Part 1: the particles and their interactions

1 LECTURE 1

1.1 Overview of the course.

In this course we will learn about the smallest particles in nature and their interactions with each other. There are four basic forces of nature and three of them have been succesfully combined to describe everything we see in the particle world. This unified theory is called the Standard Model. Gravity, the force which is not part of the Standard Model (yet) is not part of this course, although its structure can be descibed by a theory of similar construction to the Standard Model. After we have understood how the particles interact, and bind together to form composite particles, we move on to looking at nuclear physics. This addresses the behaviour of many composite particles together. Much of the nuclear physics of today is really technology: no real theory exists which predicts the behaviour of nuclei, partly because the nuclear systems are very complicated involving many particles at a time. The technology, on the other hand, is awe-inspiring. Reactors and bombs have changed the political and environmental landscape of earth irrevocably. Carbon dating has given us a chance to look way back at the past with some certainty.

There is a lot of jargon in the field of particle physics: some of it useful and some of it not. It is unavoidable however. When a new name is introduced, you will find it in bold face in the text. Make sure you go over the notes and understand what each name refers to.

1.2 Historical perspective.

The course of discovery has run in the opposite direction to that of this lecture course. We know how many fundamental particles there are, the stucture of their interactions and how they arrange themselves as the building blocks of everything we see. However, in order to uncover these fundamental constituents of matter, very high energies are needed to blast the composite structures apart. The higher energy has come with improving technology and increasing understanding of what to look for and so we have only recently discovered what is the starting point of any description of the universe.

1.3 Natural Units

In order to simplify the mathematics, particle physicists choose units called **Natural Units** in which $\hbar = c = 1$, $\hbar$ and c being fundamental constants of nature used frequently in quantum mechanics and relativity.

The implications of setting $\hbar = c = 1$ are that the unit of length will be numerically equal to the unit of time and that length and time are absolutely equivalent dimensions.

Now then, if $\hbar = 1$ then

$$E = h\nu \tag{1}$$
$$p = h/\lambda \tag{2}$$

reduces to a dimensional equation

$$[E] = [T]^{-1} = [p] = [L]^{-1} \tag{3}$$

The dimensions of $\hbar$ are therefore energy $\times$ time so that

$$[\hbar] = [M][L]^2[T]^{-1} \tag{4}$$

where M has units of mass. So if $\hbar = 1$ then $[\mathrm{M}]=[\mathrm{L}]^{-1} = [\mathrm{T}]^{-1}$. It is usaul to refer to momentum in units of GeV/c, masses in units of GeV/c^2. Some useful conversion factors are

$$1\text{kg} = 5.607\ 10^{26}\text{GeV} \tag{5}$$
$$1\text{m} = 5.068\ 10^{15}\text{GeV}^{-1} \tag{6}$$
$$1\text{s} = 1.519\ 10^{24}\text{GeV}^{-1} \tag{7}$$

The sizes of some interesting objects are shown in Table 1 and useful units are shown in Table 2.

size(m)	object
10^{27}	Universe = age $\times$ $c = 2{\cdot}10^{10} yrs \times c$
10^{23}	Intergalactic distance $2{\times}10^{25} cm$
10^{20}	Galactic size $6{\times}10^{22} cm$
10^{15}	Nearest Star $2{\times}10^{17} cm$
10^{12}	Planetary Orbit $6{\times}10^{14} cm$ (Pluto)
10^{11}	Distance to sun $1.5{\times}10^{13} cm$
10^{4}	Satellite orbit
10^{-12}	Atomic radius
10^{-15}	Nucleon radius
10^{-18}	Range of weak force

Table 1: : Sizes of certain objects

1.4 Four Vectors and Lorentz Invariance Revisited

A cornerstone of modern physics is that the fundamental laws can be written in the same form for all **Lorentz Frames**. You may remember that a Lorentz Frame is a reference frame which is moving with a uniform relative velocity. Recall that special relativity is based on the fact that c, the velocity of light, is the same in all Lorentz Frames. A

1 parsec	$3\times10^{18} cm$
1 Angstrom	$10^{-10} m$
1 fm	$10^{-15} m$
1 MeV	$10^{6} eV$
1 GeV	$10^{9} eV$
1 TeV	$10^{12} eV$
1 PeV	$10^{15} eV$
1 barn	$10^{-24} cm^2$
1 millibarn (mb)	$10^{-27} cm^2$
1 nanobarn (nb)	$10^{-33} cm^2$
1 picobarn (pb)	$10^{-36} cm^2$
1 femtobarn (fb)	$10^{-39} cm^2$

Table 2: : Useful units

Lorentz transformation relates the coordinates in two such frames (moving relative to each other). The invariant quantity is $c^2t^2 - \mathbf{x}^2$.

By definition, a set of four quantities which transform like (ct,**x**) under a Lorentz transformation is called a four vector. We use the notation

$$(ct, \mathbf{x}) \equiv (x^0, x^1, x^2, x^3) \equiv x^\mu \tag{8}$$

special relativity tells us that total energy E and momentum **p** transform like a four vector:

$$(E, \mathbf{p}) \equiv (p^0, p^1, p^2, p^3) \equiv p^\mu \tag{9}$$

and the basic invariant is $E^2 - \mathbf{p}^2 = m^2$ where m is the rest mass of the particle. We use natural units : $\hbar = c = 1$.

The scalar product of a four vector:

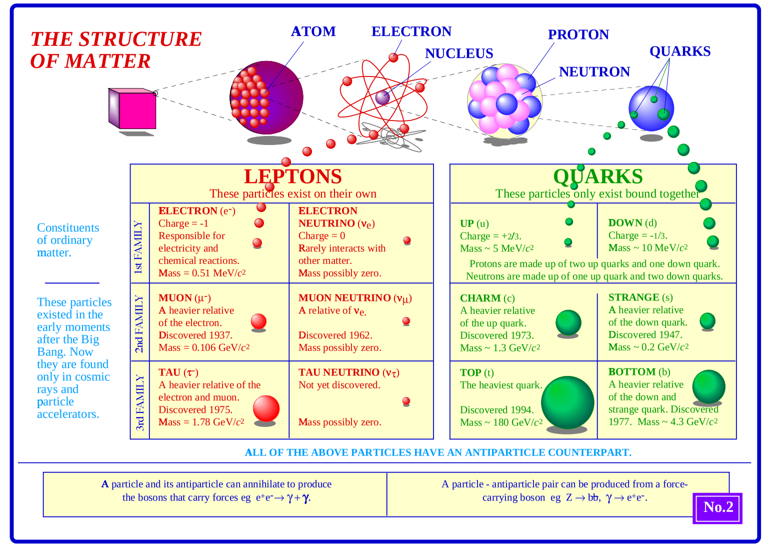
$$A \cdot B = A^0 B^0 - \mathbf{A} \cdot \mathbf{B} \tag{10}$$

Remember the metric tensor? $g_{00} = 1, g_{11} = g_{22} = g_{33} = -1$. All other components are 0.

$$A \cdot B = A_\mu B^\mu = A^\mu B_\mu = g^{\mu\nu} A_\mu B_\nu \tag{11}$$

For a *free* particle

$$p_\mu p^\mu = p^2 = m^2 \tag{12}$$



2 LECTURE 2

2.1 Introduction to the fundamental particles and forces.

There are three classifications of the fundamental particles:

- Quarks
- Leptons
- Gauge Bosons

There are three types of interaction which the fundamental particles can undergo:

- Strong via the Strong Force (or Colour Force)
- Weak via the Weak Force
- Electromagnetic (EM) via the Electromagnetic Force

The quarks interact via all three of these forces, the leptons do not interact with the strong force.

The Weak and Electromagnetic Forces are not independent but are related to each other by a factor. They are collectively referred to as the Electroweak Force. Our present understanding is that the colour force is independent, although the holy grail of particle physics is to "unify" all the forces (including gravity) such that they can all be written in terms of one fundamental coupling constant.

2.1.1 Quarks and Leptons

The quarks and leptons are **fermions** which means they have a spin quantum number which is half integer and obey Fermi-Dirac statistics. Quarks come in 6 **flavours: up, down, strange, charm, bottom** and **top**. The quarks can be arranged in three **generations** of doublets, the properties of the generations being similar, but the masses of which are successively heavier.

$$\begin{pmatrix} u^{+\frac{2}{3}} \\ d^{-\frac{1}{3}} \end{pmatrix} \begin{pmatrix} c^{+\frac{2}{3}} \\ s^{-\frac{1}{3}} \end{pmatrix} \begin{pmatrix} t^{+\frac{2}{3}} \\ b^{-\frac{1}{3}} \end{pmatrix}$$

Leptons come in 2 types and 3 flavours. The three flavours are **electron, muon and tau**. The two types of lepton are the charged leptons denoted by the symbols e,μ,τ and the neutral leptons or **neutrinos** denoted by the symbols ν_e, ν_μ, ν_τ. The leptons can also be arranged in three generational doublets whose charged lepton masses increase from the electron which has a mass of 0.511MeV to the tau which has a mass of 1.8GeV. The neutrinos have zero mass (or almost).

$$\begin{pmatrix} \nu_e^0 \\ e^- \end{pmatrix} \begin{pmatrix} \nu_\mu^0 \\ \mu^- \end{pmatrix} \begin{pmatrix} \nu_\tau^0 \\ \tau^- \end{pmatrix}$$

The quarks and leptons carry quantum numbers, which are their prescription for how they will behave in any kind of interaction, a bit like their 'DNA'. Some of these quantum numbers have to do with their type such as **Baryon Number** which is $\frac{1}{3}$for all the quarks and 0 for the leptons and **Lepton Number** which is 1 for all the leptons and 0 for all the quarks. Some of their quantum numbers determine how strongly they will couple to the Gauge Bosons. The quantum number for the Electromagnetic Force is the electric charge. For example, the charged leptons with charge $\pm e$ will couple more strongly to the EM Force than the quarks with charge of $\pm \frac{1}{3}e$ or $\pm \frac{2}{3}e$. The quantum number for the Weak Force is called the **Weak Isospin**, mainly because each of the quarks and leptons in the above doublets have weak isospin of -$\frac{1}{2}$for the upper element and $+\frac{1}{2}$for the lower element. In fact, the above doublets are called **weak iso-doublets** becasue of this. The weak interaction has the ability to span different generations. The quantum number of the Strong Force is called **colour**. Each of the quarks can come in one of three colours called red, green and blue. The leptons do not carry the colour quantum number, and the neutrinos do not carry an EM quantum number either. Free quarks are never observed because the strong force gets stronger with distance so that the quarks become more tightly bound as they get further away from each other.

A summary of the particle properties is given in Table 3. The quarks and leptons carry quantum numbers, some of which are always conserved. The quantum numbers which are conserved in all interactions are baryon number and lepton flavour number, B-L, colour

quark						lepton					
symbol	Q	B	L	I_3	mass	symbol	Q	B	L	I_3	mass
d	$-\frac{1}{3}$	$\frac{1}{3}$	0	-$\frac{1}{2}$	0.3GeV	e^-	-1	0	1	$\frac{1}{2}$	0.511 MeV
u	$\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{1}{2}$	0.3GeV	ν_e	0	0	1	$\frac{1}{2}$	≈ 0
s	$-\frac{1}{3}$	$\frac{1}{3}$	0	-$\frac{1}{2}$	0.5GeV	μ^-	-1	0	1	$\frac{1}{2}$	105 MeV
c	$\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{1}{2}$	1.8GeV	ν_μ	0	0	1	$\frac{1}{2}$	≈ 0
b	$-\frac{1}{3}$	$\frac{1}{3}$	0	-$\frac{1}{2}$	5.0GeV	τ	-1	0	1	$\frac{1}{2}$	1.7GeV
t	$\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{1}{2}$	170GeV	ν_τ	0	0	1	$\frac{1}{2}$	≈ 0

Table 3: : Table of fermion properties

and electric charge. Weak isospin is not conserved. These conservation laws are very important when it comes to drawing Feynman diagrams and calculating cross sections.

All these fundamental particles are **pointlike**, that is they have no discernable size. This is mainly what leads us to believe they are indeed fundamental and they themselves do not have substructure.

All of the quarks and leptons (the fermions) have **antiparticles**. In the case of electrically charged particles, the antiparticle has the opposite electric charge. If the particle has baryon number = $\frac{1}{3}$ then the antiparticle has B=-$\frac{1}{3}$. If the particle has lepton number L = 1, the antiparticle has L=-1. The quantum numbers are all equal and opposite. The masses are exactly the same as the particle's.

2.1.2 The Force Carriers: Gauge Bosons

The Gauge Bosons are the carriers of the forces. They interact always with the same strength, specific to the force multiplied by the appropriate quantum number carried by the fermion. They have a quantum number called spin which is = 1 (hence boson) and therefore obey Bose-Einstein statistics.

The **photon** is massless and chargeless although it carries the **electromagnetic** force. Because it is massless, the range of the electromagnetic potential is infinite, falling off like $\frac{1}{r}$, where r is the distance from the source.

The $\mathbf{W^+}$,$\mathbf{W^-}$ and $\mathbf{Z^0}$ carry the **weak** force and the $W^+$and $W^-$are also themselves weakly charged, i.e. they carry **weak isospin**). The $W^+$and $W^-$are also electrically charged. The '+' and '-' refer to their electric charge. These Gauge Bosons are very heavy, the $W^+$and W^-have mass of 84 GeV and the $Z^0$of 92 GeV. Because of their mass, the range of the weak force is very short, about 10^{-3}fm.

The **colour** force is mediated by coloured gluons. These do not carry electric or weak charge, but do carry the colour charge. There are 8 different gluons. They are massless and because of this their range is infinite. However, the color force itself is somewhat different from the other two, in that the strength of the force increases the further away you are from the source, as opposed to decreasing. Obviously, if the force gets stronger with

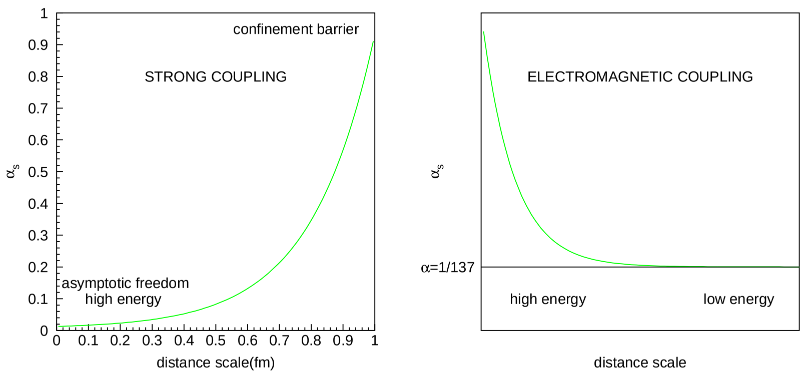


Figure 1: Behaviour of coupling constants for EM and strong forces.

distance then we could be very badly affected by quarks in another galaxy! "Confinement" and "Asymptotic Freedom" come to the rescue and ensure that gluons and quarks are bound inside about 1fm but can behave like free particles at very very small distances (large energy scales). The **strong** force manifests itself out beyond 1fm to about 2 fm in the form of bound states of quarks called pions which we shall come to later on. However, the Colour and the Strong (or nuclear) Force are just different manifestations of the same force.

The coupling constants of the forces, which will be referred to as g_F are energy (or distance) dependent and are therefore often referred to as **running couplings**. Figure 1 shows the running coupling constants of the forces as a function of distance from the source (energy transfer). The colour force (it is the colour force at these distances, mediated by massless gluons) has the most dramatic dependence. At about 1fm $\frac{g_s^2}{4\pi} \approx 1$ whereas inside 10^{-16}m the strong coupling is weaker than the EM coupling.

A summary of the characteristics of the three forces is shown in Table 4.

2.2 Introduction to Feynman Diagrams.

Once you have mastered the art of Feynman Diagrams, everything will become clear! Feynman invented the diagrams just for the purpose of Simplifying the picture of what happens in particle interactions. Figure 2 shows a generic Feynman diagram. The time axis is going up the page and space along the page. Two fermions (as this is a generic diagram, we shouldn't be more specific than that, but they could be quarks or leptons (In fact they dont *have* to be fermions at all) come close enough to each other to interact via one of the three forces we have talked about. The Gauge Boson is exchanged between them and they go on their way. Its as simple as that. We will do some examples of

Force	EM	strong	weak
Gauge Boson	γ	8g,π	W^+,W^-,Z^0
Mass	0	0,100MeV	100GeV
Quantum Number	electric charge	colour	weak isospin
Range	∞	∞,1fm	10^{-18}m
Strength	α_{EM}=0.008	$\alpha_s \approx 1.2$	α_w=0.03
Type	Abelian	Non-Abelian	Non-Abelian
High energy dependence	stronger	weaker	weaker

Table 4: : Summary of charateristics of the forces

specific interactions later. The purple blobs on the figure are important: they represent the **coupling strength** of the interaction multiplied by the appropriate qunatum number for the fermion, referred to as Q_f. At low energy transfer, the colour force has a very large(strong) coupling constant, while the weak and electromagnetic forces have very small(weak) ones. In addition, the conservation laws must be adhered to at each vertex. For example, during this interaction it is possible that the fermions have changed some of their characteristics. If the Gauge Boson is charged, then the fermions going away from the interaction will have their charges changed by the charge of the Gauge Boson because charge must be conserved at the vertex. This diagram will enable us to make an important calculation about the interaction of these two particles.

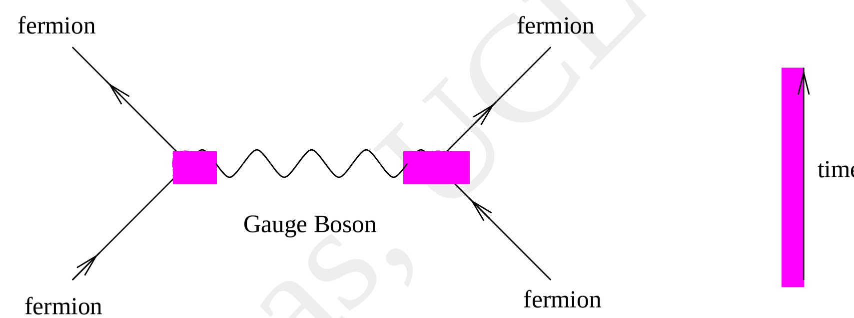


Figure 2: A generic Feynman Diagram

In the Feynman diagram, antiparticles can be represented by the particles going in the opposite direction. So, the generic Feynman diagram we looked at last time could be redrawn with the arrows going in the other direction and the interaction would have exactly the same probability of occurring for the particles as for the antiparticles as in Figure 3.

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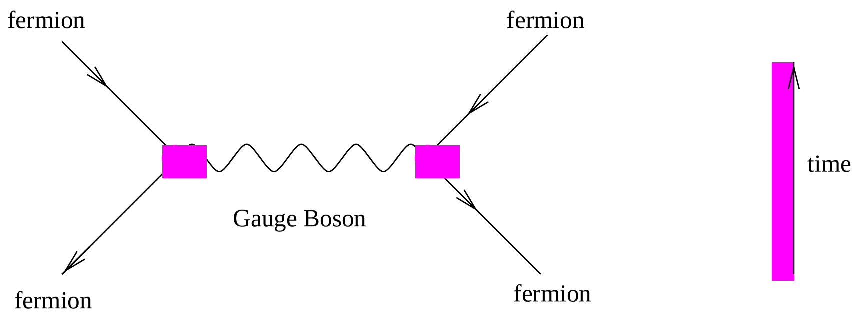

Figure 3: A generic Feynman Diagram for anti-particles

3 LECTURE 3

3.1 The Klein Gordon Equation.

In order to understand the form of these Gauge Forces, we go back to Quantum Mechanics. The two unproved postulates of quantum mechanics are:

- [E,t] = i$\hbar$
- [p,**x**] = -i$\hbar$

Trying to represent these postulates, we can choose

- $p = -i\hbar\frac{\partial}{\partial x}$
- $E = i\hbar\frac{\partial}{\partial t}$

Starting from

$$E^2 = p^2 + m^2 \tag{13}$$

and substituting in the operator expressions for E and p, we arrive at the **Klein-Gordon** equation

$$\frac{\partial^2\phi(x,t)}{\partial t^2} = \nabla^2\phi - m^2\phi(x,t) \tag{14}$$

$$(\Box + m^2)\phi(x,t) = 0 \tag{15}$$

where ϕ can be interpreted as either a wavefunction of a spinless boson with mass m, or as the corresponding classical wave of a potential. The Klein Gordon equation has solutions of the form

$$\phi = e^{i(Et - \mathbf{p}\cdot\mathbf{x})} \tag{16}$$

If there is a source of Klein-Gordon fields, such as for a fixed static potential as used to describe the forces between particles, the time derivative is zero and the equation becomes

$$(\nabla^2 - m^2)\phi(x,t) = 4\pi\rho \tag{17}$$

with

$$\rho = g_F Q_f \delta^3(\bar{\mathrm{x}} - \bar{\mathrm{x}_0}) \tag{18}$$

where g_F *is the strength of the force and* Q_f is a multiplier associated with the source of the field (the quantum number associated with the particular fermion). The Gauge Forces as manifested between fermions can be interpreted as the static potentials emanating from the fermions. The solutions have the form

$$\phi = g_F Q_f \frac{e^{-m|\mathbf{x}-\mathbf{x_0}|}}{4\pi|\mathbf{x}-\mathbf{x_0}|} \tag{19}$$

$$= g_F Q_f \frac{e^{-mr}}{4\pi r} \tag{20}$$

This simple picture is very instructive and can be immediatley applied to electrostatics which is certainly a manifestation of the Electromagnetic Force. For the EM force, the photon is massless and so

$$-\nabla^2\phi = 4\pi\rho \tag{21}$$

where we define

$$\vec{E} = -\vec{\nabla}\phi = \frac{eQ}{r^2}\hat{r} \tag{22}$$

and

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho \tag{23}$$

so that

$$\phi = \frac{eQ}{r^2}\hat{r} \tag{24}$$

and this is the equation for the scalar potential in electrostatics.

Let us consider the **potential energy** of an interaction carried by a Gauge Boson by putting a test 'charge' $g_F Q'_f$ in the potential. You can write down the general form of the potential energy

$$V(r) = g_F Q_f g_F Q'_f \frac{e^{-mr}}{4\pi r} \tag{25}$$

g is identified with the coupling constant of the interaction which is multiplied by the appropriate quantum number of the interacting fermion Q_F. The range of the force is given by the mass of the Gauge Boson: the distance over which the potential energy falls to 1/e of its strength at r=0. Now, using the conversion factors for converting mass into length you can calculate what the ranges of the forces should be given the mass of the Gauge Boson .

For the strong force, $M_\pi \approx$ 100MeV, R=2fm. For the weak force, $M_W \approx$ 100GeV, R=2$\cdot 10^{-3}$fm.

Figure 4 shows the form of the potential energy for the three Gauge Boson masses. The potential energies fall below 1MeV for the weak interaction at 10^{-17}m, for the strong interaction at 10^{-15} and at about 10^{-13}m for the EM interaction.

The characteristics of the three forces are summarized in Table 5.

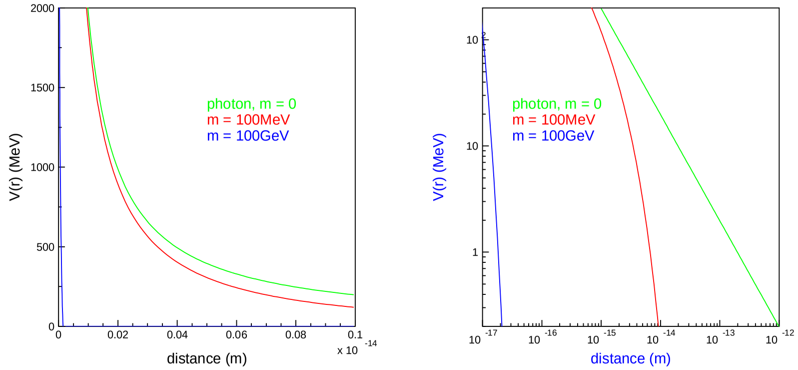

Figure 4: Potential Energy between two fermions undergoing Gauge Boson exchange with M=0, 100MeV and 100GeV

Interaction	Range	Lifetime(s)	Coupling (at $q^2 = (100GeV)^2$)
Strong or Color	1fm or $\approx \frac{1}{M_\pi}$	10^{-23}	$\frac{g_s^2}{4\pi} = 0.14$
Electromagnetic	∞	$10^{-20} - 10^{-16}$	$\frac{g_{EM}^2}{4\pi} = \frac{1}{129}(\frac{1}{137}\ at\ q^2 = 0) = 0.008$
Weak	$\frac{1}{M_W}$	$\geq 10^{-12}$	$\frac{g_w^2}{4\pi} = 0.034$

Table 5: Characteristics of the three Gauge forces

3.2 Virtual Particles

You have learned that the particles we are interested in are all point-like, but in order for the above scattering interaction to take place, at least one of the fermions must emit a Gauge boson when they are in the vicinity of each other. In fact, the fermions are surrounded by a cloud of **virtual** Gauge bosons as they propagate through space. We can use Heisenberg's uncertainty principle to help us with this concept. We know that

$$\Delta E \Delta t = \hbar \tag{26}$$

or

$$\Delta p \Delta x = \hbar \tag{27}$$

This implies that a particle of energy ΔE can survive for a time Δt without violating energy and momentum conservation. Similarly, this can be rephrased in terms of the momentum and the distance or range of the Gauge Boson. So, an electron can emit a virtual Z^0 and reabsorb it a time t later without upsetting the conservation laws.

A free particle is known as being on mass shell, or **real**. We will soon come across the

situation where particles undergoing interactions are not free, not real and not on mass shell!

3.3 Interactions between two particles

Look at the Feynman Diagram in Figure 2, the generic Feynman diagram. In this instance, two fermions come close enough to each other to exchange a Gauge boson. How do they know they are in the vicinity of each other? The Gauge bosons are responsible for producing a **field** (e.g the electromagnetic field) and they do this by constantly being emitted and absorbed from the fermions themselves. These are virtual Gauge Bosons and so can exist for a time $\Delta t \approx \frac{\hbar}{\Delta E}$.

Figure 5 shows a red u quark propagating in space. It first emits a virtual photon. Nothing about the quark quantum numbers changes. Then it reabsorbs the photon. Later it emits a W^+which changes the weak and the electromagnetic properties of the quark by one unit each. The red u quark becomes a red d quark before the $W^+$is reabsorbed. The u quark then emits a virtual red/anti-green gluon which changes the red u quark to a green u quark. After reabsorbing the red/anti-green gluon and reverting back to its original colour, the u quark emits a $Z^0$and becomes a u quark and finally reabsorbs that $Z^0$and becomes again the original red u quark! The rate at which the virtual Gauge bosons are emitted and absorbed is proportional to their coupling strengths at each of the vertices.

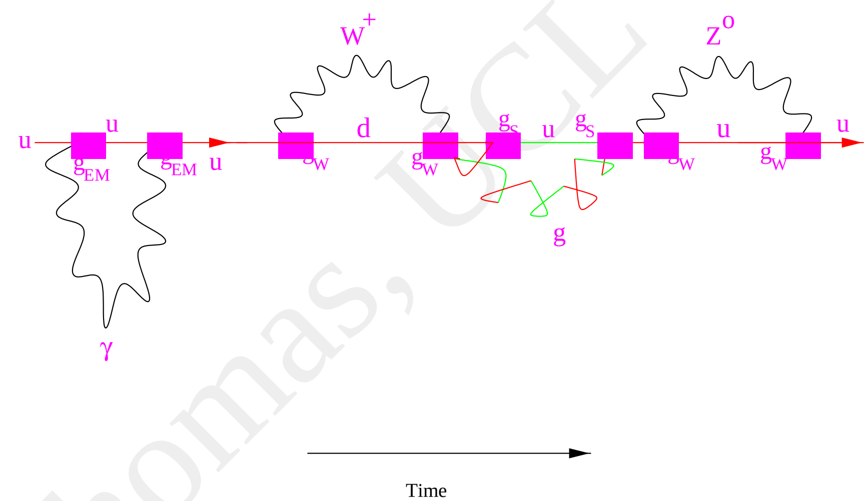

Figure 5: u quark propagating in space

Figure 6 shows what you would see if you could look at a quark coming towards you. The black lines coming out are the photons, with the longest range. The yellow lines are the weak Gauge bosons which travel only 10^{-18}m and the red,blue and green lines are the gluons. They are confined within 10^{-15}m and outside of this region the strong force is manifested by pions which are the brown lines. From the form of the uncertainty relationship, you can emit a low energy photon for a longer Δt than a high energy one. Alternatively, you can think of the low energy photons emanating a longer distance away from the fermion than the high energy ones. This is an intuitive way of looking at why it is that the electromagnetic force has an infinite range although its potential falls off like 1/r; you can imagine lower and lower energies giving longer and longer ΔTs.

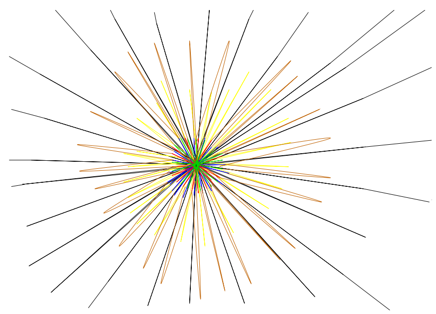

Figure 6: A quark coming out of the page: the lines represent the virtual Gauge Bosons which are emitted and reabsorbed

4 LECTURE 4

4.1 Perturbation Theory

The main quantity which is important is the **amplitude** with which the interaction will take place. The amplitude depends not only on the coupling constants at the vertices but on the characteristics of the Gauge Boson being exchanged. It is possible to construct an **Invariant Amplitude** for a process which is Lorentz invariant and will therefore have the same value in any Lorentz Frame.

The calculations for the Invariant Amplitude rely on the use of a method called **Perturbation Theory**. As long as the couplings are small compared to $\sqrt{4\pi}$, the interactions between the particles can be thought of as perturbations on a free particle wave function. This is true for the weak and electromagnetic interactions, but not always for the strong interaction. It is not always possible to think of quarks in the interaction as being free because they are always tightly bound together, but at very small distances, as the force becomes weaker, then it is indeed possible to use perturbation theory.

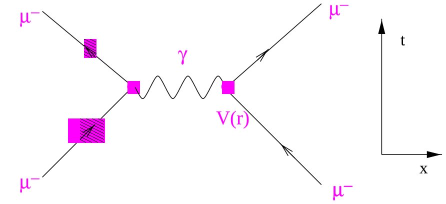

Figure 7: Interaction being considered for the amplitude calculation

A free particle is described by the solution to Schrodingers Equation

$$\psi(\mathbf{p},t) = \frac{e^{i(\mathbf{p}\cdot\mathbf{r}-Et)}}{(2\pi)^{3/2}} \tag{28}$$

sometimes we are interested in the time independent part of this wave function: $\phi(\mathbf{r}) \propto \psi(t)e^{i\mathbf{pr}}$ and sometimes the time dependent part: $\phi(\mathbf{t}) \propto \psi(\mathbf{r})e^{-iEt}$. The wave function includes all the information which the uncertainty principle allows us to know about the particle.

4.2 The Invariant Amplitude

We can derive the amplitude starting from the form of the force:

$$V(r) = \frac{g}{4\pi r}e^{-Mr} \tag{29}$$

where

$$g = g_F^2 Q_f Q_f' \tag{30}$$

The potential in coordinate space will have an associated amplitude in momentum transfer space, f($\mathbf{q}$), for scattering of a particle of momentum $\mathbf{p}_1$ to $\mathbf{p}_2$ with resultant momentum transfer $\mathbf{q} = \mathbf{p}_2$-$\mathbf{p}_1$. The situation is described in a Feynman Diagram in Figure 7.

This amplitude is simply the Fourier transform of the potential (just as the angular distribution of light diffracted from an object in classical optics is the Fourier transform of the object). So for a central potential, multiplying the potential by the particle's time independent part of the wave function (the same as a Fourier Transform),

$$V_{fi} \equiv \int \phi_f^*(\mathbf{r})\mathbf{V}(\mathbf{r})\phi_i(\mathbf{r})d\mathbf{r} \tag{31}$$

$$f(\mathbf{p_1}-\mathbf{p_2}) = \int_{-\infty}^{+\infty} e^{-i\mathbf{p_2}\cdot\mathbf{r}}V(r)e^{i\mathbf{p_1}\cdot\mathbf{r}}d\mathbf{r} \tag{32}$$

$$f(\mathbf{q}) = \int_{-\infty}^{+\infty} V(r)e^{i\mathbf{q}\cdot\mathbf{r}}d\mathbf{r} \tag{33}$$

(34)

where g_0 is the intrinsic coupling strength of the particle to the potential.

$$\mathbf{q}\cdot\mathbf{r} = qr\cos\theta \quad (35)$$
$$d\mathbf{r} = r^2 d\phi \sin\theta d\theta dr \quad (36)$$
$$(37)$$

This changes the integration limits from $-\infty \to \infty$ to $0 \to \infty$.

$$f(\mathbf{q}) = 4\pi \int_0^\infty V(r)\frac{\sin qr}{qr} r^2 dr \quad (38)$$
$$f(\mathbf{q}) = -\frac{g_F^2 Q_f Q_f'}{|\mathbf{q}|^2 + M^2} \quad (39)$$

In this example, the scattered particle has lost no energy, but more usually energy E as well as three-momentum **q** will be transferred. This result also holds when q is the four-momentum transfer.

In mathematical terms, the Gauge Boson exchange is known as a **propagator** and its magnitude (the probability that it can be produced) is always proportional to $\frac{1}{q^2 - M^2}$. (q refers to the four-momentum transferred between the fermions and M is the mass of the Gauge Boson) The four-momentum of the propagator is just the four-momentum transferred between the two fermions. The amplitude for producing a given propagator is inversly proportional to the square of the mass of the Gauge Boson. Look what happens if $q^2 = M^2$. The Gauge Boson is **on-shell**. This is the 4-momentum criteria for a free particle.

So, now you know everything you need to know to calculate this amplitude of the interaction.

$$\mathcal{F}, \mathcal{M} = g_f Q_f \frac{1}{q^2 - M^2} g_F Q_f' \quad (40)$$

where the first g_F is the coupling constant of the Gauge Boson (remember that Gauge Bosons couple to *all* particles with the strength of their associated gauge force quantum numbers) to the emitting fermion, and g_F is the coupling of the Gauge Boson to the second fermion. See Table 4 which lists the ranges for the various forces. M and p refer to the mass and four-momentum of the Gauge Boson.The **Invariant Amplitude** is denoted by the letter $\mathcal{F}$or $\mathcal{M}$. It is known as the Invariant Amplitude because it is Lorentz Invariant. It will have the same value in any frame which has a uniform relative velocity.

The Invariant Amplitude contains *all* the physics of the interaction in it. Quantities which are important to study are :

- The coupling constant
- The behaviour of the coupling "constant" with momentum transfer
- The mass of the Gauge Boson

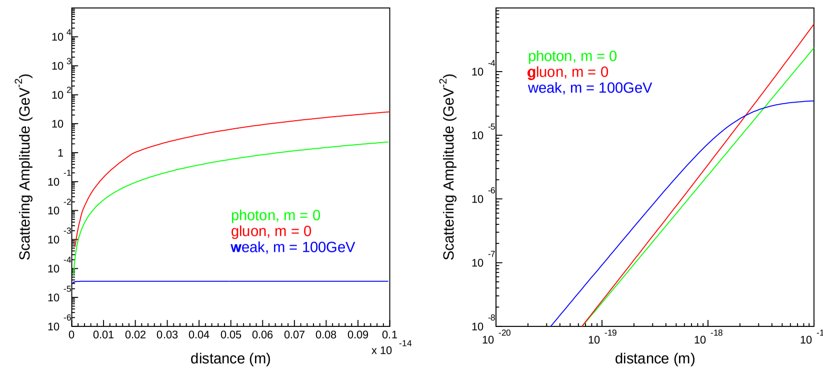

Figure 8: Invariant amplitudes for the Gauge interactions.

The study of particle physics is the quest to relate experimental (i.e. measurable) quantities to this Invariant Amplitude.

The Invariant Amplitude is the same for all interactions of a given type. For example, if the Gauge Boson is a Z^0, and the interaction is e^+e^- annihilation, then all the quarks with the same quantum numbers (u,c,t) or (d,s,b) will have the same Invariant Amplitude. However, if the quarks are very massive compared with the total energy available, i.e. two top quarks, then this reaction is suppressed compared to the others, although the Invariant Amplitude is the same. Figure 8 shows the Invariant Amplitude ($q^2 - M^2$) as a function of q^2 for the three forces.

We now try to relate the Invariant Amplitude to a measurable quantity.