5 LECTURE 5

5.1 The relationship between the theory and the measurables

There are actually not so many **measurables**. We will study two of them, the **cross section** and the **lifetime**. The cross section is essentially the probability of an interaction. If you had a beam of electrons and a beam of muons and you wanted to see how often they interacted (and thus find out something about the Invariant Amplitude) you might set up an experiment similar to that shown (schematically) in Figure 9.

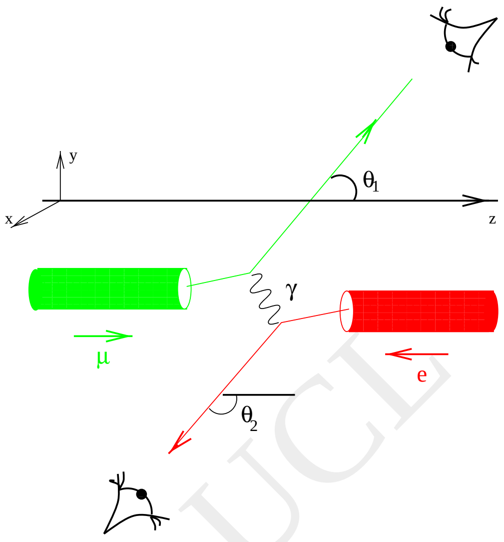

Figure 9: conceptual experimental set up for measuring the differential cross section

5.2 Interaction Kinematics

When studying an interaction, the kinematics are very important in understanding what is actually going on. Lets look at how the angles and energies of the particles in an interaction are related to each other. We have to choose a frame in which to do the calculation. Let us choose the frame where the muon is at rest. The system is depicted in Figure 10 for this particular frame of reference..

Conservation of energy tells us that $E_i = E_f$. In addition to conservation of energy, conservation of momentum (three momentum) leads to the invariant 4-momentum expression:

$$p^2 = E^2 - \mathbf{p}^2 = m^2. \tag{41}$$

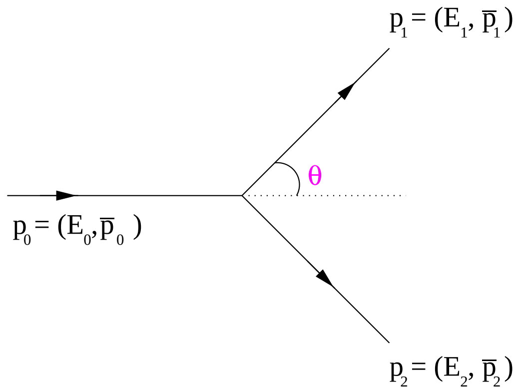

Figure 10: Diagram of interaction in the rest frame of the muon.

Similarly, the four-momentum *transfer* is invariant:

$$\begin{aligned} q^2 &= (E_0 - E_1)^2 - (\mathbf{p_0} - \mathbf{p_1})^2 && (42) \\ &= 2p_0p_1\cos\theta - 2E_0E_1 && (43) \\ &= -2p_0p_1(1-\cos\theta) && (44) \\ &= -4p_0p_1\sin^2\frac{\theta}{2} && (45) \end{aligned}$$

where the masses of the particles have been neglected because $m_f^2 << E_f^2$. Clearly, the magnitude of the scattered particles momenta are related to their scattering angle.

5.3 The Search for New particles

We have learned that there are 6 different quarks and 6 different leptons arranged in 3 generations of increasingly massive weak isospin doublets. It took a long time (some 20 years) from the time that the structure of quarks was recognized until the final quark was discovered in 1995. The reason for this delay is that more and more energy is needed in order to create the new quark states as they become more massive ($E^2 = \mathbf{p}^2 + m^2 : E \equiv m$ for $\mathbf{p} = 0$). This has meant the construction of very large machines called accelerators which are used to produce the interactions in which new particles are produced. We will learn more about accelerators in the next section.

5.4 The Center of Mass System

The **center of mass system** (CMS) is defined as a reference frame in which the total 3-momentum is zero. For example, if a particle decays at rest, then the center of mass

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system is the same as the laboratory system or **lab frame**. Alternatively, if a particle decays in flight, then the CMS is a frame which travels at a velocity such that the particle is at rest in that frame. In the case where two particles are in collision with each other the total $(4\text{-momentum})^2 (p^2)$ of the system is

$$\begin{aligned} p^2 &= (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 && (46) \\ &= m_1^2 + m_2^2 - 2\mathbf{p}_1 \cdot \mathbf{p}_2 + 2E_1 E_2 && (47) \end{aligned}$$

If the total energy in the CMS is E^*, then also $p^2 = E^{*2}$ If the target particle (2) is at rest then $E_2 = m_2$ and $\mathbf{p}_2 = 0$.

$$E^{*2} = m_1^2 + m_2^2 + 2m_2 E_1 \tag{48}$$

Now then, if you were to want to create a new particle of mass m^* from this system, you can calculate what energy is needed to create such a massive particle.

$$\begin{aligned} E_1^{th} &= \frac{m^{*2} - m_1^2 - m_2^2}{2m_2} && (49) \\ &\approx \frac{m^{*2}}{2m_2} && (50) \end{aligned}$$

In the case where two fundamental particles are travelling towards each other in opposite directions then

$$\begin{aligned} E^{*2} &= p^2 = m_1^2 + m_2^2 + 2p_1 p_2 + 2E_1 E_2 && (51) \\ &\approx 4E_1 E_2 \text{ if} && (52) \\ E_1, E_2 &>> m_1, m_2 && (53) \end{aligned}$$

(Remember that $\mathbf{p}_1 \cdot \mathbf{p}_2 = -1$) . If the two particles in collision are not fundamental, but are, for example, two protons which contain quarks inside them, then the center of mass energy is given by:

$$\begin{aligned} E^{*2} &= p^2 && (54) \\ &\approx 4\mathrm{x}_1\mathrm{E}_1\mathrm{x}_2\mathrm{E}_2 \text{ if} && (55) \\ \mathrm{x}_1\mathrm{E}_1, \mathrm{x}_2\mathrm{E}_2 &>> m_1, m_2 && (56) \end{aligned}$$

where $\mathrm{x}_1, \mathrm{x}_2$ is the fraction of the proton's momentum carried by the struck quarks. This is usually about $\frac{1}{6}$ because the protons momentum is equally shared between the quarks and the gluons inside it and because the momentum carried by the quarks is shared between three of them. The lesson to be learned from these examples is that a system where the particles are both moving in opposite directions will have more energy available to make a new state than if one particle is at rest.

5.5 $\mathrm{e}^+\mathrm{e}^- \rightarrow \mu^+\mu^-$ ($\mathrm{e}^+\mathrm{e}^-$ annihilation)

The Feynman Diagrams we have looked at so far for two particle interactions have been scattering processes. There is a second type of interaction called annihilation. This is a special case of the second center of mass system example above. If we replace the outgoing

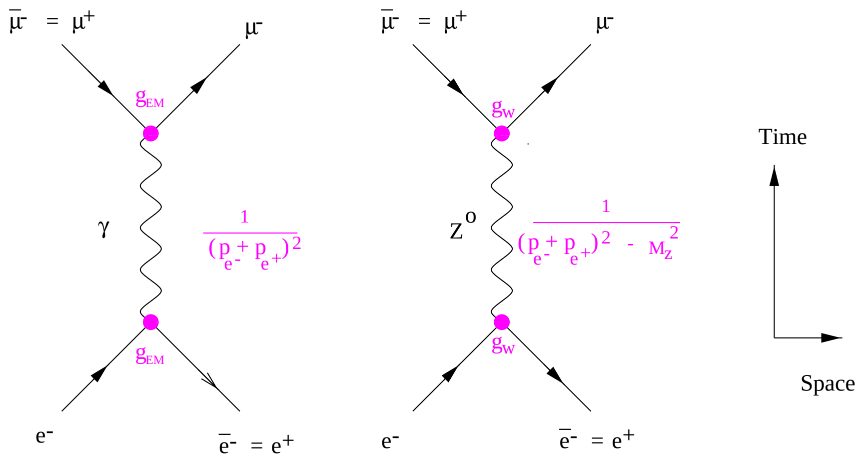

Figure 11: Annihilation Diagram

electron with an incoming anti-electron and the incoming muon with an outgoing anti-muon then we have an annihilation. See Figure 11. The only other difference is that now the Gauge Boson goes up the page. Now the propagator is given by either $\frac{1}{(p_{e+} + p_{e-})^2}$ or $\frac{A}{(p_{e-} + p_{e+})^2 - M_Z^2}$ depending on whether the Gauge Boson created is a Z^0 or a γ, i.e. whether the interaction is a Weak or an EM interation.

The process of $\mathrm{e}^+\mathrm{e}^-$ annihilation has provided a very good environment for searching for new particles. If the electron and the positron have exactly equal and opposite momentum,charge, lepton number and weak isospin (as they must if they are a fermion antifermion pair) the *neutral* Gauge Boson is produced at rest and will decay into a fermion antifermion pair which also have equal and opposite momentum. The produced fermion-antfermion pair could also carry equal but opposite baryon number and color. The only allowed neutral Gauge bosons for this process are the photon and the Z^0. Gluons are not produced because electrons do not carry the colour quantum number.

Any fermion which has a mass which is lower than half the center of mass energy can now be produced if it couples to the Gauge Boson. This constitutes the entire set of fermions: all the fermions have *either* an electromagnetic interaction (i.e. they carry electric charge) *or* a weak interaction (they carry weak isospin) and so if the energy of the interaction is high enough, all the fermions can be made by this process. And once produced, they can be studied. As the energy available for the interaction increases, if there are more quarks existing at higher energies, they can start to be produced.

In fixed target or pp collisions ,often there are **remnant** quarks (i.e. quarks which do not take place in the fundamental interaction which we study) which only serve to muddy the water. However, in e^+e^-annihilation, there are no remnant initial state particles and this is another reason why e^+e^-annihilation gives such a good platform for the study of

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new particles.

The photon is always virtual. The Z^0 can be a real Z^0. Why is this? The photon has $m_\gamma = 0$ and so by its very nature must travel at the speed of light. However, in the case of e^+e^- annihilation, the total energy $= 2E_{beam}$ but the total momentum $\mathbf{p}_{beam} + -\mathbf{p}_{beam} = 0$. How can a photon of energy $E = 2E_{beam}$, mass m=0 have $\mathbf{p} = 0$, if $E^2 = p^2 + m^2$? It comes back to the uncertainty principle. The photon of energy $2E_{beam}$ can exist for $\Delta t = \frac{\hbar}{2E_{beam}}$ before it must decay into some constituents which obey the energy and momentum conservation laws.

5.6 Fermion Production in $p\overline{p}$ annihilation

In $p\overline{p}$ collisions a quark from the proton can collide with an anti-quark from the anti-proton. The anti-proton has quark composition : $\overline{d}\ \overline{u}\ \overline{u}$. Quarks unlike charged leptons also feel the strong force and can thus couple to gluons. $q\overline{q}$ annihilation can thus produce an intermediate gluon state as well as an intermediate γ, Z state as shown in Figure 12. The annihilation via a gluon is favoured since the strong coupling constant $(g_s^2/4\pi)$, at high energies, has a value of ~ 0.1, whereas the weak or electromagnetic coupling constant is ~ 0.007. The quantum numbers at the annihilation vertex also favour annihilation to gluons since in the gluon case we have colour factors of 1.0, whereas in the photon case we have charge factors of $\left(\frac{1}{3}\right)^2$ or $\left(\frac{2}{3}\right)^2$, depending on the quark type. The Feynman diagram for $q\overline{q} \rightarrow e^+e^-$ via a γ, Z is the exact time reversal of the $e^+e^- \rightarrow q\overline{q}$ diagram. (The cross section thus has the same dependence on coupling and energy i.e. $\sigma \propto \frac{\alpha^2}{x_1 E_p x_2 E_{\overline{p}}}$ (see next section)). However in comparison to the e^+e^- case we now have factors of x_1 and x_2 which are the fractions of the proton's and anti-proton's momentum four-vectors carried by the quark and anti-quark that annihilate. The energy in the CMS for such collisions is thus $2\sqrt{x_1 E_p x_2 E_{\overline{p}}}$. In the e^+e^- case, where the e^+ and e^- annihilate directly x_1 and x_2 are 1. On average one observes that each of the three valence quarks in the proton (or $\overline{p}$) carry around 1/6 ($x = 1/6$) of the proton's total momentum. The remaining 50 % is carried by gluons. In colliding beams, protons can be accelerated to much higher energies than electrons and thus although the energy in the CMS for $q\overline{q}$ collisions is reduced by the $\sqrt{x_1 x_2}$ factor, it is still possible to achieve high energies. For example it is presently not possible to produce top quark pairs in e^+e^- collisions since the top quark mass is 175 GeV and the highest energy $e^\pm$ beams only have energies of $\sim$ 100 GeV. However by colliding 900 GeV protons and anti-protons it has been possible to achieve enough energy in the $q\overline{q}$ CMS to produce top quark pairs via an intermediate gluon.

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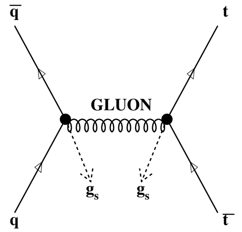

Figure 12: Feynman diagram for top production in $p\overline{p}$ collisions at the Tevatron

6 LECTURE 6

6.1 The Cross Section: a measurable.

If we know the number of electrons per unit time per unit area (n_e)coming in the -z direction (this is called the **flux** in the jargon) and the number of muons per unit area per unit time (n_μ) in the +z direction and you measured the number of muons seen as a function of θ_1 and the number of electrons seen as a function of θ_2 you would have measured the **differential cross section**, denoted by $\frac{d\sigma}{d\cos\theta}$. From the discussion of kinematics above, you can see that the energy of the scattered electron and its angle are related and that they are in turn related to the angle and energy of the scattered muon for a given total energy. So, it would be possible to infer the momentum of the outgoing muon from knowing the total energy and the angle. The total energy would be $E_{e_i} + E_{\mu_i} = E_{e_f} + E_{\mu_f}$ where subscripts i and f refer to inital and final respectively.

The total cross section is just the sum of the differential cross section over all theta:

$$\sigma = \int_{-1}^{1} \frac{d\sigma}{d\cos\theta} d\cos\theta \tag{57}$$

Now for the crucial step. How do we relate the cross section that we have measured to the Invariant Amplitude? If there are n_e electrons per unit *volume* passing through n_μ muons per unit *area* in unit time, then the number of interactions per unit area per unit time is $n_e n_\mu \sigma v_i$ where σ is the total cross section and v_i is the relative velocity of the e and the μ. Per μ, the transition rate

$$W_{fi} = \sigma n_e v_i = \sigma\phi \tag{58}$$

where ϕ is the flux.

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6.2 Fermi's Golden Rule

Fermi's Golden Rule states that the **transition rate** of the interaction is given by the $|\text{Invariant Amplitude}|^2$ multiplied by the **density of states**.

$$W_{fi} = |\mathcal{M}_{fi}|^2 \rho(E_f) = \sigma n_e v_i \tag{59}$$

For a fixed p_{e_i} and fixed p_{μ_i} there is a large number of different allowed p_{e_f} and p_{μ_f} which will depend on the scattering angle and the masses of e and μ as seen above. However, the number is not infinite, owing to the fact that energy comes in quanta. We call this a density of states where $\rho(E_f)dE_f$ is the number of states in the energy interval E_f and $E_f + dE_f$.

The cross section is given by

$$\sigma = \frac{|\mathcal{M}|^2 \delta^4(p_{e_i} + p_{\mu_i} - p_{e_f} - p_{\mu_f})(2\pi)^4 \rho_{E_f}}{\phi} \tag{60}$$

$$\sigma = \frac{|\mathcal{M}|^2 \delta^4(p_{e_i} + p_{\mu_i} - p_{e_f} - p_{\mu_f})\rho_f}{\phi} \tag{61}$$

and has units of area. The delta function just represents momentum and energy conservation. ρ_f is a phase space factor which absorbs the density of states and the numerical constants.

6.3 Phase Space or Density of States

The final state which is measured is only one of a set of allowed final states. For example, for a total energy of 2GeV, the final state particles could have 1Gev each, 0.5Gev and 1.5Gev, 0.2GeV and 1.8GeV etc etc. The number of states in this energy interval is proportional to the number of different allowed decays. In Figure 13, the number of different allowed decays depends on the energy of the incoming particles. So if there are 4 different final state channels allowed, the density of states will be 4 times $\rho(E_f)$ if there were only one allowed channel.

The cross section for $e^+e^- \to \mu^+\mu^-$ via the electromagnetic interaction at E_{CMS} >10GeV

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{12E_{e^+}E_{e^-}} \tag{62}$$

where $\alpha = \frac{g_{EM}^2}{4\pi}$. You can see that apart from constants, this is just the simple expression for the $|\text{InvariantAmplitude}|^2$ multiplied by the density of states (in this case for two final state particles, E^2) and we have ignored the masses of the electrons and positrons which are very small compared to their energies.

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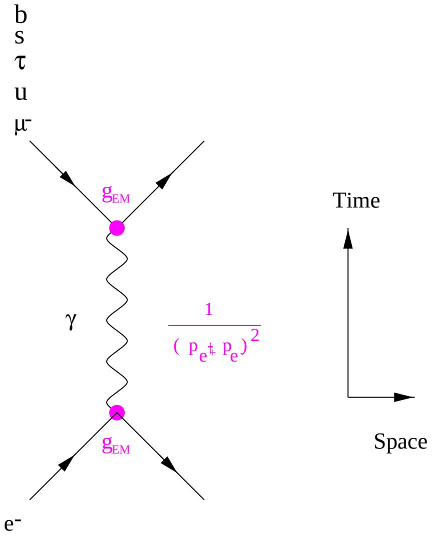

Figure 13: The annihilation of e^+e^- to a number of possible final states.

6.4 R$\equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$

There turns out to be a very dramatic prediction from the very simple experimental ratio from e^+e^- annihilation via the electromagnetic interaction:

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\Sigma e_q^2 \tag{63}$$

where e_q is the electric charge on the quarks and the factor of 3 comes from the fact that there are 3 colours of quark. Figure 14 shows the experimental measurement of R from various e^+e^-annihilation experiments.

6.5 First and Second Order Feynman Diagrams

In many interactions, there is not only one Feynman Diagram which represents the interaction. There can be a series of Feynman Diagrams which, although they have the same initial and final states go through different intermediate states. *All these intermediate states can be represented by specific Feynman Diagrams.* These different intermediate states have the opportunity to **interfere** with each, either destructively or constructively and can therefore modify the cross section which would be calculated naively from one of the Feynman Diagrams. Such an example is given in e^+e^-annihilation where the intermediate state can be either through a Z^0 or a γ. Look again at Figure 11. In order to

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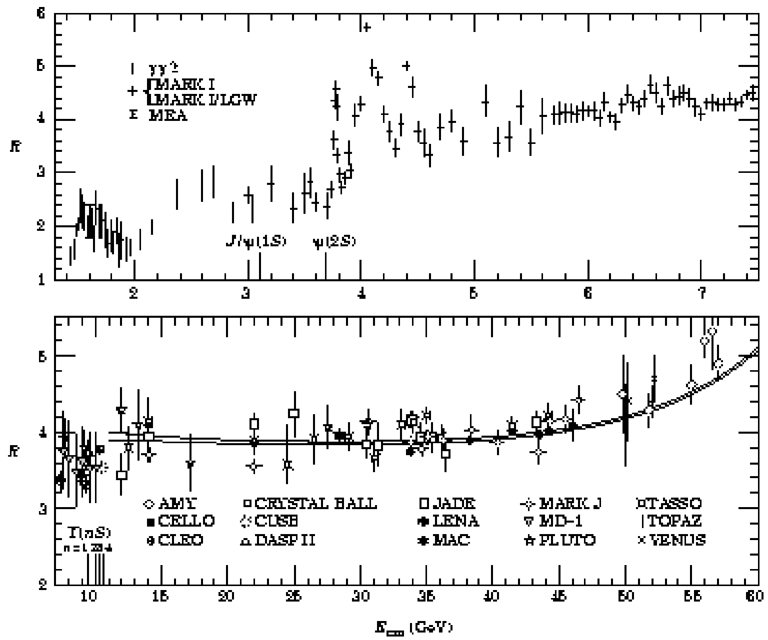

Figure 14: R distribution from e^+e^- annihilation experiments

properly calculate the cross section, the amplitudes for these two different states must be added together before they are squared to give the cross section and then the interference terms (the cross terms) are properly taken into account.

6.6 Fermion Production in e^+e^-annihilation

The experimental cross section for e^+e^-annihilation is shown in Figure 15 where the entire energy range of experiments is shown. The value of the Electromagnetic coupling constant, g_{EM}, is the charge on the electron, e. The value of the Weak coupling constant, g_W is related to g_{EM} by a factor:

$$g_W = g_{EM} \sin\theta_W \tag{64}$$

where θ_W is known as the weak mixing angle, or sometimes, the Weinberg angle. θ_W has been measured extensivly at LEP and the present value of the measurement is 0.223 $\pm$ 0.002. In e^+e^-annihilation, if the interaction goes via a virtual photon (i.e. it is an electromagnetic interaction) then the fermions e,μ and τ are produced more often

fermion	d	u	s	c	b	e	μ	τ	ν_e	ν_μ	ν_τ
coupling	$\frac{-1}{3}$	$\frac{2}{3}$	$\frac{-1}{3}$	$\frac{2}{3}$	$\frac{-1}{3}$	-1	-1	-1	0	0	0
production	$3 \cdot \frac{1}{9}$	$3 \cdot \frac{4}{9}$	$3 \cdot \frac{1}{9}$	$3 \cdot \frac{4}{9}$	$3 \cdot \frac{1}{9}$	1	1	1	0	0	0

Table 6: : Relative production for fermions in e^+e^-annihilation at 35 GeV

than one particular flavour of quark. However, the quarks come in three colours so there is three times the probability that a particular quark flavour of *any* colour be produced. Table 6 shows the relative production cross sections for the fermions which can be produced in e^+e^-annihilation at 35 GeV center of mass energy. At higher energies, the interaction can go via a virtual Z^0, and for this the fermions have different quantum numbers still, Q_W which are given by

$$Q_W = I_3 - \sin^2\theta_W Q_f^{EM} \tag{65}$$

where I_3 is the weak isospin and has a value of $\pm\frac{1}{2}$ for the weak iso-doublets or 0 for the iso-singlets (the right-handed fermions).

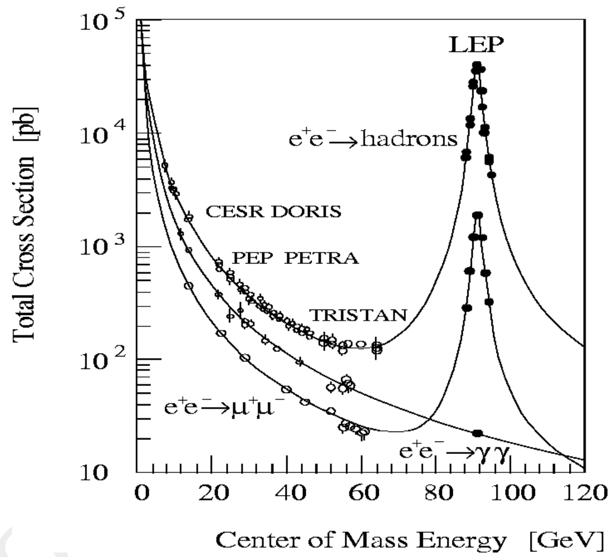

Figure 15: Cross section for e^+e^-annihilation showing photon and Z peaks

7 LECTURE 7

7.1 Introduction to Hadrons

Up until now, we have spoken of fundamental particles, those which are pointlike and have no composite stucture. Another set of particle which we need to know about are the **hadrons** which are bound states of the quarks. (The word hadron comes from the greek word meaning heavy. The word lepton comes from the greek word meaning light: leptons are light compared to the composite hadrons).

One of the quantum numbers which hadrons carry which is useful in characterizing them is spin which is defined as the hadrons angular momentum in the center of mass frame of the constiutuent quarks. Th spin can be either integer of half intgere while the z component can have any of the 2J+1 possible values.

7.2 Baryons and Mesons.

There are two types of hadron: **baryons** which are made up of three quarks and **mesons** which are made up of a quark and an anti-quark. These composite particles are **colour singlets** which means that they have overall colour = 0. The baryons have total baryon number = 1 and the mesons have total baryon number = 0. They can be charged or neutral and they carry weak isospin. We will now study the two most common groups of hadrons: pions (mesons) (π^+, π^-, π^0) and nucleons (protons(p) and neutrons(n)) (baryons). The proton and neutron are collectively called **nucleons**.

7.2.1 Nucleons.

Nucleons are the most common hadron on the earth. All the atoms have a nucleus which contains some number of protons and neutrons. The quark content of a proton is (u,u,d). The quark content of a neutron is (u,d,d).

$$\begin{array}{cccccccccc} p & = & u & + & u & + & d & & \\ Q & = & \frac{2}{3} & + & \frac{2}{3} & + & \frac{-1}{3} & = & 1 \\ B & = & \frac{1}{3} & + & \frac{1}{3} & + & \frac{1}{3} & = & 1 \end{array} \qquad \begin{array}{cccccccccc} n & = & u & + & d & + & d & & \\ Q & = & \frac{2}{3} & + & \frac{-1}{3} & + & \frac{-1}{3} & = & 0 \\ B & = & \frac{1}{3} & + & \frac{1}{3} & + & \frac{1}{3} & = & 1 \end{array}$$

7.2.2 Pions

The lightest and most stable mesons are the pions. There are two of these: the charged pion and the neutral pion designated π^+ and π^0 respectively. These are made up of a linear combination of the lightest of the quarks: $u\bar{d}$ and $u\bar{u}$-$d\bar{d}$.

$$\begin{array}{cccccccc} \pi^+ & = & u & + & \bar{d} & & \\ Q & = & \frac{2}{3} & + & \frac{1}{3} & = & 1 \\ B & = & \frac{1}{3} & + & \frac{-1}{3} & = & 0 \end{array} \qquad \begin{array}{cccccccc} \pi^0 & = & u\bar{u} & - & d\bar{d} & & \\ Q & = & 0 & + & 0 & = & 0 \\ B & = & 0 & + & 0 & = & 0 \end{array}$$

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7.2.3 Kaons

Kaons are mesons (like pions) which have one strange quark instead of a d quark. Their masses are all in the region of 500 MeV.

You can make up many different baryons and mesons starting from the 6 quarks (and 6 anti-quarks). If the spin of the quarks is 1/2, then you can arrange the spins to give baryons of 1/2 or 3/2 and mesons to give spins of 0 or 1 depending on how the spins line up.

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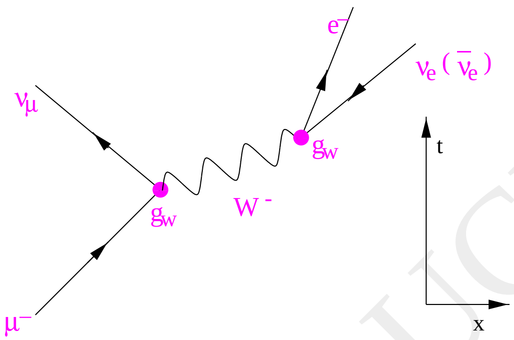
8 LECTURE 8

8.1 The Lifetime: another measurable.

As mentioned earlier, the everyday fundamental particles are the first generation of quarks and leptons. These are also **stable** which means that they do not **decay** to some other particle if left alone. The second and third generation of fermions decay via the weak interaction to the next generation down. A Feynman diagram of a muon decaying to an electron and two neutrinos is shown in Figure 21.

8.2 Conservation Laws.

We can do some examples of allowed processes and look at the conservation of quantum numbers. Figure 16 shows the Feynman diagram for muon decay. We can write down the process:



μ^-	$\rightarrow$	ν_μ	$\bar{\nu}_e$	e^-
B: 0	$\rightarrow$	0	0	0
Q: -1	$\rightarrow$	0	0	-1
L: 1μ	$\rightarrow$	1μ	-1e	+1e
C: 0	$\rightarrow$	0	0	0

Figure 16: Feynman Diagram for muon decay and its associated conserved quantum numbers.

8.3 The Spectator Model

The quarks decaying inside the hadrons are thought to do so independently of the other quarks(s) and treating them this way has proved to be a very successful tool. Look as an example at the D^0 decaying **semileptonically** i.e. via a charged W to a charged lepton and a neutrino (in a similar way to the inclusive Kaon decay of the previous example). This is known as a semileptonic decay because one quark decays to leptons while the other quark in the meson does nothing. It is the spectator. This decay is shown in Figure 17. This type of decay is especially prevalent for the heavy quarks, c and b.

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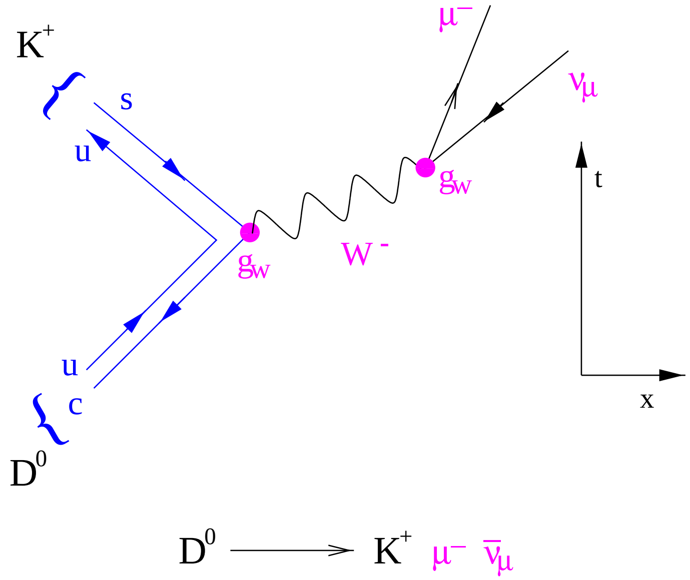

Figure 17: A Feynman diagram showing the semileptonic decay of a neutral D meson according to the spectator model

8.4 Kaon Decay

The mesons containing one strange quark are called Kaons for historical reasons. The Kaons have lifetimes of about 10^{-8}s which is commensurate with their mode of decay via the weak interaction. The reason why their decay is only via the weak is that the only interactions which to not conserve flavour are the weak interactions. The only way a quark can decay to another flavour of quark is via the emission of a charged W. It is experimentally observed that there are no neutral flavour changing interactions.

Two different decay modes of the kaon are shown in Figure 18. They both are for $K^+ \rightarrow \mu^+ + \nu_\mu + X$ where sometimes $X = 0$(left) and sometimes $X = \pi^0$(right).

The **branching fractions** for these decays are shown in the figures. Mostly, kinematics favour the "annihilation" decay where an energetic muon and neutrino are produced and the initial quarks annihilate to for a lone W. In the spectator decay, the same energy is shared out between the muon,neutrino and pion.

Some other quark decay diagrams are shown in Figure 19 and 20. The first one is a charm quark decaying via a W^+ to a strange quark within the same weak isodoublet. The second diagram is that for the decay of a bottom quark to a charm quark via a W^-: this decay spans two different weak isodoublets.

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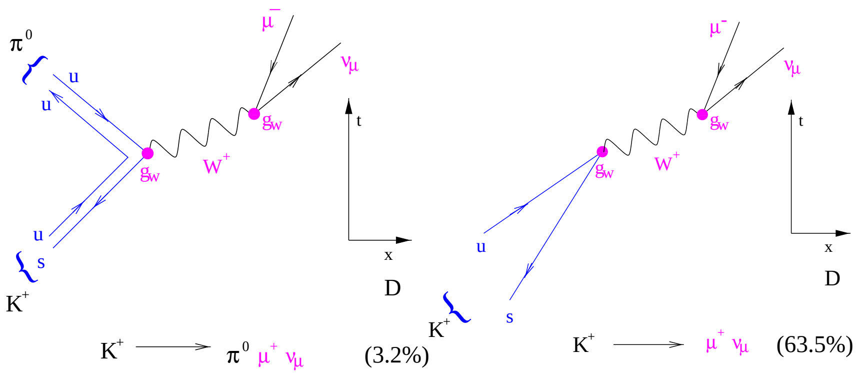

Figure 18: Spectator decay(left) and annihilation(right) kaon decay

BQLC. Remember these conservation laws must be satisfied for all processes.

8.5 Muon Decay

We can calculate the **lifetime** of the muon from what we have learned already.

The lifetime is given by the inverse of the **decay rate**. The decay rate is given by

$$\Gamma = -\frac{dN/dt}{N} \tag{66}$$

So, for example if the decay rate were 1 decay per second out of 1000000 particles then the lifetime would be $(\frac{1}{1000000})^{-1}s^{-1}$: i.e. 10^6s. There is an exponetial decay law for the number of particles:

$$N(t) = N_0 e^{-\Gamma t} \tag{67}$$

and Γ^{-1} is the lifetime of the particle.

Now then, in a similar fashion to the treatment of cross sections, we have to relate the *experimental* measurement of the lifetime to the Invariant Amplitude which contains all the theory about the forces at work.

So now we have to calculate what Γ is from our Feynman diagram. The initial state is a muon, the final state is an electron and two neutrinos. The expression for the transition rate is exactly the same as in 59 with units of s^{-1}. Then

$$\Gamma \quad = \quad \int |\mathcal{M}|^2 \delta^4 (p_{\mu^-} - p_{\nu_\mu} - p_{e^-} - p_{\bar{\nu}_e}) \rho_f d\Omega \tag{68}$$

The total decay rate, Γ is the sum of the rates for all the decay channels. dΩ is the solid angle containing the final state particles. Now the delta function in the expression ensuring energy and momentum conservation reflects the fact that the muon is the initial

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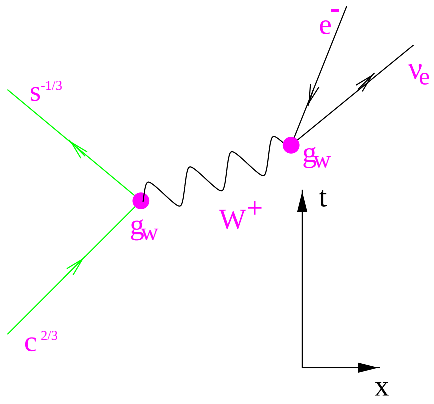

$c^{+2/3}$	$\rightarrow$	$s^{-1/3}$	$\bar{\nu}_e$	e^-
B: 1/3	$\rightarrow$	1/3	0	0
Q: +2/3	$\rightarrow$	-1/3	0	1
L: 0	$\rightarrow$	0	-1e	1e
C: g	$\rightarrow$	g	0	0

Figure 19: Feynman Diagram for weak decay of a green charm quark and the associated conserved quantum numbers.

state and the two neutrinos and the electron are the final state particles. ρ_f is the phase space factor which absorbs all the density of final states and numerical constants.

If you measure the lifetime of a state by measuring the particle's distance before decay, you will learn something about its width. In some cases, the lifetime of a state is so short that it is very difficult to measure experimentally. In this case, measuring the width can be possible if the state can be produced resonantly.

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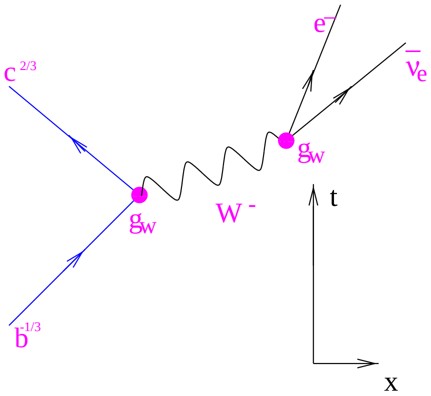

$b^{-1/3}$	$\rightarrow$	$c^{+2/3}$	$\bar{\nu}_e$	e^-
B: 1/3	$\rightarrow$	1/3	0	0
Q: -1/3	$\rightarrow$	+2/3	0	-1
L: 0	$\rightarrow$	0	-1e	1e
C: b	$\rightarrow$	b	0	0

Figure 20: Feynman Diagram for weak decay of a blue bottom quark and the associated conserved quantum numbers.

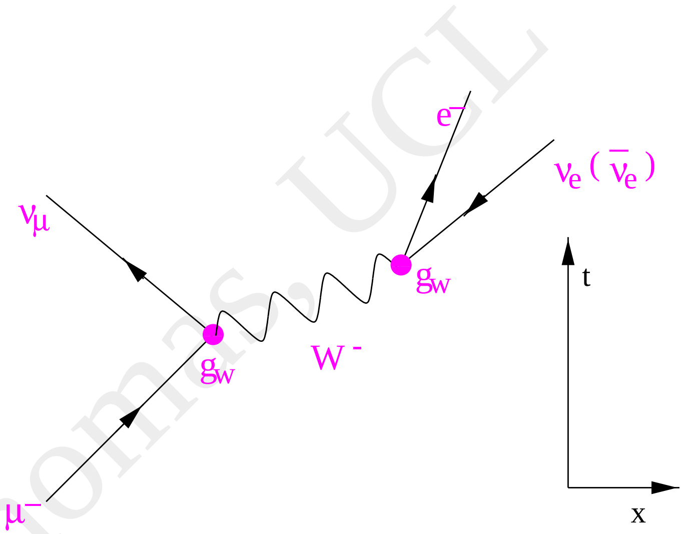

Figure 21: Feynman Diagram for muon decay

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