23 LECTURE 24

23.1 Fission

We can use the SEMF to find out about the energetically allowed decays of nuclei. From the graph of the binding energy per nucleon in Figure 60(left) it is apparent that the most stable nuclei will have A≈60. Actually, ^{56}Fe is the most stable nucleus. It is also apparant that energy can be released by reducing the mass if A> 60 (called fission) or increasing the mass if A< 60 (called fusion). If Q is the difference in the binding energy between the decay products and the parent nucleus;

$$Q(A,Z) = B(A-X, Z-Y) + B(X,Y) - B(A,Z) \tag{169}$$

then if Q is positive, the decay is energetically favourable. Following on from this equation

$$\begin{aligned}
\Delta B &= u_v\left[A - X + X - A\right] && (170)\\
&+ u_s\left[A^{2/3} - X^{2/3} - (A-X)^{2/3}\right] && (171)\\
&+ u_a\left[\frac{(A-2Z)^2}{A} - \frac{(X-2Y)^2}{X} - \frac{(A-X-2Z+2Y)^2}{(A-X)}\right] && (172)\\
&+ u_c\left[\frac{Z^2}{A^{1/3}} - \frac{Y^2}{X^{1/3}} - \frac{(Z-Y)^2}{(A-X)^{1/3}}\right] && (173)
\end{aligned}$$

The first term is obviously zero. This means that the attractive part of the strong force which comes from the bulk attraction, i.e. number of nucleons is not involved in the balance of energy from fission. The Coulomb interaction is obviously repulsive and energy will be released, the surface energy will always prefer a smaller "drop" of nuclear material and the asymmetry term may or may not be important depending on what the decay products are. This is summarised in Figure 64 which shows the relative contributions to the SEMF from these different terms as a function of A.

The SEMF predicts that the energy release is a maximum when the two fragements are of equal size, but there is usually a striking difference between the sizes of the products which are thought to be due to shell structure effects. For fission into two equal parts

$$\begin{aligned}
Q &= -u_s A^{2/3}[2\left(\frac{1}{2}\right)^{2/3} - 1] - u_c\frac{Z^2}{A^{1/3}}\left[2(\frac{1}{2})^{5/3} - 1\right] && (174)\\
\frac{Z^2}{A} &> \frac{u_s(2-2^{2/3})}{u_c(2^{2/3}-1)} && (175)\\
\frac{Z^2}{A} &> 18 && (176)
\end{aligned}$$

This condition is satisfied by β stable nuclei heavier than $^{98}_{42}$Mo. A large amount of energy is released in symmetric fission for example for ^{235}U, about 170MeV per fission is given out which is about 10^7 times greater than heat given out in a combustion process. An **order of magnitude** calculation can be performed to demonstrate this fact using only electromagnetic considerations. This is a particularly simple calculation which is valid to within a factor of two (the difference between u_v and u_c).

Consider a chemical reaction such as occurs in an explosion of TNT. A valence electron will be torn out of its shell and replaced around another atom or compound. The energy emitted is given by

$$E = \frac{e^2}{R} \tag{177}$$

which can be derived from 25. R in the case of the atomic radius is $R_{atom}=10^{-10}$m which is 10^4 times the size of a Uranium nucleus. The electron can be considered as moving away to infinity from R_{atom}.

Uranium has 92 protons so $E_{parent} = (92)^2/R_{nucleus}$ and $R_{nucleus}$ is 10^{-14}m. After the decay to two equal sized pieces, the total energy of the system is

$$2E_{daughter} = \frac{2.1.26.(46)^2}{R_{nucleus}}. \tag{178}$$

The factor of 1.26 comes in because the two daughters now have smaller radii than the parent by a factor $1/^3\sqrt{2}$. The difference between the original energy and the final energy is $3\cdot10^{17}$ (in some units) compared with $1\cdot10^{10}$ in the case of the chemical reaction. This shows that the nuclear reaction gives out on the order of a whopping 10^7 times more energy than a chemical process. In the case of explsoives, 1kg of ^{235}U≡ $3\cdot10^3$TONS of TNT.

The transuranic elements can be made manifestly unstable by bombarding them with neutrons. A schematic diagram of what happens to a nucleus while undergoing fission is shown in Figure 68. If the nucleus is given a certain excitation energy, it will oscillate between an oblate and a prolate ellipsoid (between the points A and B). If enough energy is given to the nucleus to overcome the potential barrier then the nucleus will fission. The Coulomb barriers inhibiting spontaneous fission are in the range 5-6MeV for A≈240. Therefore, if a neutron with zero kinetic energy enters a nucleus, the excitation energy will be equal to the neutron binding energy. Now for ^{235}U, a zero kinetic energy neutron will have a binding energy of 6.46MeV which is enough to push the nucleus over the Coulomb potential and fission quickly occurs (within 10^{-14} seconds). However, for ^{238}U, the binding energy of the last neutron is only 4.78MeV which is not enough to cause the nucleus to fission.

The differences in the binding energy of the last neutron in even-A and odd-A nuclei are incorporated in the SEMF in the pairing term. The odd A nuclei

$$^{233}\text{U}, ^{235}\text{U}, ^{239}\text{Pu}, ^{241}\text{Pu}$$

are fissile, i.e. fission is induced by a zero kinetic energy neutron whereas the even A nuclei

$$^{232}\text{Th}, ^{238}\text{U}, ^{240}\text{Pu}, ^{242}\text{Pu}$$

need an energetic neutron to induce fission.

One decay chain of ^{235}U is

$$^1n + ^{235}U \rightarrow ^{236}_{92}U^* \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^1n + Q$$

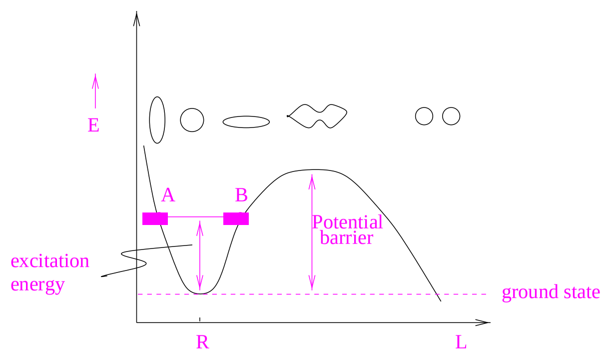

Figure 68: Energy levels and break up of a heavy nucleus

The importance of the neutrons in this decay chain cannot be understated. One neutron is needed to induce a fission in a nucleus but three are produced in the fission itself, leading to the possibility of inducing three more nuclei to fission. This is called a chain reaction. The neutrons have a certain absorption length which depends on their energy. The energy distribution of neutrons from ^{235}U decay is shown in Figure 69 (left) and the mean energy is about 2 MeV. Their absorption cross section for fission is shown as a function of this energy in Figure 69 (right).

Lets make a few definitions:

<u>E</u>LASTIC SCATTERING
$^{23X}\mathrm{U} + {}^1_0\mathrm{n} \rightarrow {}^{23X}\mathrm{U} + {}^1_0\mathrm{n}$
where 23X could be 235 or 238.

<u>I</u>NELASTIC SCATTERING
$^{23X}\mathrm{U} + {}^1_0\mathrm{n} \rightarrow {}^{23X}\mathrm{U}^* + {}^1_0\mathrm{n}$
where 23X could be 235 or 238

<u>R</u>ESONANT CAPTURE
$^{23X}\mathrm{U} + {}^1_0\mathrm{n} \rightarrow {}^{23X+1}\mathrm{U}^* + \mathrm{m}\gamma$
where 23X could be 235 or 238.

A 2 MeV neutron has a low cross section for interaction with a ^{235}U nucleus and the probability that it will induce fission is only 18% compared with resonant capture. However, it will undergo elastic collisions with nuclei and lose energy, thereby increasing its probability of interaction. The **mean free path** of a 2MeV neutron in U (the average distance it travels before capture) is 3cm and it does this in $1.5{\cdot}10^{-9}$s. The average number of collisions is ≈ 6, and assuming a random walk, the distance it travels from where it started is $\sqrt{6} \cdot 3\mathrm{cm} = 7\mathrm{cm}$ and this takes about 10^{-8}s.

The energy release in fission can be separated into two parts: the fast component which is given out in 10^{-14}s and the slow component which is emmitted at anytime after about 13s, and is sometimes delayed by decades!

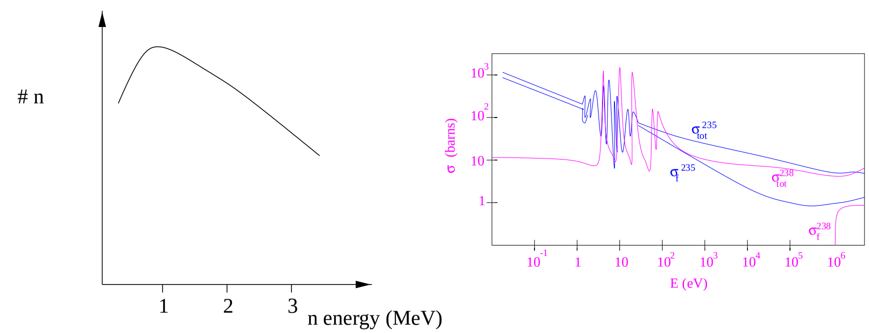

Figure 69: Left: Energy of neutrons from fission of ^{235}U . Right: Fission and total absorption cross section for neutrons.

24 LECTURE 25

24.1 Why ^{238}U is safe

Figure 69(right) shows the fission absorption cross section for neutrons in ^{238}U . There is essentially zero probability that a neutron will cause fission in ^{238}U unless its energy is above 1.4MeV which accounts for about 75% of the emitted neutrons. Of these neutrons which have enough energy, 95% of them are slowed down to below this 1.4MeV threshold by just a couple of inelastic collisions before being absorbed while 5% of them produce a fission. Therefore, there is a factor of 0.037 times the number of neutrons emitted in a fission. This average number of neutrons per fission is 2.2 (for all U chains) so the **effective** number of neutrons per fission for ^{238}U is 2.2 times $0.04 \approx 0.09$. Not enough to produce a chain reaction. In order to bump up the effective number of neutrons to above 1, you need a further 0.9 neutrons per fission from the ^{235}U in the sample. Naturally occurring U has only 0.7% ^{235}U so in order to produce enough neutrons, the U must be **enriched** by some percentage: i.e. the proportion of ^{235}U in the sample must be increased.

(In a previous version of this section, 25% of the neutrons were assumed to produce fission rather than 5%. This was due to finding the numbers in an old text where the cross sections were not so well known as they are today.)

24.2 Critical Mass in ^{235}U

The fact that neutrons have a mean free path in the Uranium before they are absorbed leads to the concept of a critical mass. If the block of ^{235}U is big enough such that every neutron will travel its 7cm and then be absorbed, the chain reaction will continue. If, on the other hand, the block of Uranium is so small that a substantial fraction of the neutrons escape the Uranium altogether before being absorbed, then if the number of neutrons actually reabsorbed is less than 1 per fission the chain reaction will stop. Because fissile material is very expensive, it is useful to be able to minimize the size of the critical mass m_c necessary to sustain a chain reaction.

In 1kg of ^{235}U , there are about $2.5{\cdot}10^{24}$ nuclei, so it will take about 80 generations to fission the entire kg which will take 0.8μs (80 times 10^{-8}s from the neutron path). However, as the chain reaction starts, the mass of Uranium will be heating up because each fission gives out 170 MeV. The temperature rises dramatically and the mass will expand, the density will therefore be lowered and the neutron mean free path will be increased until too many neutrons escape and the chain reaction stops. If only 1% of the available nuclei fission, the average velocity of the nucleons is 10^8cm s^{-1} and an expansion of a few cm will stop the reaction. This implies that the whole reaction must occur in $5{\cdot}10^{-8}$s. This looks like there is only time for about 5 further generations of fission after 1% of them have aready fissioned.Because the last generations give out the most energy, it is not possible to slow the neutrons down because of time limitations, and so they must be used as they are.

If the probability that a newly created neutron induces fission is q, then each neutron leads to (on average) $\nu q - 1$ additional neutrons in time $t_p \approx 10^{-8}s$. If there are $n(t)$ neutrons present at time t then:

$$n(t + \delta t) = n(t) + (\nu q - 1)n(t)\frac{\delta t}{t_p} \tag{179}$$

and if δt is small then

$$\frac{dn}{dt} = \frac{(\nu q - 1)}{t_p}n(t) \tag{180}$$

which has the solution

$$n(t) = n(0)e^{(\nu q - 1)t/t_p} \tag{181}$$

so the number of neutrons will increase or decrease *exponentially* depending on whether $\nu q < 1$ or $\nu q > 1$. For ^{235}U fission, the average number of neutrons $\nu = 2.2$, so the number of neutrons will increase exponentially if

$$q > \frac{1}{\nu} \approx 0.4 \tag{182}$$

If the mass is too small, i.e. sub-critical, the reaction damps down exponentially too. The critical radius for ^{235}U is ($\nu q = 1$) is about 8.7cm corresponding to a mass of 52Kg.

Plutonium (^{239}Pu) is more efficient because there are 3 neutrons on average given out in the fission instead of 2.2 in Uranium. It is produced in the decay:

$$^{239}_{92}\mathrm{U} \rightarrow ^{239}_{93}\mathrm{Np} + \mathrm{e}^- + \bar{\nu}_e$$
$$^{239}_{93}\mathrm{N} \rightarrow \mathrm{p}^{239}_{94}\mathrm{Pu} + \mathrm{e}^+ + \nu_e$$

The critical mass of Plutonium is 11Kg.

24.3 Thermonuclear Devices

The property of fission has been utilized in producing thermonuclear devices. The critical mass is effectively reduced by using a **tamper** which surrounds the fissile material to reflect the escaping neutrons and also to retard the expansion. Critical masses for Plutonium and Uranium are shown in Table 8 with and without a tamper. For the tamper, an extremely dense material is needed such as Au, W (Tungsten), Re (Rheunium) or ^{238}U .

	Pu	Pu(with Tamper)	^{235}U	^{235}U (with tamper)
Critical Mass(kg)	11	5	56	15

Table 8: Critical masses for fissile materials

Using fissile Plutonium in a bomb reduces the critical mass substantially over ^{235}U , but the efficiency is still quite low because of the time considerations.

To detonate the critical mass, two sub-critical masses must be brought together very quickly along with a neutron source to produce the first neutrons for the chain reaction.

The damage caused by a nuclear explosion is terrifying. There are two effects: the shock wave which destroys everything within miles but following that is air incandescence: where the air itself is set alight. A 1kg device 0.3s after detonation at 425ft will produce a temperature of 7000C (the temperature at the sun's surface is only 5000C). 1 mile from the explosion, brightness will be 3.5 times that of the sun.

25 LECTURE 26

25.1 Measurements of γ rays from cruise missiles

The problems of nuclear decomissioning are manyfold, but nevertheless the various treaties which are being negotiated to ensure the non-proliferation of nuclear weapons are of paramount importance.

One issue which concerns physicists is the identification of particular warheads during the process of decomissioning. All parties agree that the weapons must be destroyed but at the same time, neither 'side' wants to give away their nuclear technical secrets. As will be seen in the following, gamma ray templates of nuclear weapons can be made, and kept secret, so that the identity of a particular warhead can be verified without loss of sensitive information.

The following describes highlights of an experiment done on the Slava warship on the Black Sea which was equipped with a single nuclear warhead in the outside forward launcher on the starboard side.

A germanium detector kept at low temperature was used to measure the energy spectrum of gamma rays. The detector had energy resolution $\sigma(E)$, of 2KeV at $E_\gamma \approx 1000KeV$. Measurements were taken at four different positions

- 24 minutes on the launch tube
- 10 minutes on empty tube
- 27m from launch tube
- 32m from launch tube

Several software packages were used to identify the peaks in the spectrum. An energy calibration was done on the detector using the two peaks from ^{60}Co. As can be seen from Figure 70, there are many lines due to ^{235}U or ^{239}Pu. The presense of either suggests some nuclear warhead present.

Some interesting detective work was undertaken. For example, the presense of ^{232}U as indicated from the peaks at 583KeV and 2614.3KeV, show that the Uranium used for the missile had come from a reactor: ^{232}U is not naturally ocurring. The statistical analysis performed showed that the maximum distance that such measurements could be performed would be between 4-6m. Beyond that, the signal would not be significant above the background.

26 LECTURE 27

26.1 Reactors

Fission also provides the possibility of power generation. Figure 71 shows a schematic diagram of a **thermal reactor**. A more complete picture is shown in Figure 72. Thermal reactors have the advantage of utilizing natural Uranium because the neutrons are slowed down to **thermal** energies (<.1eV). The neutrons are slowed down by the **moderator** which is usually graphite or heavy water (D_2O). The moderator must have low mass number and a low neutron absorption cross section in order to maximize the energy loss of the neutron per collision while at the same time not absorbing the neutron completely.

The fuel rods are usually made of ceramic Uranium Dioxide.

The thermal neutrons must then go out and look for ^{235}U in which to induce fission as they slow down. The probability for the neutron producing fission in ^{238}U is very small until it has slowed down to $< 0.1eV$ kinetic energy where it is below the ^{238}U fission threshold. In fact, if the neutron slows down in Uranium and not in a specially selected moderator, it is more likely to be resonantly captured by ^{238}U to make $^{239}U + \gamma$ and there are no further neutrons. This is another manifestation of the fact that ^{238}U is safe.

A second type of reactor is the **fast breeder** reactor. In this type of reactor, fission is induced directly by the fast neutrons and so either the Uranium must be enriched to 20% ^{235}U or a fuel rod containing 20% ^{239}Pu is used in order to sustain a constant rate of fission. This does not utilize a moderator, but relies on the fact that some fraction of the neutrons are resonantly captured by the ^{238}U which eventually produces ^{239}Pu after some days:

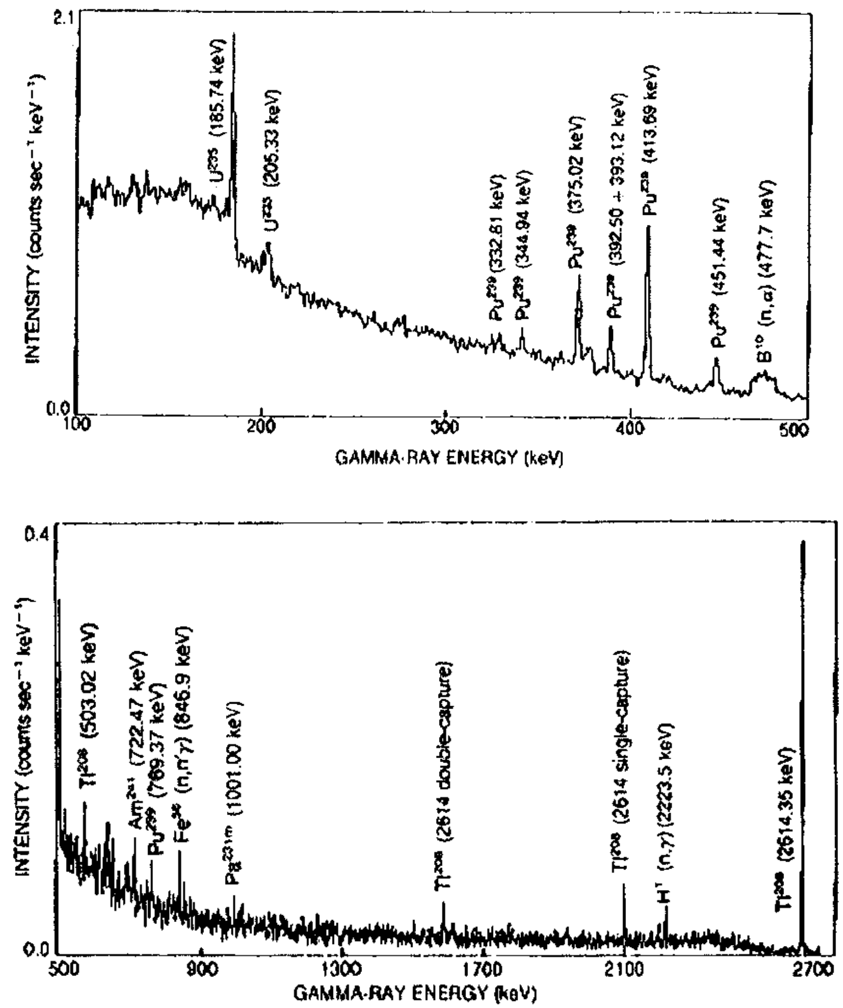

Figure 70: Energy spectrum of gamma rays measured on the launch tube

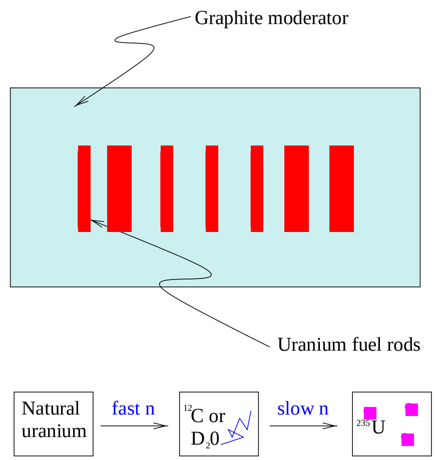

Figure 71: Schematic diagram of a thermal reactor

$$^{238}\mathrm{U} + {}^{1}\mathrm{n} \rightarrow {}^{239}_{92}\mathrm{U} + \gamma$$
$$^{239}_{92}\mathrm{U} \rightarrow {}^{239}_{93}\mathrm{Np} + \beta^- + \bar{\nu}_e \ \mathrm{T{=}23.5min}$$
$$^{239}_{93}\mathrm{Np} \rightarrow {}^{239}_{94}\mathrm{Pu} + \beta^- + \bar{\nu}_e \ \mathrm{T{=}2.35\ days}$$
$$^{239}_{94}\mathrm{Pu} \rightarrow {}^{235}_{92}\mathrm{Pu} + {}^{4}_{2}He \ \mathrm{T{=}24{,}360\ years}$$

This Pu is easily extracted from the Uranium because it is a different *chemical* and so it can be chemically extracted (lets the electrons do the work). Enriching Uranium is much harder because the ^{235}U must be extracted from ^{238}U which is the same chemical and so chemical procedures cannot be utilized and centrifugal or mass spectrometry technology must be used.

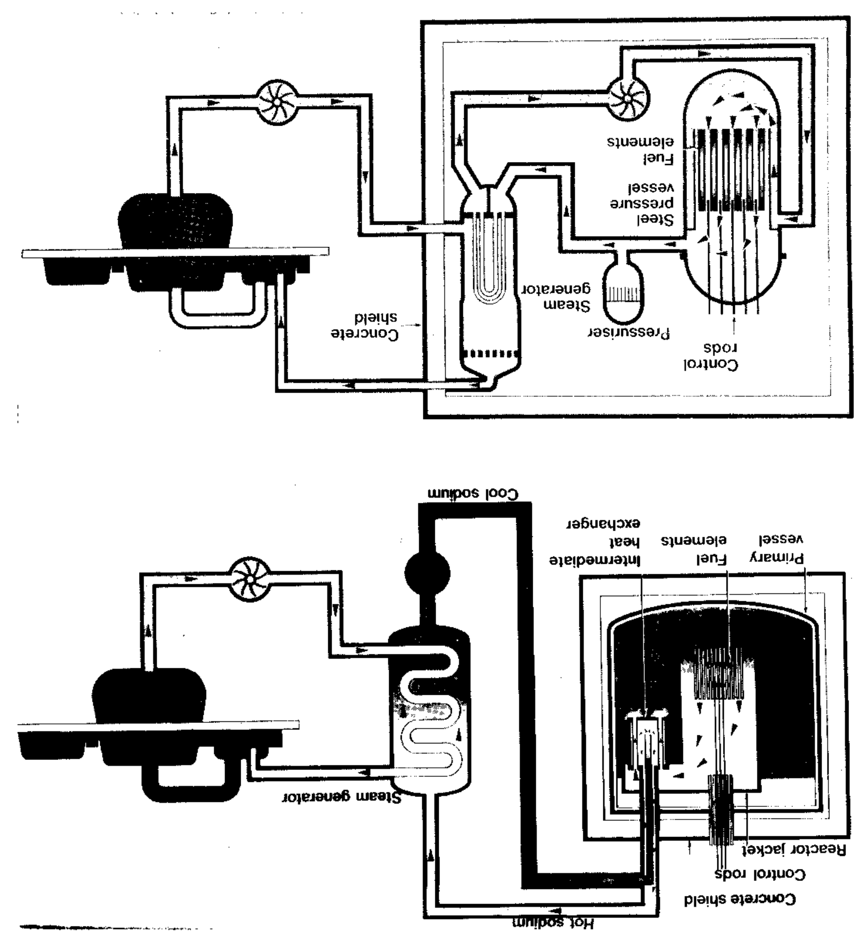

Figure 72: Thermal and Fast breeder reactors