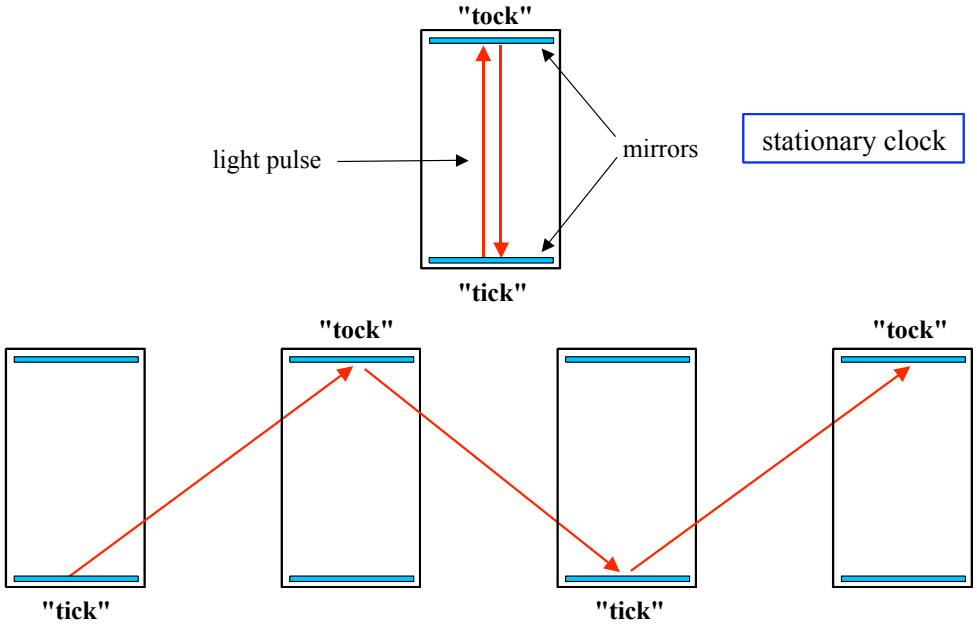
The Constancy of the Speed of Light

- Every important consequence of special relativity flows from the assertion that :

> The speed of light is constant (c) for all observers in all inertial reference frames.

Time Dilation

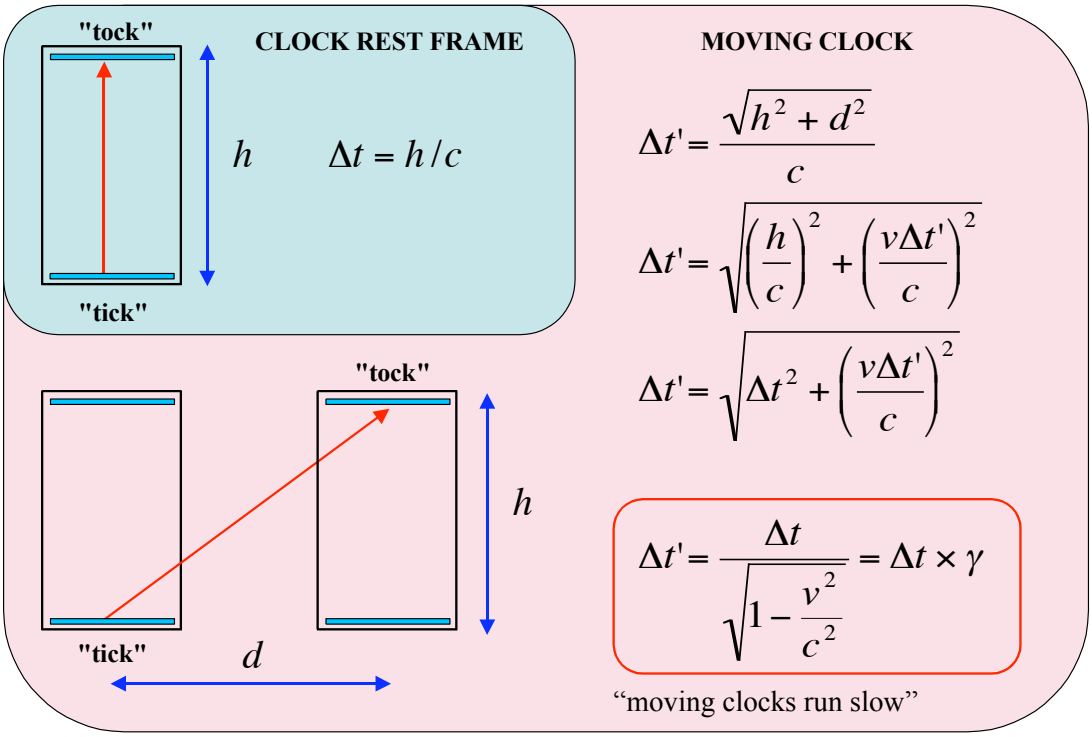
- Consider a "light clock" :



moving clock : light travels further between "ticks" ***at the same speed***.

Time Dilation

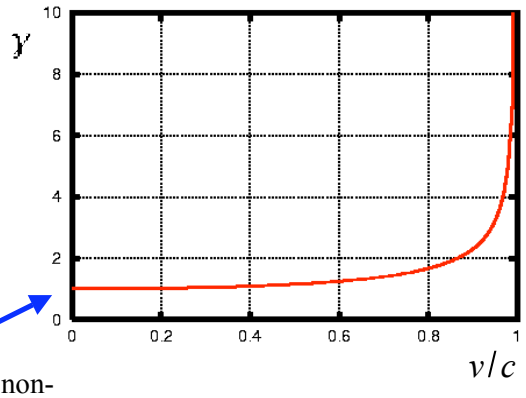
- Time between “tick” and “tock” :



Lorentz γ Factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lorentz γ Factor



Approaches ∞ in the ultra-relativistic limit $v \rightarrow c$.

Approaches 1 in the non-relativistic limit $v << c$.

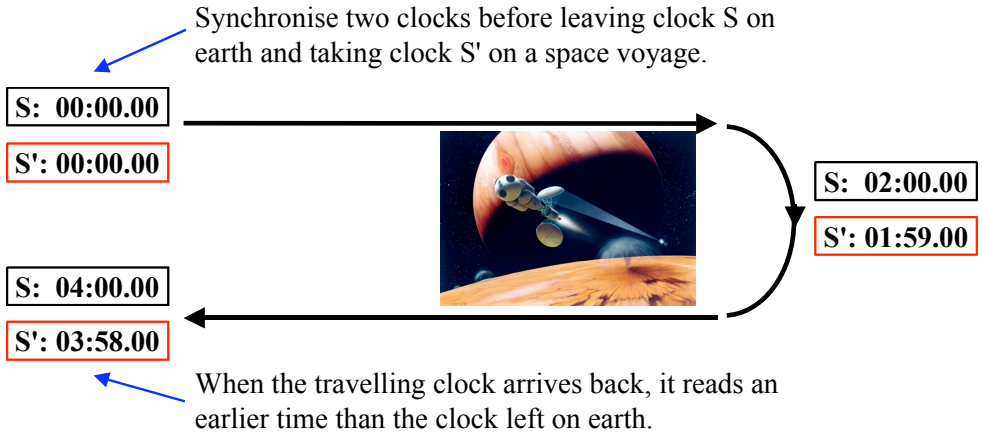
Time Dilation & Proper Time

- The time elapsed in the rest frame of an object is called the **proper time**.
- The proper time is the amount of time that the object has "aged", regardless of the time indicated in other inertial frames.

person's rest frame	**1**	**Jan**	**2000**	**…**	**2**	**Jul**	**2090**
some other inertial frame	**1**	**Jan**	**2000**	**…**	**4**	**Oct**	**3052**

Time Dilation & Time Travel

- Time dilation implies that it is possible to travel into the future.
- You just have to travel fast !



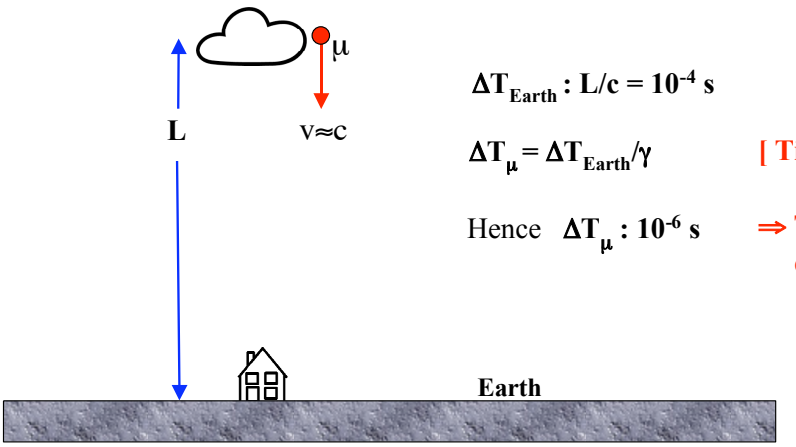
- However, the effect is usually tiny, e.g. Apollo spacecraft to the moon travelled at ≈ 10,000 ms^{-1} :

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 1.0000000005$$

➨ Clocks on earth would have gained 1 second after a journey of 60 years !

Length Contraction

- Consider the fate of muons, created in the upper atmosphere and travelling towards earth.
- They have a lifetime of a few microseconds and travel at close to the speed of light; say, with a gamma factor of 100.
- Analysing this in the Earth's rest frame :



$\Delta T_{Earth} : L/c = 10^{-4}$ s

$\Delta T_{\mu} = \Delta T_{Earth}/\gamma$ [Time Dilation]

Hence $\Delta T_{\mu} : 10^{-6}$ s ⇒ The muons reach earth before decaying

Length Contraction

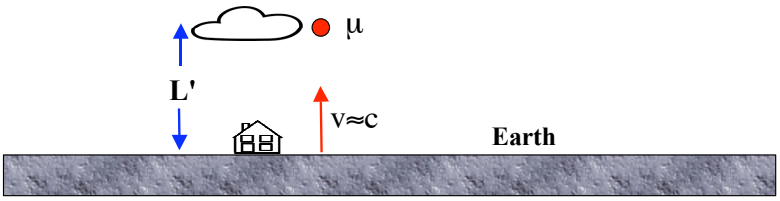
- But in the muon's rest frame, there is no time dilation.
- What's the explanation of the muon's survival in the muon rest frame ?

ΔT_{Earth} : $L/c = 10^{-4}$ s

ΔT_{μ} : $L'/c = 10^{-6}$ s $\Rightarrow L' = L/\gamma$

Length Contraction

Hence ΔT_{μ} : 10^{-6} s $\Rightarrow$ **The earth reaches the muons before they decay.**



Lorentz Transformations

- More formally, time dilation and length contraction (and other consequences of relativity) follow from a new set of coordinate transformations :

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - vx/c^2)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lorentz Transformations

Speed of Light as a Limiting Speed

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}} = \Delta t \times \gamma$$

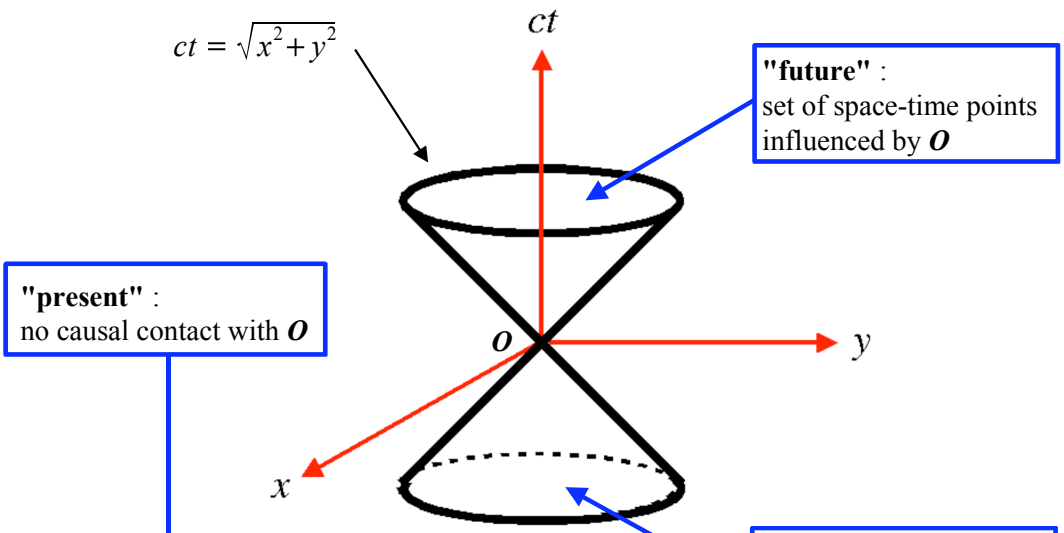
➢ Time dilation : moving clocks run slow.

$$L' = L \times \sqrt{1 - \frac{v^2}{c^2}} = L / \gamma$$

➢ Length contraction : moving objects are compressed

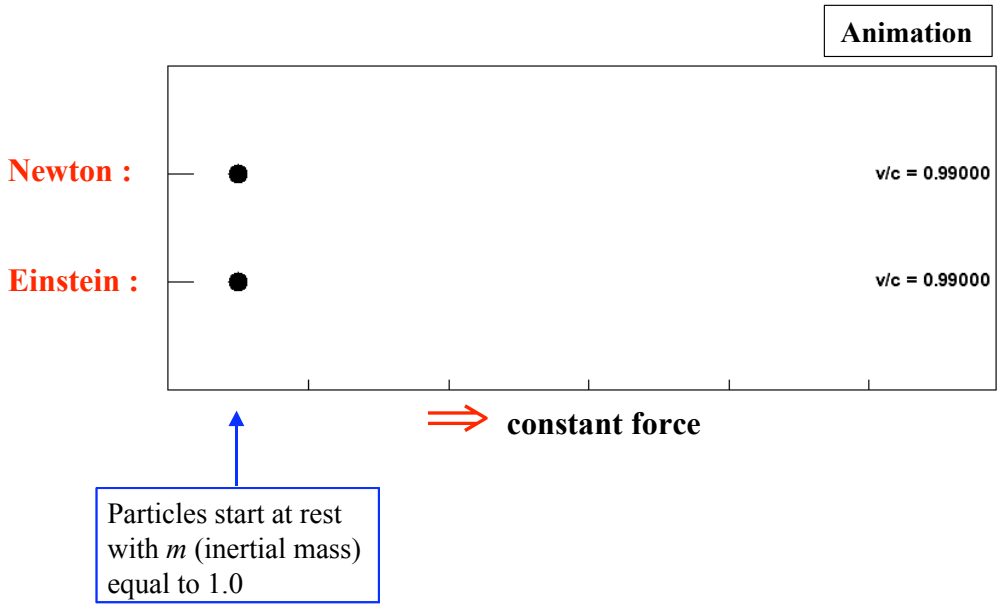
- $v > c$ would not make sense : no body can move faster than the speed of light.
- No ***signal*** or ***cause*** can move faster than the speed of light.

Mapping Space-Time



Accelerating Bodies

- So what happens when we try to accelerate bodies beyond the speed of light ?



E=mc²

- Where does the energy go ?
- Consider accelerating a body moving with $v \approx c$. Since the velocity cannot increase, it must be the mass that changes :

work = force × distance — momentum = mass × velocity

$$\Delta E = F \times d = \frac{\Delta p}{\Delta t} \times d = \Delta m \times c \times \frac{d}{\Delta t} = \Delta m \times c \times c = \Delta m c^2$$

force = rate of change of momentum — speed = distance / time

- Einstein then made the leap to suggest all mass is equivalent to energy :

$$E = mc^2$$

… including that which a body would have at rest. In fact "***m***" is now speed dependent :

$$E = \gamma m_0 c^2$$

gamma factor — rest mass

$E=mc^2$

- What does this equation mean ?

- Example applications.

Antimatter

$$E = mc^2 = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

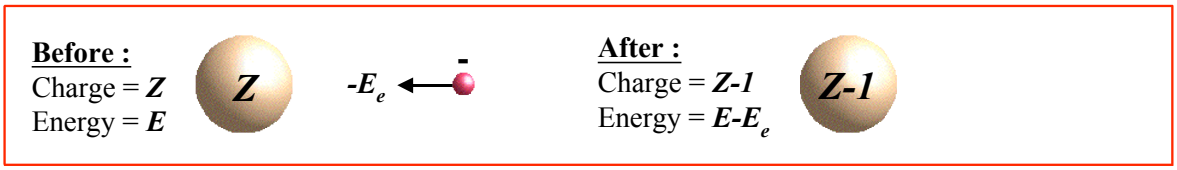
- Every square root has a negative solution as well as a positive solution .
- Can I just ignore the negative energy solutions ?

- It was a curiosity for some years, until Paul Dirac realised that those negative energy solutions were actually ***required*** in order to be able to extend quantum mechanics to describe relativistic (fast moving) electrons.

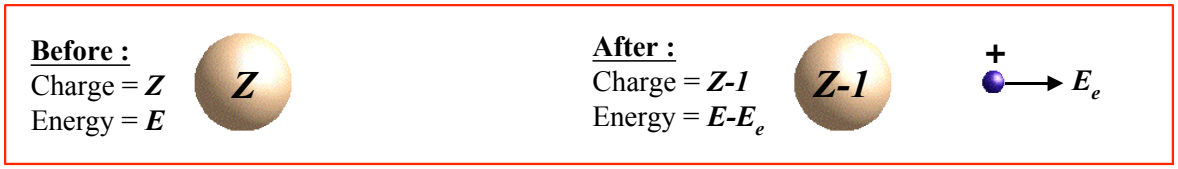
- What does it mean ?

How to Interpret Negative Energies ?

- But a (negatively charged) electron with **negative** energy and momentum behaves exactly like a (positively charged) positron with **positive** energy and momentum.
- For example, imagine a nucleus absorbing a negative energy electron :



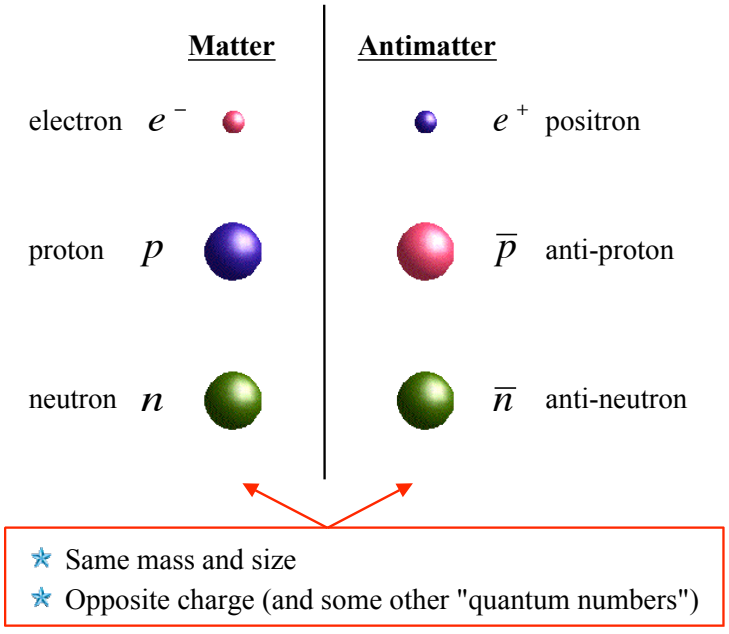
- This is exactly the same as emitting a positive energy positron :



- Some nuclei (e.g. ^{64}Cu) decay in exactly this manner.

Antimatter

- So, relativity and quantum mechanics together predict the existence of antimatter particles corresponding to every known matter particle.



Summary

- The principle of relativity states that :

  The laws of physics are the same in all inertial reference frames

- Newton's laws are invariant under Galilean transformations between inertial reference frames :

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

- The laws of electromagnetism, with their prediction of a constant light speed, violate Galilean relativity. But they are invariant under a new set of coordinate transformations - the Lorentz transformations. Highly counter-intuitive consequences flow from these transformations :
  - Different observers do not agree on which spatially separated events are simultaneous. The end of Newtonian absolute time.
  - Moving clocks run slow by a factor of γ .
  - Moving objects appear contracted by the same factor γ .

Summary

- Einstein's famous equation

$$E = mc^2 = \gamma m_0 c^2$$

  tells us that mass and energy are equivalent. Mass can be converted into energy (nuclear reactor) and energy can be converted into mass (particle accelerator).

- The negative energy solutions are interpreted as describing antimatter. We'll discuss antimatter in more detail in the particle physics session.

Exercises

1) A train 110m long passes a station platform 100m in length. Use the length contraction formula $L' = L/\gamma$ to calculate how fast the train must be moving in order to fit into the station. What happens if the train stops to let passengers off ?

2) A time traveller is prepared to leave earth and travel for 1 year of his own time, in order to arrive back on earth 2 years into the future :

   i. How fast must he travel ?

   ii. What energy is required to accelerate his spaceship to the required speed (as a multiple of its rest mass energy) ?

Exercises

3) The nuclear reaction powering the sun is :

$$4\left({}^{1}\mathrm{H}\right) \rightarrow {}^{4}\mathrm{He} + 2e^{+} + 2\ \text{neutrinos} + \text{energy}$$

The masses of the hydrogen and helium atoms are :

$$m(\mathrm{H}) = 1.00794 \quad \text{amu}$$

$$m(\mathrm{He}) = 4.002602 \quad \text{amu}$$

where 1 amu $= 1.66 \times 10^{-27}$ kg

i. What is the energy liberated in this reaction ? (You can ignore the positrons and neutrinos)

ii. Given that this reaction takes place 10^{38} times per second, at what rate is energy generated inside the sun ?