Part 3: Nuclear Physics

19 LECTURE 20

19.1 Terminology of Nuclear Physics

Protons and neutrons bind together to make nuclei via the strong force. Protons and neutrons are collectively called nucleons. The notation is usually

- Z - Atomic Number - number of protons
- A - Mass Number - number of protons + neutrons
- N - Neutron number - number of neutrons

The notation for a nuclide is ${}^A_Z$X or just ^{A}X. The charge on the atom is +Ze where e is the absolute value of the charge on the electron.

Nucleides of a given chemical can occur with different masses: these are called **isotopes** which have Z fixed but different A. All chemical properties are derived from the electrons of an atom and are therefore electromagnetic in nature. Because atoms are neutral, the number of protons defines the chemical name of a nuclide.

Nucleides of the same mass number but different Z and N are known as **isobars**. An example of two isobars is ^{3}H and ^{3}He.

The simplest atom is Hydrogen and is made up of 1 proton and 1 electron. This is a purely electromagnetic entity: the Electromagnetic force binds the electron to the proton and so nothing will be learned about nuclear physics from a study of Hydrogen. However, there is something interesting about the Hydrogen atom. The mass of the Hydrogen atom is slightly less than the sum of the electron and proton masses. This so called **binding energy** is only 13.6eV, but it is striking evidence to support Einstein's theory that matter and energy are equivalent. The electron and proton are still in tact because the electron can be liberated from the protons grasp, but when they are bound together, their masses are reduced.

It is usually the *atomic* weight which is measured and used for the mass number. This is done using atomic ions: atoms with one or more electrons stripped off which will then bend in the magnetic field of a mass spectrometer. The difference between the atomic weight and nuclear weight is very small: $E_{electronic} \approx 20.8Z^{7/3}eV$.

19.2 The Nucleons

The proton(p) and the neutron(n) make up the set of nucleons.

$$M_p = 938.28MeV(/c^2)$$
$$M_n = 939.57MeV(/c^2)$$

Usually in nuclear physics terminolgy masses are given in units of MeV/c^2 but we will repect natural units as before and allow $c = 1$. The first thing to notice is that the neutron is the heaviest of the two nucleons. The charge on the proton is exactly equal an oposite to the charge on an electron. The underlying symmetry which ensures that the charge of the three quarks making up a proton be exactly equal to the electron charge is as yet undiscovered and why this is so is one of the fundamental questions still left open in particle physics. The charge distribution on the proton is not pointlike (neither of course is the proton itself pointlike) but distributed around the center of the proton out to a radius of about 0.8fm. For the neutron, even though it is a charge zero object, it is made up of three charged objects and its charge distribution is measured to have the positive charge concentrated in the center with the negative charge around the edge.

The matter radius of the nucleons is also about the same size as the charge radius, 0.8fm

Both p and n have magnetic dipole moments:

$$\mu_p = 2.79284(e\bar{h}/2m_p)$$
$$\mu_n = -1.91304(e\bar{h}/2m_p)$$

and the measurement of these was clear evidence that the nucleons were not fundamental.

The nucleons are ground states of composite systems. The photon energies which are typical of *nuclear* transitions between any of these excited states are 100s of MeV to be compared with KeV and below for characteristic energies of atomic transitions.

The electrical energy associated with the charge distirbutions of p and n are:

$$\approx \frac{e^2}{4\pi\epsilon_0 R_p} \approx 2MeV$$

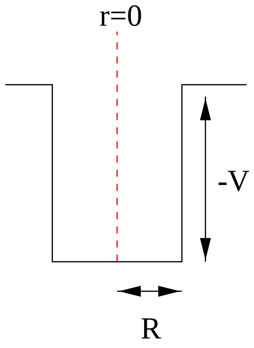
19.3 The Deuteron

The Deuteron is the lightest nuclear entity. This consists of a proton and a neutron. The neutron is not charged and so it is bound together with the proton solely by the nuclear force. The Deuteron also has a characteristic binding energy, but this time it is much bigger than in the case of the electromagnetic object.

$$m_D = m_p + m_n - 2.2\text{MeV} \tag{116}$$

indicating that the Deuteron is more tightly bound than the proton and the electron of the hydrogen atom. However, on the nuclear scale, this binding energy is rather small. The Deuteron provides us with a particularly good environment in which to do a calculation, it being very simple in terms of the forces in play. We can derive the wavefunction of the Deuteron starting from energy conservation and the simple picture of a potential square well around the proton. The square well is not really exactly the correct shape of the potential (it has curved corners rather han square ones), but it does give a very good approximation to the problem.

$$V(r) = -V; r < R \tag{117}$$
$$V(r) = 0; r > R \tag{118}$$



For the ground state (triplet state),

$$V(r) \;=\; -25.5MeV; R = 2.04 \tag{119}$$

So, lets start out from energy conservation:

$$\left[\frac{p_n^2}{2m_n} + \frac{p_p^2}{2m_p} + V(r)\right]\psi = E\psi \tag{120}$$

In the center of momentum, $p_n = -p_p$, the reduced mass is given by $\mu = \frac{m_p m_n}{m_n + m_p}$ and so

$$\left[\frac{p^2}{2\mu} + V(r)\right]\psi = E\psi \tag{121}$$

from this it can be shown that if $\phi = \frac{u}{r}$

$$-\frac{\hbar^2}{2\mu}\frac{\partial^2 u}{\partial r^2} + (V - E)u = 0 \tag{122}$$

E is the binding energy of a bound state and so E=-|E|. It can be shown that if

$$u \;=\; A\sin kr; r < R \tag{123}$$
$$u \;=\; Be^{-\gamma(r-R)}; r > R \tag{124}$$

then

$$k \;=\; \sqrt{\frac{2\mu}{\hbar}(V - |E|)} \tag{125}$$
$$\gamma \;=\; \sqrt{\frac{2\mu}{\hbar}|E|} \tag{126}$$
$$k \;=\; \gamma\frac{\sin kR}{\cos kR} \tag{127}$$

You get A by normalizing

$$\int_{r=0}^{r=+\infty} \psi^*(r)\psi(R)r^2 dr = 1 \tag{128}$$

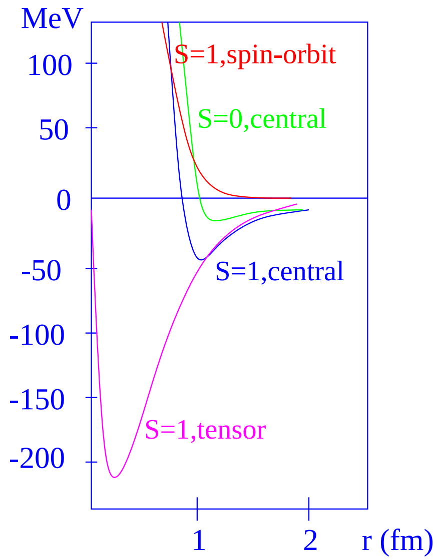


Figure 59: The Paris Potenial

19.4 The Nuclear Force

The simple square well picture works well for describing the bound deuteron system as shown above. However, the nuclear force is rather complicated (not at all simple like the gauge forces) and one reason for that is that there are a multitude of particles which act as the exchange particles for the nuclear force. we mentioned earler that pions acted as the exchange particle for the nuclear force outside of the baryon diameter of about 1fm but there are several particles such as $\rho(770)^{\pm,0}$,$\omega(782)$ which have spin 1 and larger mass than the pion which can mediate the nuclear force. Different masses lead to different ranges of force and different spin leads to different behaviour of the force. This is supposed to set the scene such that you are not surprised when you find out that the parameterization of the nuclear force is rather complicated!!

A large amount of data has been collated by groups in Paris from p-p and p-n scattering experiments. The form of the nuclear force has been parametried in a particular way and then fit against the deuteron (the simple bound n-p system) to give values of the parameters.

The form of the force depends on the relative spin alignment of the nucleons. If they have their spins anti-parallel then there is only one graph to consider which is a central attractive potential although it is not deep enough to bind together to nucleons shown in Figure 59. This is the reason why the deuteron is the bound state of two nucleons with spins aligned parallel to each other. In this case there are three different components to the potential. There is an attractive central potential (even shallower than for the S=0 case), a tensor attractive potential and a repulsive spin-orbit coupling component. The first two components are responsible for the binding of the Deuteron. The tensor force should be equal to zero when averaging over all directions but it is important for the Deuteron where there is a well defined axis but less so for heavier nuclei. The central potentials have a repulsive core inside of 0.8fm and are attractive out to a radius of about 2fm.

There are no bound n-n or p-p states because of the shallowness of the S=0 central potential. The Pauli exclusion principle prevents a neutron and a proton from being in the same state and so if they are to be bound in the ground state they would have to have their spins aligned anti-parallel. This means they are subject to the S=0 cental potentialwhich is not large enough to bind them together. Another way of saying this is that the nuclear force is spin dependent.

It is very difficult to do calculations for nuclei heavier than the Deuteron.

20 LECTURE 21

20.1 Heavier Nuclei

To bring two protons together, it is necessary for them to overcome the Coulomb repulsion between them. Effecively, the attratcive nuclear force must overcome the potential barrier produced by this Coulomb repulsion. An extremely stable 'bundle' of nuclear matter is that containing two neutrons and two protons. This was called the alpha particle, due to its stability and ability to exist as if it were a particle in its own right. The alpha particle has a binding energy of 28.3 MeV which is very large considering the total energy of the bundle as will be seen in the next section. It is necessary to have at least two neutrons present in order to make the strong attractive force large enough to bring two protons together. As will be seen later, nuclei always have the same number of, or more, neutrons than protons.

20.2 The Measurables

The strength of the binding of nuclei is an important **measurable** which we can use to try to understand more thoroughly what is going on. Figure 60(left) shows the binding energy per nucleon as a function of atomic number. The dots represent some actual data whereas the line is an approximation to the data. Only the gross features of this plot should be considered at this stage. The binding energy per nucleon peaks at about mass number 60 at a B/A of about 8.7 MeV and slowly falls off to between 7 and 8 MeV per nucleon. Another empirical piece of information is found in the observation of *which* stable nuclei exist. There is a steady increase in the number of neutrons relative to the number of protons as A increases. Furthermore, above A>209 there are *no* stable nuclei. However, a new element has been produced in Russia in the last month which has 114 protons (and at least 280 neutrons) which has a lifetime of 30s!!

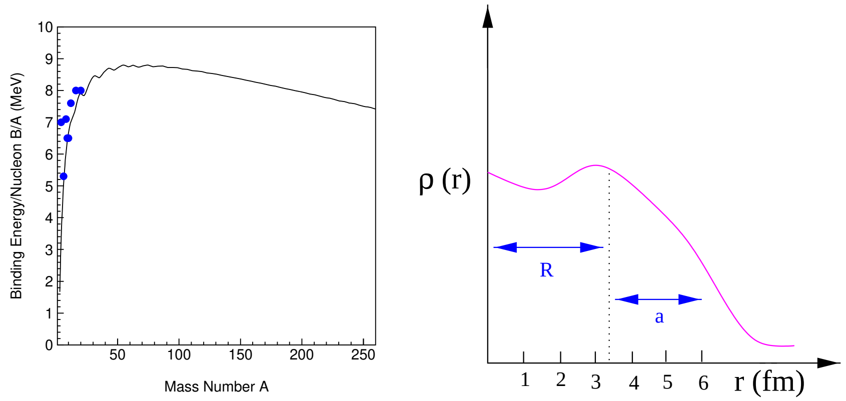


Figure 60: Left:Binding energy per nucleon as a function of the mass number A. Right: Charge density of the nucleus (schematic)

Another experimental measurement that can be made is the study of particle scattering off nuclei. From *charged particle* scattering off nuclei can be measured the distribution of charge in the nucleus. A schematic diagram of the charge radius of a nucleus is shown in Figure 60(right). It has a slightly non-uniform shape due to the fact that the charge is not evenly distributed inside the nucleus but tends to cluster around the edges in order to minimize the Coulomb repulsion. It is somewhat difficult to go backwards from scattering data to the charge density shape $\rho_{ch}(r)$. However, it is possible to assume a plausibe shape for the $\rho_{ch}(r)$ described by a simple mathematical expression:

$$\rho_{ch}(r) = \frac{\rho(0)}{1 + e^{(r-R)/a}} \tag{129}$$

From *neutron scattering* off different nuclei can be ascertained not only the size of the nucleus but from studying many different nuclei, the density of nuclear matter can be determined. The size of a nucleus is measured to be

$$R = r_0 A^{\frac{1}{3}} \tag{130}$$

where $r_0 \approx 1.5\ 10^{-15}$. This reflects the fact that nuclear density is found to be constant. This is depicted in Figure 61 where the matter distribution for three nuclei is shown. Inside the nucleus, the density is very similar no matter what the mass of the nucleus. This behaviour is very similar to that of a liquid and this fact did not go unnoticed. The surface of the nucleus is a well defined quantity for nuclei with A>20, but is not really applicable to light nuclei. The shape is probably spherical, but observation of electric quadrupole moments show some deviation from sphericity. For more information about the experiments undertaken, see Cottingham or Williams.

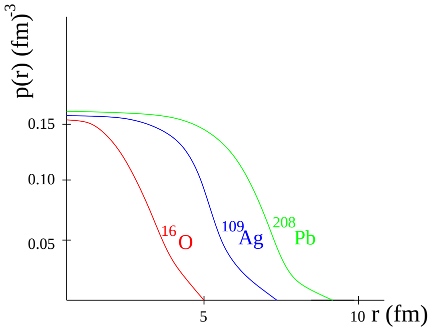


Figure 61: Nuclear density for three different nuclei.

20.3 Liquid drop model

Calculations for heavier nuclei are too hard to do. However, a model called the **liquid drop model** is invoked which turns out to be a very successful picture of the nucleus. In order to bring two protons together, at least two neutrons are needed to overcome the Coulomb (electromagnetic) repulsion. The forces at work inside nuclei are very strong and not necessarily simple. However, a simple consideration of the strong (attractive) force overcoming the electromagnetic (repulsive) force does give an intuitive picture. The liquid drop model, as its name suggests, treats the nucleons as if they were molecules in a liquid. Nuclear matter is pretty much incompressible, like a liquid, and the forces at work inside a liquid are also strong and complicated.

21 LECTURE 22

21.1 The Semi Empirical Mass Formula of Weizsaecker

The liquid drop model can be described mathematically by the Semi Empirical Mass Formula (SEMF). It is called *Semi* Empirical because although the constants used to describe nuclei in this way are derived from the data, the dependence of these constants has an intuitive root as we will see now. The SEMF predicts the binding energy of nuclei which has several components.

21.2 The Volume Term

The strength of the binding of a nucleus is proportional to the number of nucleons inside. Although the binding energy *per nucleon* falls slightly with increasing A above A≈60, the overall binding energy is increased. The contribution from this effect to the binding

energy is given by

$$-B_{volume} = -u_v A \tag{131}$$

21.3 The Surface Term

A liquid drop, in the absence of external fields, will adjust its shape to minimize its energy which leads to a spherical shape in order to minimize the surface tension. Nuclei are thought to be approximately spherical, but observation of electric dipole and quadrupole moments show some deviation from sphericity. The concept of the nuclear surface is not really applicable to nuclei with A<20.

Nucleons at the surface of the nucleus will feel less attraction than those in the middle and therefore the total potential energy is lowered compared to what you would expect if you just considered a sphere of condensed nuclear matter (i.e. this will decrease the binding energy or increase the energy needed to keep the nucleus bound). This will be most important for light nuclei and the magnitude of the effect will be proportional to the surface area.

$$-B_{surface} = u_s A^{\frac{2}{3}} \tag{132}$$

21.4 The Coulomb Term

As nucleons are brought together, they must overcome the Coulomb repulsion between the protons. Once the separation of the protons is about 1fm, the repulsion is over compensated. This term will decrease the binding energy (increase the energy necessary to keep all the nucleons bound).

$$-B_{Coulomb} = \frac{Z(Z-1)e^2}{A^{\frac{1}{3}}} \tag{133}$$

$$\approx \frac{(Ze)^2}{r} \tag{134}$$

For large Z it is useful to use the approximation $Z^2 \approx Z(Z-1)$

21.5 The Asymmetry Term

The strong force is an exchange force. Exchange forces are **attractive** if the wave function is symmetric with respect to space exchange and are **repulsive** if the wavefunction is anti-symmetric. Therefore, anti-symmetric pairs do not contribute to the energy because they have the wrong sign for binding. This is the reason why most stable nuclei are the ones with fewest anti-symmetric pairs. Look at two cases for the A=16 isobar as depicted in Figure 62. The lowest energy system will be the ^{16}O which is completely symmetric between n and p. The higher energy system will be ^{16}N which has an extra neutron. Because of the Pauli exclusion principle, the extra neutron must inhabit the next energy level. This leads to a term in the binding energy expression:

$$-B_{asymmetry} = u_a(N-Z)^2 A^{-1} \tag{135}$$

$$= u_a T^2 A^{-1} \tag{136}$$

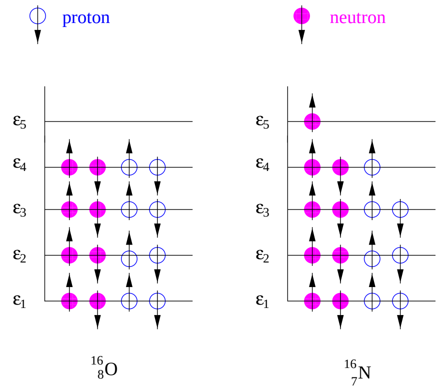


Figure 62: Left: Energy levels for ^{16}O. Right: Energy levels for ^{16}N

The SEMF can thus far be written:

$$\begin{aligned} U(A,Z) &= Zm_p + Nm_n - B_{volume} - B_{surface} - B_{Coulomb} - B_{Asymmetry} \qquad (137) \\ &= Zm_p + Nm_n - u_v A + u_c Z^2 A^{-\frac{1}{3}} + u_s A^{\frac{2}{3}} + u_a T^2 A^{-1} \qquad (138) \end{aligned}$$

where U represents the total energy of the nucleus including its rest mass.

21.6 The Pairing Term

Following on from what we have learned about the symmetry of the exchange force, a plot of U(A,T) versus T for isobaric nuclei is a parabola with respect to T as shown in Figure 63(left), the minimum of the parabola being at T_{min}.

The SEMF so far excludes the possibility of more than one stable isobar. Experimentally, there is only one stable isobar for odd mass numbers but there are two and sometimes three for even mass numbers. These stable nuclei are always of the even-even variety with the odd-odd ones in between being beta-unstable in both directions (see Section 22.3). This is indicated in Figure 63(right) where U as a function of T is given for FIXED A. In this case, the difference between even-even and odd-odd nuclei is given purely by

$$\left(\frac{u_a}{A} + u_c A^{-1/3}\right) \qquad (139)$$

The bulk terms being the same for a given A. The final term in the SEMF should be one depending on the type of the nucleus. The term added is

$$\pm\frac{\delta}{2A} \qquad (140)$$

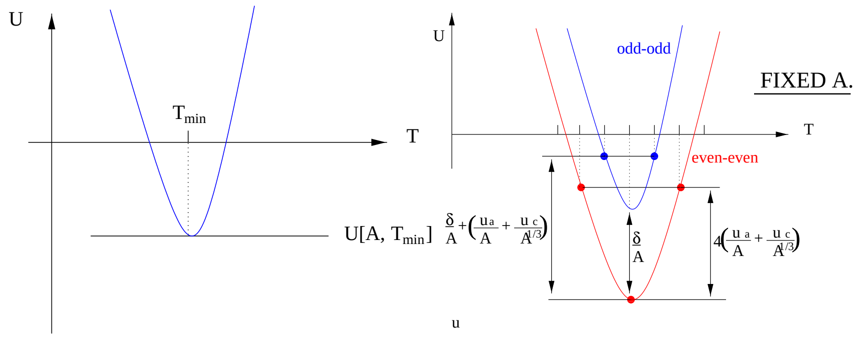


Figure 63: Left: U as a function of T from the SEMF. Right: U as a function of T for odd-odd and even-even nuclei for fixed A. The figure shows the case most favourable to the occurrance of three stable isobars (all of them even-even), with the spacing parameter δ chosen so large as to make the three even-even isobars stable.

with the minus sign for even-even nuclei and the plus sign for odd-odd. This reflects what we learned earlier about the Deuteron: n-p pairs can be bound together because they can be in an S=1 state (their spins aligned parallel to each other) whereas n-n and p-p pairs to not add to the binding energy because the S=0 potential is not enough to bind the nucleon pairs. If there are an even number of protons and neutrons, this implies that there is maximal pairing between n and p nucleons whereas with odd-odd nuclei, it is at least guaranteed that there be an odd n and an odd p which are not paired. So even-even nuclei will have a larger binding energy than odd-odd nuclei given a fixed A. $\delta = 0$ for odd A. The proportionality to A^{-1} is assumed, since this is a symmetry effect. $\delta \approx 270$MeV for medium weight nuclei. Several texts quote the pairing term as

$$\pm\frac{\delta}{A^{1/2}} \qquad (141)$$

with δ=11.2MeV but either will give a sufficienctly accurate determination of the binding energy. This demonstrates the purely empirical nature of this term.

The quoted values of these parameters are the ones which give the best agreement between the theory and the measured data for some particular method of determination. There are many such sets of these parameters, determined in different ways. In the literature, you will find them to vary from text to text. For example, in Williams:

$$\begin{aligned} u_v &= 15.56\text{MeV} \\ u_s &= 17.23\text{MeV} \\ u_c &= 0.70\text{MeV} \\ u_a &= 23.28\text{MeV} \\ u_p &= 12.00\text{MeV} \end{aligned}$$

The relative contributions of all the terms to the binding energy of nuclei as a function of A is shown in Figure 64.

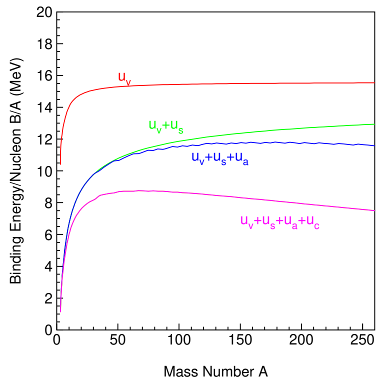


Figure 64: N vs Z for stable nuclei

21.7 Stability conditions of nuclei

There are two main effects which determine the stability of a particular nucleus and a third effect which happens rarely:

- The symmetry effect: the charge independence and the exchange character of nuclear forces, together with the Pauli exclusion principle, strongly depresses the energy of nuclei with equal or nearly equal numbers of protons and neutrons
- The charge effect: the effect of the Coulomb repulsion of the protons favours nuclei with fewer protons than neutrons. The neutron proton mass difference also enters the charge effect, but only as a small correction. The charge effect increases in importance with increasing Z
- In a few special cases, the spin dependece of nuclear forces favours parrallel spin over anti-parrallel spin of a pair of (extra) nucleons.

22 LECTURE 23

22.1 Spontaneous Activity: Radioactivity and Fission

A group of nucleons can be bound if Z≈N. No nuclei exist which are all n or all p because of the Pauli exclusion principle and the exchange nature of the strong force. Some nuclei are unstable against a split into two or more parts. This happens if the binding energy of the parent nucleus is smaller than the sum of the binding energy of the daughter nuclei. How can these nuclei then be held together in the first place? Figure 65 shows

three possibilities. At large separations the energy reaches a constant level. The energy reaches a maximum at about 1fm from the Coulomb repulsion and then at distances smaller than about 1fm, the energy drops off. In the top graph, the energy level at very close range is lower than the energy at infinite separation and so the nucleus is completely stable. In the second case, the small separation energy is higher than the large separation energy, but in order for a spontaneous decay to occur, the decaying particle must either tunnel through this potential (Coulomb) barrier in order to reach the lower energy state or energy must be supplied to the nucleus to overcome the barrier. The first case scenario is indeed what happens in all forms of spontaneous decay. In the lower diagram, no bound nucleus is ever formed. The probability of this tunnelling goes down as a function of the energy of the penetrating particle and therefore spontaneous fission is a very rare process, but spontaneous alpha decay is fairly frequent. The second case scenario, where energy is supplied to the nucleus, is what happens in induced fission.

In the case of bound nuclei, the energy difference between the small separation and the large separation is known as the separation energy. Because of the equality of neutron-neutron and proton-proton forces, the separation energy

$$S_n \approx S_p; N \approx P \tag{142}$$

Nucleons are fermions and so Pauli requires that no two n or two p are in the same state. The energy levels are filled up to E_n^F and E_p^F (where F stands for Fermi) for neutrons and protons respectively. In $A \leq 40$, N≈Z and so $E_n^F \approx E_p^F \approx 38$MeV. In heavier nuclei, the value of E_n^F can be much higher. The energy to detatch a neutron is given by

$$S_n(Z,N) = B(Z,N) - B(Z,N-1) \tag{143}$$

and hence is about 8MeV. Therefore, the total depth of the neutron well is ≈46MeV. The most stable nucleus will have $E_n^F \approx E_p^F$: if these are very different the nucleus will be unstable against beta decay.

22.2 Radioactivity

Radioactivity is the name given to several distinctly different processes which we can now identify in terms of the fundamental forces which we have learned about. You may be familiar with the terms alpha, beta and gamma radiation, somewhat historically named.

Alpha radiation: (α) Alpha radiation is the spontaneous emission of a helium nucleus. This is a strong interaction process.

Beta radiation: (β) Beta radiation is just the weak decay of one of the quarks in a nucleon to an electron and an electron anti-neutrino. The electron is emitted (the beta ray) while the neutron(udd) becomes a proton(uud) via a W^- or under certain energetically favoured situations a proton becomes a neutron via a W^+. See Figure 66.

Gamma radiation: (γ) Gamma radiation is just another name for a particular energy of photon. These energies are in the MeV range and the photons come from the nucleons inside the nucleus rearranging themselves into a more energetically favourable configuration with the emission of the resultant energy given off as a photon. X rays on the

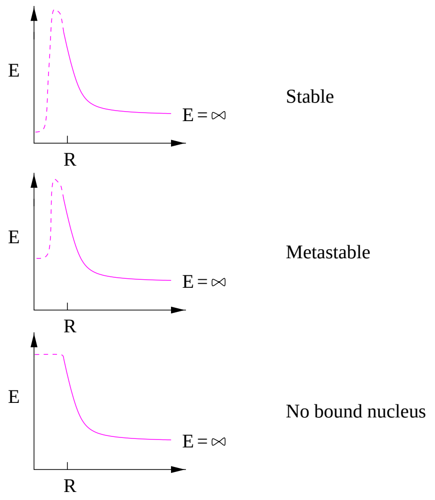


Figure 65: Three different cases for the binding of nuclei

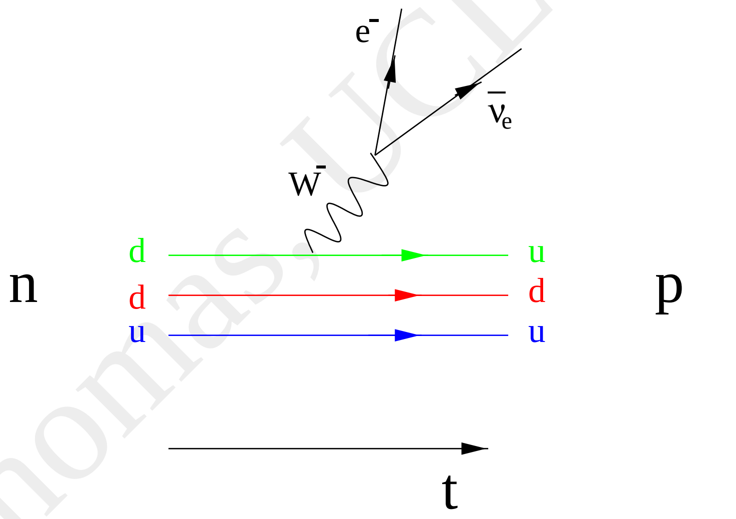


Figure 66: Feynman diagram of 'beta decay': a weak decay of a neutron to a proton, electron and anti-neutrino.

other hand, come from electrons in the atoms rearranging themselves and dropping down across energy levels giving out photons. These photons are typically in the KeV range of energies.

22.3 Energy considerations for stability against α and β radioactivity

Although the liquid drop model of the nucleus has its shortcomings, the SEMF which has been derived using the model describes several gross features of nuclei such as the binding energy curve. It can be used to estimate the stability conditions of nuclei.

α decay occurs when the binding energy of the parent nucleus is less than the sum of the binding energies of the daughters in a decay:

$$B_1 < B_2 + B_3 \tag{144}$$

Because of the gradual fall off of B/A with increasing A, this may be true for high A nuclei into two intermediate A nuclei. For any beta stable nucleus with A>85 this is true. α particles are the most stable clump of nuclear matter with 2 neutrons,2 protons and a binding energy of 28.3MeV. The spontaneous emisssion of an α particle is energetically favourable for nuclei above A≈191.

The difference in energy between the parent and daughter nuclei for β decay can be written:

$$\begin{aligned}
\Delta U_{Z\to Z+1,N\to N-1} &= U(Z,N) - U(Z+1,N-1) && (145)\\
&= B(Z+1,N-1) - B(Z,N) + (m_n - m_p) && (146)
\end{aligned}$$

We can look at three different general examples:

$$\begin{aligned}
{}_ZX &= {}_{Z+1}Y + e^- + \bar{\nu}_e; \Delta > 0.511\text{MeV} && (147)\\
{}_{Z+1}Y &= {}_ZX + e^+ + \nu_e; \Delta < -0.511\text{MeV} && (148)\\
{}_{Z+1}Y + e^- &= {}_ZX + \nu_e; \Delta < 0.511 - \epsilon\text{MeV} && (149)
\end{aligned}$$

where ϵ is the binding energy of the electron in the state from which it is captured. Any excess of energy appears as kinetic energy of the emitted electron and neutrino. Therefore we see that a nucleus is *only* β stable if either:

$$\begin{aligned}
\Delta U_{Z\to Z+1,N\to N-1} &< 0.511\text{MeV or} && (150)\\
\Delta U_{Z\to Z-1,N\to N+1} &< -0.511\text{MeV} && (151)
\end{aligned}$$

this leads to the interesting result that two *stable* isobars *must* differ by more than one unit in Z. X and Y isobars differing by one unit cannot be stable against β decay *unless* $\Delta U = 0.511$MeV. The lifetimes for this process are on the order of a fraction of a second which is very slow. This is because it is a weak interaction and this force is very weak.

The SEMF can be used to determine whether a particular nucleus is stable against spontaneous decay. The SEMF can be written as

$$U = \Delta + \Lambda Z + \Gamma Z^2 \tag{152}$$

where

$$
\begin{aligned}
\Delta &= Am_n - u_v A + u_s A^{\frac{2}{3}} + u_a A \pm \frac{\delta}{2A} && (153)\\
\Lambda &= m_p - m_n - 4u_a && (154)\\
\Gamma &= \frac{u_c}{A^{\frac{1}{3}}} + \frac{4u_a}{A} && (155)
\end{aligned}
$$

For odd isobars, $\delta = 0$ because there is only one parabola of stable nuclei as shown in Section 21.6. Now, if the weak decay arguments above allow only one stable isotope (which has the smallest mass for a given Z) which is an even-even combination even isobar (the odd-odd combination even isobars being beta unstable) then $\frac{dE}{dZ} = 0$ at the minimum Z:

$$
\begin{aligned}
\left(\frac{\partial U}{\partial Z}\right) &= \Lambda + 2\Gamma Z && (156)\\
&= m_p - m_n - 4u_a + 2\frac{u_c}{A^{1/3}}Z + 8\frac{u_a A}{Z} && (157)\\
&= 0 && (158)\\
Z &= \frac{-\Lambda}{2\Gamma} && (159)\\
&= \frac{A}{2}\left[\frac{m_n - m_p + 4u_a}{u_c A^{2/3} + 4u_a}\right] && (160)\\
&= \frac{A}{2\left[\frac{4u_a + u_c A^{2/3}}{4u_a + m_n - m_p}\right]} && (161)
\end{aligned}
$$

This expression can be reduced by retaining only the leading terms and neglecting the neutron-proton mass difference, to

$$Z \approx \frac{A}{2} + \frac{u_c}{4u_a} A^{\frac{5}{3}} \qquad (162)$$

The ratio of u_c to u_a enters because the stability against β decay is determined by the competition between the charge effect which favours large neutron excesses and the asymmetry effect which favours $T = 0$. For the same reason the dependence on the mass number is $A^{\frac{5}{3}}$: the charge effect varies like $A^{\frac{2}{3}}$ and this must be divided by the symmetry effect which varies like A^{-1}. The effect of this consideration is shown in Figure 67 where the stable odd-A nuclei are shown as dots on the graph of Z vs N and the line is N=Z. The value of T_{min} determines the relationship between N and Z.

Another set of constants for the SEMF IS:

- $u_v = 14.1$MeV
- $u_c = 0.15$MeV
- $u_a = 18.1$MeV
- $u_s = 13.1$MeV

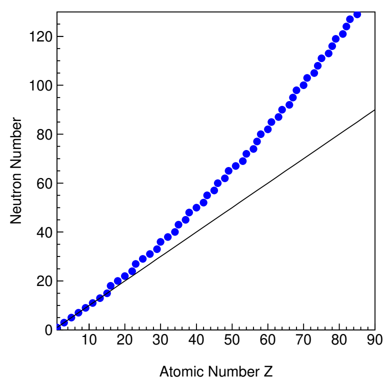


Figure 67: N vs Z for stable nuclei

If you know all the constants then equation 160 becomes:

$$Z \approx \frac{A}{2(0.98 + 0.002A^{2/3})} \qquad (163)$$

which falls short of A/2 and more so as A increases as shown in the figure. The reason why there are several sets of the constants is that in order to measure them, a fit has to be done to the binding energy per nucleon curve in terms of the 5 parameters, and different approaches to dong this fit result in different values of the constants.

To look at the stability of a nucleus against α decay, we differentiate the SEMF with resepct to A and Z.

$$
\begin{aligned}
\Delta U &= U_{parent} - (U_{daughter} + U_\alpha) && (164)\\
&= U(Z,A) - U(Z-2, A-4) - U(2,4) && (165)\\
&= \frac{\partial U}{\partial A}\Delta A + \frac{\partial U}{\partial Z}\Delta Z - U(2,4) && (166)\\
&= \frac{Z}{A^{1/3}}(\chi_1 - \chi_2\frac{Z}{A}) + \frac{\chi_3}{A^{1/3}} - \chi_4(1 - \frac{2Z}{A})^2 - 28.1MeV && (167)
\end{aligned}
$$

where $\frac{\partial U}{\partial Z}$ is given by equation 156, ΔA=4, ΔZ=2 and

$$\frac{\partial U}{\partial A} = m_n - u_v + \frac{2}{3}u_s A^{-1/3} - \frac{1}{3}u_c Z^2 A^{-4/3} \qquad (168)$$

In order to get the expression for ΔU in (167 you need to know that the binding energy of an alpha particle is 28.3MeV and that $u_v = 14.1MeV$ and $-4u_v + 28.3$MeV = -28.1MeV.