9 LECTURE 9

9.1 Resonances

Measuring the cross section of a process such as $e\mu$ scattering gives us information about the Invariant Amplitude. However, there is a special case called resonant production, where a real intermediate state is formed by the incoming particles and then decays after some time to other particles. This has been seen often in e^+e^-annihilation. In Figure 14, it is clear that close to the point where R changes dramatically, there are spikes in the cross section corresponding to these **resonances**. In fact, at high enough energies, a real Z^0 *Gauge Boson* can be produced this way. In order to calculate the Invariant Amplitude for resonant production, attention has to be paid to the time dependent part of the wave function. Before, when calculation $\mathcal{M}$, the time dependent part of the wavefunction was ignored:i.e. the assumption was made that the time taken for the interaction was short compared to the time evolution of the wave function. For a resonant state, this is not necessarily true.

9.2 The Breit-Wigner Formula

The **width** Γ and the lifetime τ are related by the now familiar uncertainty relation

$$\Delta E \Delta t = \hbar \tag{69}$$

where ΔE is known as the width. Figure 22 shows the width Γ. This means that there is an energy dependence of the invariant amplitude which is given by transforming this time dependent wave function into an energy dependent amplitude function, E and t being fourier transform pairs. Starting from the exponential decay law:

$$\begin{aligned}
|\psi(t)|^2 &= |\psi(0)|^2 e^{-\Gamma t} \text{ then} && (70)\\
\psi(t) &\propto e^{-iE_R t} e^{-\frac{\Gamma t}{2}} && (71)\\
\phi(t) &= \psi(0) e^{-iE_R t} e^{-t/2\tau} && (72)\\
&= \psi(0) e^{-t[iE_R + \Gamma/2]} && (73)
\end{aligned}$$

The Fourier transform is

$$\begin{aligned}
\chi(E) &= \int_0^\infty \phi(t) e^{iEt} dt && (74)\\
&= \psi(0) \int_0^\infty e^{-t[\Gamma/2 + iE_R - iE]} dt && (75)\\
&= \frac{K}{(M-E) - i\Gamma/2} && (76)
\end{aligned}$$

This is now the energy dependent Invariant Amplitude. However, it doesn't really look all that much like what we had before! Actually, it is almost the same. Start with the form of our time independent Invariant Amplitude:

$$\frac{1}{q^2 - M^2} = \frac{1}{E^2 - M^2} \quad (\mathbf{p} = 0 \text{ for e}^+\text{e}^- \text{ annihilation}) \tag{77}$$

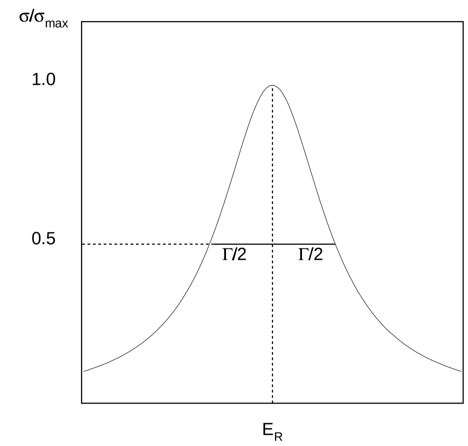

Figure 22: The Breit-Wigner distribution showing the lifetime, Γ.

$$\begin{aligned}
&= \frac{1}{(E-M)(E+M)} \text{ at resonance E} \approx \text{M so} && (78)\\
&= \frac{1}{-2M(M-E)} && (79)
\end{aligned}$$

$-2M$ can be absorbed into the proportionality constant K. So the above expression only differs from the resonant case by $-i\frac{\Gamma}{2}$ which is just the reason that it has to be treated more carefully.

So, if you measure the cross section for a particular process as a function of energy close to the resonant energy (mass M) of an intermediate state then the lifetime of this state is given by Γ^{-1}. The function will be a maximum at $E = M$, i.e.

$$\begin{aligned}
\chi^* \chi &= 1 && (80)\\
|\chi(E)|^2 &= \frac{K^2}{(M-E)^2 + \Gamma^2/4} && (81)\\
K &= \Gamma/2 && (82)
\end{aligned}$$

$$\sigma(E) = A|\chi(E)|^2 \tag{83}$$

where A is a number incorporating the number of states available for the decay and the spin of the decaying and final states and the constant K. Looking at the example of Z^0 decay, Figure 23 shows what the cross section would look like for three different hypotheses for the number of light neutrinos. Ascertaining that this number was 3 was one of the major products of the LEP collider running, and it was determined by comparing the hypotheses with the data.

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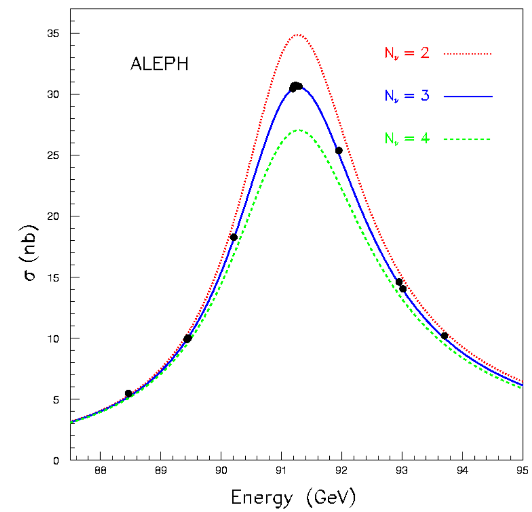

Figure 23: The Z^0 resonance showing its shape given different numbers of light neutrinos.

9.3 The Δ^{++}

The Δ^{++} is a baryon with mass of 1.232 GeV. Experimental measurements of its decay angular distribution reveal its spin to be $\frac{3}{2}$. It is produced in pion-nucleon scattering according to the Feynman diagram in Figure 24. Its mass is extremely close to that of the neutron and the proton and the decay products also show no sign of the strange quark (such as a Kaon). It is clear that the constituent quarks of the Δ^{++} must be [uuu]. This means that there are three fermions, with the same spin (all spins aligned), with the same flavour in this meson. This was a piece of very hard evidence in favour of the colour quantum number. There must be another degree of freedom associated with these fermions and that was found to be colour: r+g+b = 0.

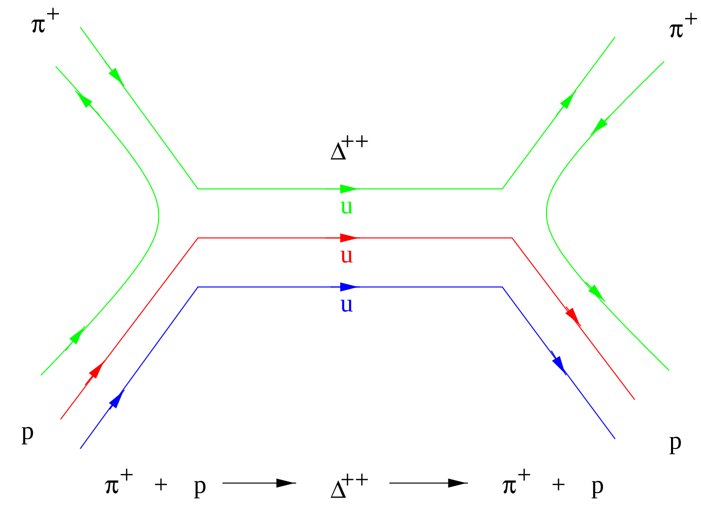

Figure 24: The Feynman diagram of the productaion and decay of the Δ^{++}

10 LECTURE 10

10.1 The Weak Interaction : some important topics

There are several fundamental differences between the Weak Force and the other two Forces. The Weak Force is mediated by massive vector Gauge Bosons whereas the EM and Strong Force are mediated by massless vector Gauge Bosons. The Weak Interaction maximally violates parity.

10.1.1 Helicity

Up until now, spin has been mentioned fleetingly. Spin is the quantum number which determines whether a particle must obey Fermi-Dirac statistics or Bose-Einstein statistics. The fermions have spin s=$\frac{1}{2}$ and the Gauge Bosons have spin s=1. Helicity is the dot product of the momentum vector of the particle with this spin direction normalised by E+m.

$$H = \frac{\sigma \cdot \mathbf{p}}{E + m} \tag{84}$$

Helicity is a quantity that can be *measured*. However, the reason it is being introduced here is because it is, in certain circumstances (massless fermions), identical to the *chirality* of a fermion and usually (for relativistic fermions) very similar. Chirality is the fundamental quantity as far as the weak interaction is concerned, but it is helicities that we can measure. Particles with H=+1 are called **positive helicity** and particles with H=−1 are called **negative helicity**.

The spin vector, σ, is called an **axial vector** whereas **p** is a **polar vector**. An axial vector does not change sign under reflection but a polar vector does so the quantity $\sigma \cdot \mathbf{p}$ also changes sign under reflection. This is illustrated in Figure 25.

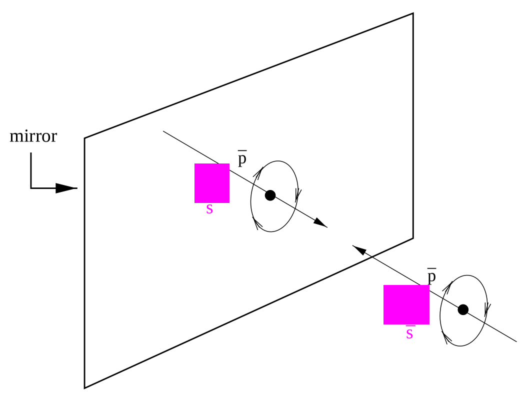

Figure 25: A diagram showing how spin and momentum change under reflection (or parity)

You can see that changing a particles momentum under the parity operation will change its helicity. For a massive fermion, one cannot be prevented from boosting to a Lorentz frame where the momentum (and therefore the helicity) is reversed. This implies that there *must* be two available helicity states for a massive fermion. By contrast, a massless particle travelling at c cannot have its helicity flipped without violating special relativity i.e. we would need a frame travelling at a velocity $> c$.

10.2 Parity Violation in the Weak Interaction

One of the biggest surprises of this century was the observation that the Weak interaction violates parity. That is, it seems to favour interactions with −ve helicity particles and +ve helicity anti-particles. Actually, more specifically, the weak interaction only interacts with **left-handed** particles (or **right-handed** anti-particles). Left and Right handed refer to the chiral state of the fermion which is the Weak Eigenstate of the fermion. The Weak interaction projects out the left-handed chiral state of a fermion. This means in the eyes of the weak interaction, right-handed and left-handed fermions are actually *different species* of fermion. The relationship between helicity and chirality can be written down as follows:

$$\begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \chi'_- \\ \chi'_+ \end{pmatrix} \tag{85}$$

The value of the angle θ is related to the mass of the fermion : as $m \to 0, \theta \to 0$. This means that a massless particle in a left chiral state is also 100% in a −ve helicity state. As the mass is increased, a left chiral state also receives a small +ve helicity contribution. The left chiral state of a non-massless particle is a superposition of a −ve helicity state and +ve helicity state. The amount of +ve helicity increases as the mass increases. Since the weak interaction only sees left chiral states, if the particle in the interaction has +ve helicity (e.g. to satisfy conservation of momentum), then this interaction will be heavily suppressed because the +ve helicity contribution to a left chiral state is small. Since the neutrino is massless and only interacts via the weak force, it will *only* ever be observed in a −ve helicity (=left chiral) state. +ve helicity neutrinos have never been observed.

The experimental verfication of these ideas first predicted in 1957 by Lee and Yang was provided by Madame Wu. Ms. Wu observed the angular distrubution of electrons from the nuclear beta-decay of ^{60}Co. The ^{60}Co was put in a very high magnetic field at very low temperature such that all the spins of the nucleons were aligned via their magnetic moments coupling to the magnetic field. The electrons from the beta decay were found to be emitted preferentially in the direction opposite to the B-field / spin direction. No electrons are observed in the direction of the B-field. This is a direct consequence of the anti-neutrino from the beta decay only existing in a +ve helicity (right chiral) state. Conservation of momentum then dictates that the electron must be in −ve helicity state and this was observed. If the weak force coupled equally to left and right chiral states i.e. if a −ve helicity anti-neutrino were allowed to exist then it would be possible for electrons to be emitted in the same direction as the B-field. This does not happen and is referred to as "parity violation". If one applies the parity operator to the ^{60}Co experiment, one would expect electrons to be emitted in the same direction as the B-field. Under the parity operator the spins/B-field are unchanged since they are axial vectors but the momenta are reversed. This would mean that the anti-neutrino should have negative helicity. It does not happen - thus parity is said to be violated.

The right-handed fermions do not see the weak interaction and must have zero weak quantum numbers (I_3) : this is shown in table 7. Left handed fermions necessarliy have non zero weak quantum numbers.

	I_3	Q	B	L	colour
u_R	0	$\frac{2}{3}$	$\frac{1}{3}$	0	r,g,b
d_R	0	$\frac{-1}{3}$	$\frac{1}{3}$	0	r,g,b
e^-_R	0	-1	0	1e	0
ν_{e_r}	0	0	0	1e	0

Table 7: : The right handed fermions and their quantum numbers

10.3 Pion Decay

Pions decay to a muon and a muon neutrino. The mass of the pion is 139.6 MeV while the mass of the muon is 105.7 MeV. Simple consideration of phase space factors would prefer that the pion decay to an electron and an electron neutrino, there being almost a factor of 5 advantage energetically over the decay to a muon. However, this does not happen and it does not happen because of parity violation in the weak interaction. Helicity

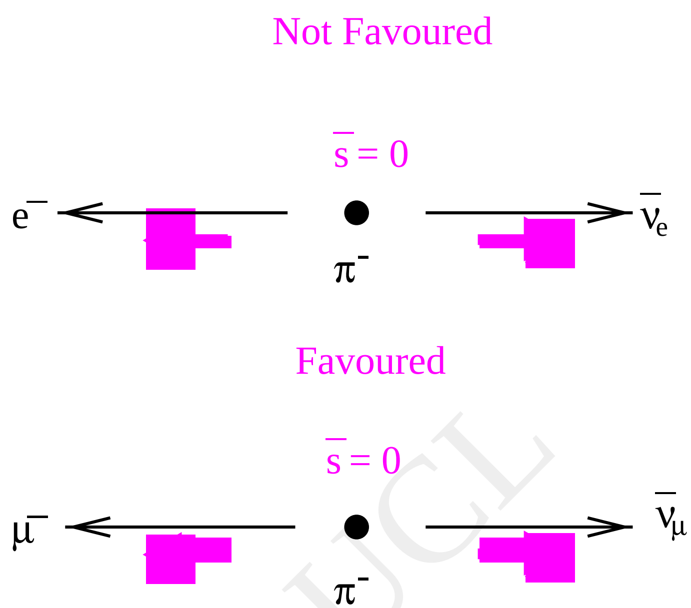

Figure 26: Spin considerations favour $\pi \rightarrow \mu\bar{\nu_\mu}$ over $\pi \rightarrow e\bar{\nu_e}$

CANNOT be conserved in this interaction after taking into account that the neutrinos must be left-handed. Figure 26 shows the spin alignment of the decay products of the pion in the rest frame of the pion. Both charged leptons *have* to be in the suppressed state of left chirality and right-handed helicity. The amount of right-helicity in a left chiral state increases with mass. The decay to a muon is thus favoured because a muon has a larger right-handed component in the left chiral state compared to the electron. Experimentally it is found that the π^- decays 10^4 times more often to $\mu^- + \overline{\nu_\mu}$ than $e^- + \overline{\nu_e}$ although simple-minded consideration of the available energy would favour the electron decay channel.

10.4 Muon Decay

A further example of the selective nature of the weak interaction is given by muon decay. As we've just seen a relativistic muon (μ^-) will have H=+1 when produced from the decay of a pion. The electrons of highest energy from the decay of a μ^- are emitted preferentially in the same direction as the muon was originally travelling. The reason for this is seen from the two diagrams of Figure 27. Again this is due to the fact that neutrinos can only exist in a certain helicity state and conservation of spin and momentum that "force" the electrons to be in a certain helicity state. If this electron helicity state is the same as the chiral state (bottom picture i.e. electron is left chiral & −ve helicity) then the reaction is favoured over the case where the electron must be in the state : left chiral & +ve helicity, since as we've said before +ve helicty states are a small part of a left chiral state and the weak interaction only involves left chiral particles.

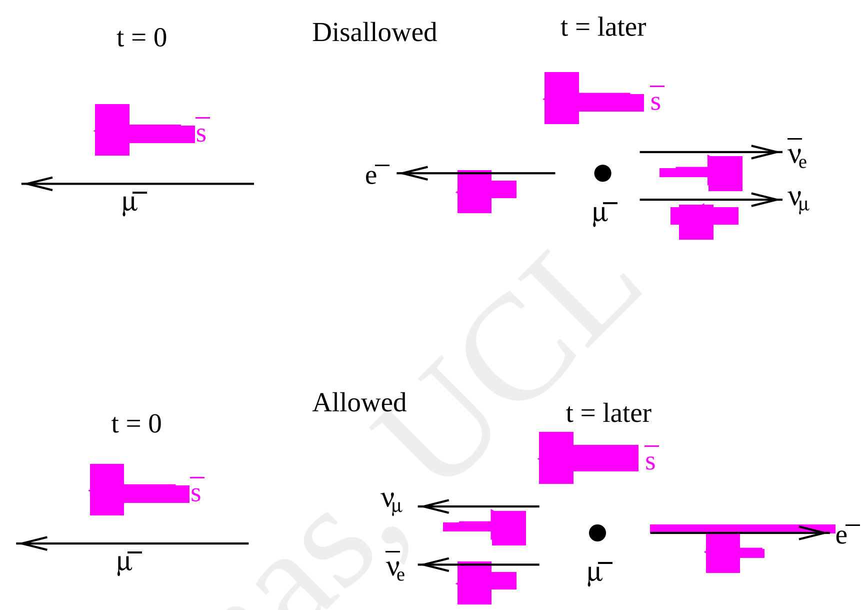

Figure 27: Two diagrams showing why there is a forward backward asymmetry in muon decay at rest due to parity violations in the weak interaction

11 LECTURE 12

11.1 The thing about neutrinos

The amazing fact about neutrinos is that they only interact via the weak interaction. They have no charge or color and so are only produced in weak interactions. This means they are only produced in the left-handed state. If they had a mass, they could be boosted into a frame where their helicity was opposite and they would never again interact, having no interaction allowed them. For this reason, neutrinos are thought to be massless.

If neutrinos do have a finite mass, they certainly have very small masses. This has been verified experimentally, the lowest limit on the mass of the electron neutrino is < 1eV! One of the most exciting prospoects in particle physics is to find some evidence that the extremely successful Standard Model is not entirely correct, and the neutrino mass problem presents such a possibility. Although impossibly difficult to measure directly, the neutrino mass might be detected another way. It has been proposed by Pontecorvo in 1967 that neutrinos might **oscillate** from one flavour to another if they were to have some non-zero mass. This is actually a direct consequence of quantum mechanics. If the mass eigenstates (these are the eigenstates which propagate through space) are not the same as the weak eigenstates (the left-handed neutrinos which must be produced in the weak interaction), then for example, the weak electron neutrino eignestate can be written as a linear combination of three mass eigenstates:

$$\nu_e(t) = c_1 e^{-iE_1 t}|\nu_1> + s_1 c_3 e^{-iE_2 t}|\nu_2> + s_1 s_3 e^{-iE_3 t}|\nu_3> \qquad (86)$$

The probability that one type of neutrino will oscillate into a different type after some time t, can therefore be written:

$$P(\nu_e \to \nu_\mu) = |<\nu_\mu|\nu_e(t)>|^2 \qquad (87)$$

$$P = 1 - 2c_1^2 s_1^2 c_3^2[1 - cos(E_1 - E_2)t] - 2c_1^2 s_1^2 s_3^2[1 - cos(E_1 - E_3)t] - 2s_1^4 s_3^2 c_3^2[1 - cos(E_2 - E_3)t] \qquad (88)$$

and this is expressed pictorially in Figure 28.

The various coefficiencts are the mixing matrix elements made up of the neutrino mixing angles of which there are three, and one phase:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 c_3 + s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 s_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad (89)$$

If the mass eigenstates have the energy eigenvalue:

$E_i^2 = p^2 + m_i^2$ then for p>>m_i

$$E_i = p + \frac{m_i^2}{2p} \qquad (90)$$

$$E_i - E_j = \frac{(m_i^2 - m_j^2)}{2p} \qquad (91)$$

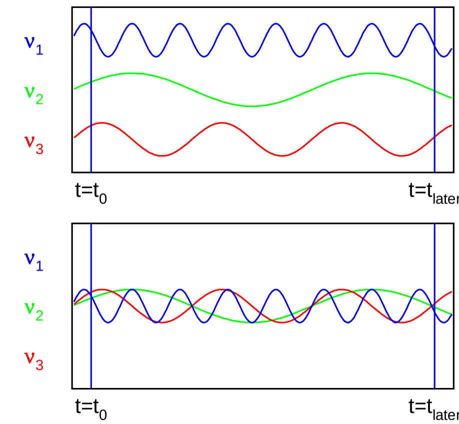

Figure 28: A diagram showing schematically the reason for neutrino flavour oscillations

therefore, the oscillation probability depends on the difference between the squares of the masses of the oscillating neutrino mass eigenstates. An experiment which observed the flavour of the neutrino changing would be evidence of neutrino mass and therefore concrete evidence that there was more to out picture of particle physics than the Standard Model suggests.

The expression for the probability of neutrino oscillation is somewhat simplified if only two of the neutrino types dominate the oscillation. It then becomes:

$$P(\alpha \to \beta) = \sin^2 2\theta \sin^2(1.27\Delta\mathrm{m}^2\frac{\mathrm{L}}{\mathrm{E}}) \qquad (92)$$

where $\sin^2 2\theta$ is the strength of the mixing between the two neutrino mass eigenstates,Δm^2 is the difference in the square of their masses, L is the distance they travel and E is their energy.

11.2 The MINOS proposed experiment

There is a proposed experiment which will provide muon neutrinos from the Main Injector at Fermilab (Il, U.S.A) and point them at a very large (10kTon) detector in an unused iron mine in Minnesota, 720 km away. It is due to start taking data in the year 2001. The neutrinos will have energy of about 10GeV (produced from pion decay at Fermilab) and so for this distance, or **baseline**, it will be sensitive to a range of values of Δm^2. The mixing angle is thought to be large from previous experiments which have found what could be interpreted as evidence for neutrino oscillations in neutrinos from cosmic rays. A schematic of the layout is shown in Figure 29.

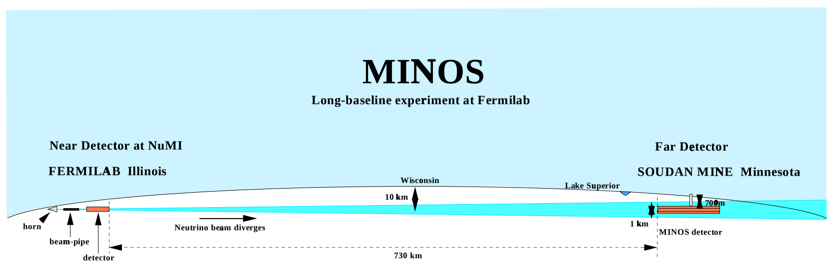

Figure 29: A diagram showing the set up of the MINOS proposed experiment

11.3 Cosmic Rays

The earth is being constantly bombarded by high energy cosmic ray showers. These start out as protons entering our atmosphere and interacting with a nucleon in some gas molecule. The result of this high energy interaction is usually several pions which start to travel towards the earth and decay in flight to a muon and a muon anti-neutrino. Very often these high energy muons make it as far as earth before decaying themselves, whereas some of them do decay to electrons and electron anti-neutrinos. If you hold out your hand, there will be, on average, one cosmic ray muon passing through your hand every second.

11.4 Evidence for neutrino oscillations from Cosmic Rays

There have been several experiments set up to measure the ratio of muon neutrinos to electron neutrinos from cosmic rays. Cosmic rays are a very good source of particles: 1 charged particle from a cosmic ray goes through your outstretched hand every second! The decay chain of a typical cosmic ray, is shown in Figure 31. Sometimes, the muon will decay to an electron; sometimes the charged pion will make it all the way to earth before decaying at all. But on average, it is expected that there should be approximately twice as many muon neutrinos as electron neutrinos reaching the earth's surface. The neutrinos are detected and their flavour identified by the weak interaction with a neutron (or proton for an anti-neutrino) shown in Figure 30.

The experiments have nearly all measured a dearth of muon neutrinos compared to electron neutrinos. One solution to this puzzle is that the muon neutrinos are oscillating on their way through the atmosphere into tau neutrinos. Because the average energy of the muon neutrinos is $\leq$ 1GeV, the tau neutrino cannot interact and make a charged tau: there is just not enough energy, the charged tau having a mass of about 1.8GeV.

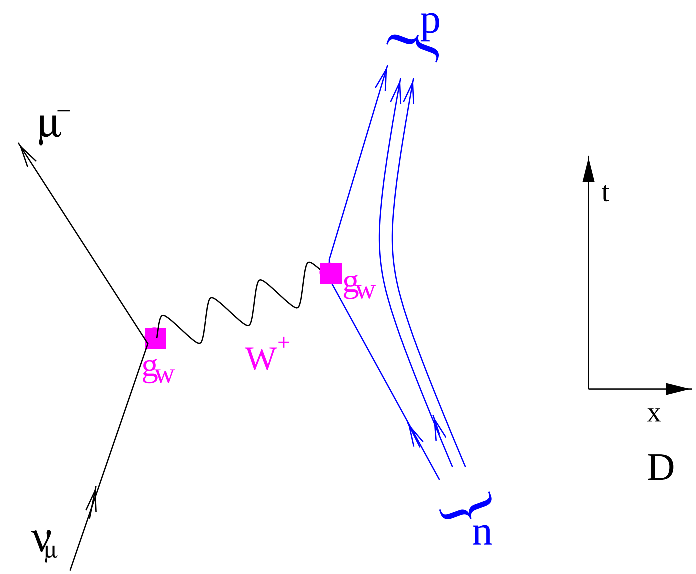

Figure 30: Interaction of a neutrino with a neutron in the matter in its path.

11.5 Summary of fundamental particles and their interactions

At this point in the course you should have come to grips with the following concepts.

- The fundamental particles are Gauge Bosons, leptons and quarks
- Fundamental means that they are not composite (i.e. they are pointlike)
- Forces are transmitted between the quarks and leptons via virtual Gauge Bosons
- If there is enough energy transferred, massive Gauge Bosons can be real.
- The amplitude of an interaction is a combination of the strength of the coupling contant, the momentum transfer and the range of the force (given by $\frac{1}{m_{GB}}$)
- At momentum transfers of about 0.1 GeV, the strong interaction has the largest amplitude, the EM has the next highest amplitude and the weak interaction has the lowest amplitude. At distance scales of order 10^{-18}, all the amplitudes have about the same magnitude.
- The colour force gets stronger with increasing distance
- The electromagnetic force gets weaker with increasing distance

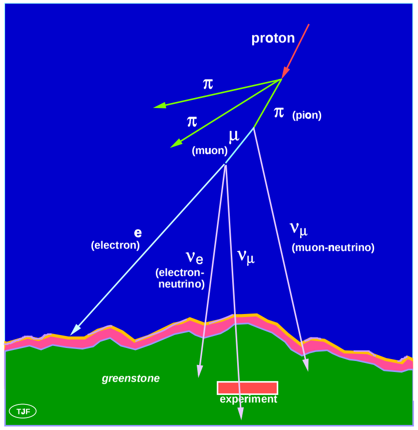

Figure 31: Typical cosmic ray decay chain

- The coupling strength of the colour interaction becomes so strong at about 1fm distances that free quarks are never seen: they are always bound inside hadrons which have sizes of about 1fm
- At very small distances < 1fm, the colour force becomes very weak and quarks can be considered as free inside hadrons: this allows perturbation theory to be used for colour interaction calculations
- Feynman Diagrams as a pictorial technique: BLQC conserved at the vertices
- A Fourier transform produces the 3-momentum amplitude from the free particle wave function assuming Perturbation theory holds
- The four-momentum *Invariant* Amplitude has a similar form
- The Invariant Amplitude contains all the physics of the interaction
- The Invariant Amplitude near to a resonance has an extra $\frac{-i\Gamma}{2}$ in the denominator which can be ignored everywhere else
- The expression for the Invariant Amplitude near to a resonance can be extracted from a Fourier Transform of the time dependent part of the wavefunction
- The cross section is a *measurable* quantity which relates experiment to the Invariant Amplitude (i.e. the theory)
- The lifetime is a *measurable* quantity which relates experiment to the Invariant Amplitude (i.e. the theory)
- Baryons are made up of three quarks. Anti-Baryons are made up of 3 anti-quarks

- Mesons are made up of a quark and an anti-quark
- Heavy Mesons (those containing a heavy quark) decay mostly according to the spectator model via the weak interaction: the weak interaction is the only way to change quark flavour
- Parity is violated in the Weak Interaction: only left-handed particles and right-handed antiparticles feel the Weak Force
- Pions decay to muon and a muon neutrino because there are only left handed neutrinos
- Neutrinos must be massless in the Standard Model: ν_R has no interaction at all and cannot be produced