27 LECTURE 28

27.1 Fusion in the Sun

In stars, gravity, the weak, EM and strong interactions all play a role. Gravity is a long range and universally attractive force. Any homogenous mass of gas at low temperature will contract to form a star under gravity. The rate of collapse is determined by the extent that the build up of heat and pressure balances gravity. Some important parameters of the sun are:

- M = 1.99 $\dot{1}0^{30}$Kg
- R = 6.96 $\dot{1}0^{8}$m
- $\mathcal{L}$ = 3.86 $\dot{1}0^{26}$W

The collapse will stop when the pressure and gravity balance. This happens when the interior becomes hot enough to ignite the Hydrogen burning cycle:

$$\mathrm{p+p} \rightarrow {}^2_1\mathrm{H+e^+}+\nu_e\mathrm{+0.42MeV}$$

This reaction occurs *very* rarely. It not only must overcome the coulomb repulsion between the two protons but it must also rely on the weak interaction (which has very slow time scales because its so weak) for the beta decay. Thanks to this fact, the sun is still burning today. This reaction is so rare in fact that it has never been observed in a laboratory environment.

27.2 Fusion in the Lab

Fusion is potentially a very great source of energy. The binding energy per nucleon for nuclei with masses lower than Fe is lower than that for Fe and so the possibility of producing energy by fusing nuclei together presents itself. Two possible reactions for producing fusion in the lab are:

$$\begin{array}{c} {}^2_1\mathrm{H} + {}^2_1\mathrm{H} \rightarrow {}^3_2\mathrm{He} + \mathrm{n} + 3.27\ \mathrm{MeV} \\ {}^2_1\mathrm{H} + {}^2_1\mathrm{H} \rightarrow {}^3_1\mathrm{H} + \mathrm{n} + 4.03\ \mathrm{MeV} \end{array}$$

One reason for trying these reactions is that 0.015% of all hydrogen is deuterium; for example it is abundant in sea water. In addition the isotope separation is easy because of the mass ratio of 2:1. Then, the final reaction can be attempted:

$${}^2_1\mathrm{H} + {}^3_1\mathrm{H} \rightarrow {}^4_2\mathrm{He} + \mathrm{n} + 17.6\ \mathrm{MeV}$$

We can do a neat calculation. How many neutrinos reach the earth from the Sun assuming that only the PPI chain is responsible for the luminosity of the sun (not a bad assumption)?

There are two neutrinos + 26.7 MeV per Helium nucleus. The power is given by

$$\begin{aligned} \mathcal{L} &= \frac{3.86\mathrm{x}10^{26}\mathrm{Js}^{-1}}{1.602\mathrm{x}10^{-12}\mathrm{JMeV}^{-1}} \\ &= 2.4\mathrm{x}10^{39}\mathrm{MeVs}^{-1} \end{aligned} \tag{183}$$

The number of Helium nuclei produced is

$$No.(He) = \frac{2.4\mathrm{x}10^{39}\mathrm{Hes}^{-1}}{26.7MeV} \tag{184}$$

and the number of neutrinos is twice this. Therefore, the number of neutrinos intersecting the earth is

$$\begin{aligned} \nu/m^2/s &= \frac{1.8\mathrm{x}10^{38}\mathrm{s}^{-1}}{4\pi r^2_{earth-sun}m^2} \\ &= 6.37\mathrm{x}10^{14}\mathrm{m}^{-2}\mathrm{s}^{-1} \end{aligned} \tag{185}$$

This is a large number! The neutrinos are mostly very low energy but they travel a very long way and actually produce a very good experimental arena to look for neutrino oscillations. There are several experiments which have reported many fewer neutrinos from the sun than expected from the Standard Solar Model. This is thought to be further evidence for neutrino oscillations. There is another explanation. Can you think of one?

Figure 73 shows the relative cross-sections (in arbitrary units) for the d-d and the d-t reactions as a function of energy. There is a resonance of an excited state of ${}^5_2$He at about 10MeV which increases the d-t reaction probability and it is this which experimenters have been trying to take advantage of at JET for example in Culham near Oxford. The disadvantage is that tritium must be manufactured and has no natural abundance because of its beta decay with a half life of 17.7 years. In order to produce the temperatures necessary to reach an energy of 60 keV (where the d-t cross section is a maximum) it is necessary to make a **plasma**. This is a very hot gas which is essentially an ion gas because all the electrons have been boiled off their respective nuclei. These temperatures are very hard to sustain and would melt any container which tried to confine the gas. Therefore a pulsed device is used which heats the plasma for short bursts and at the same time the plasma is confined with a pulsed magnetic field (this, at least, cannot burn!) A second method called inertial confinement has also been attempted. This confines the gas in mm sized balls which are pulsed by laser beams to heat them up under pressure. The engineering problems associated with controlled fusion are immense and a sustained fusion has not yet been achieved.

to acheieve temperature T in a d-t plasma, the energy that must be input to the plasma is given by:

$$E_{IN} = 4\rho_d(3k_BT/2)/\mathrm{unitvolume} \tag{186}$$

where ρ_d is the density of deuterium and tritium ions ($\rho_d = \rho_t$). The electron density is $2\rho_d$ and therefore there are $4\rho_d$particles/unit volume. The reaction rate in the plasma is

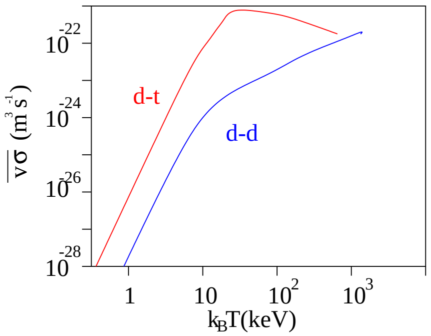

Figure 73: Relative cross-sections for d-d and d-t fusion

$\rho_d^2 \bar{\sigma v}$. If the plasma is confined for time t_c, then the ratio of (fusion energy out)/(energy out), is given by

$$\begin{aligned} & \frac{\rho_d^2 \bar{\sigma v} t_c (17.6 MeV)}{6 \rho_d k_B T} && (187) \\ \approx \; & 10^{-19} m^3 s^{-1} \rho_d t_c && (188) \\ & && (189) \end{aligned}$$

and this is clearly an inefficient process. A 'useful' device would need (fusion energy out)/(energy out)>1, and for this to be true,

$$\rho_d t_c > 10^{19} m^{-3} s$$

that is, the number desnity has to be very high, or the length of time the confinement lasts must be long. This is known as the *Lawson Criterion.*

28 LECTURE 29

28.1 Nucleosynthesis in Stars

When all the Hydrogen in a star is 'burned', the star will contract. The increased pressure caused by this contraction under gravity will increase the temperature until the Helium is ignited. After the Helium, further stages of nuclear burning set in until Iron and Nickel are formed. At each stage, the higher temperature needed to overcome the higher Coulomb barrier is provided by gravity. Most of the energy, however, is released in the hydrogen burning to Helium, in fact a massive 7.1MeV/nucleon. There is only a further 1.7MeV per nucleon to be released in the complete burning to Iron. Eventually the star becomes a neutron star and a supernova explsion occurs. It is thought that during neutron star formation the heavy elements such as Uranium are formed.

28.2 Resonance Enhanced Neutron Capture for Waste Transmutation

We have seen that one of the failings of the nuclear reactor as a power generator is the very long lived radioactive waste which must be stored for many years away from civilization. The idea of Resonance Enhanced Neutron Capture for Waste Transmutation is to bombard radioactive waste with neutrons produced in an accelerator to turn transuranic elements(TUR) and long lived Fission Fragments(FF) into less toxic and more importantly, stable elements. This process makes use of the resonant capture by FF and TUR at certain specific neutron energies.

The radioactive waste is made up of TUR(1%), FF(4%) and Fuel Cladding(95%). TUR is the most dangerous, but this can be induced to fission and used in the reactor itself. The FF can be separated into stable, short and long lived parts (for example ^{90}Sr and ^{137}Cs have a half life of about 30 years). The fuel caldding does not present a danger and can be disposed of simply.

In order to create the exactly correct energy for the neutrons for a given resonant capture, the choice of the medium which the FF is prepared in is of paramount importance. The medium must be transparent to neutrons (i.e. cannot absorb neutrons itself) but must be able to slow the neutrons to the correct energy by either:

- undergoing many inelastic scatters such as happens in D_2O or graphite which quickly makes the neutrons thermal (energies of 1eV or less)
- undergoing many *elastic* scatters with the nuclei in the medium of Pb or some other heavy nuclei which slowly reduces the neutron energy by a given amount per collision and thus sweeps out the entire energy spectrum from original energy to thermal

It is indeed fortuitous that adding an extra neutron to a long lived FF can turn it into either a short lived isotope or even a completely stable element.

We can look at one example: that of ^{99}Tc (Technecium) which is one of the worst offenders of radioactive waste. Technecium is soluble in water with a half life of $10\dot{1}0^5$ and is retained in the stomach,blood, salive and the thyroid gland. This obviously presents a hazard to the biological life in the vicinity of the stroed nuclear waste, especially if it is under water, but ^{99}Tc has a large resonant cross section for neutron capture leading to the completely stable isotope ^{100}Ru. The energies for resonant neutron capture are shown in Figure 74.

The number of neutrons needed to convert the huge amounts of radioactive waste is very large and the best sources of neutrons are either those from a spallation source (this is where protons produced in an accelerator knock out neutrons from a particular target) or from reactors themselves. However, because the neutron energy must be carefully matched, the waste products must be prepared in their appropriate medium and cannot be just left in the fuel rods to capture the neutrons. It is estimated that the power requirements to convert the waste would be about 10% of the electricity produced by a light water reactor.

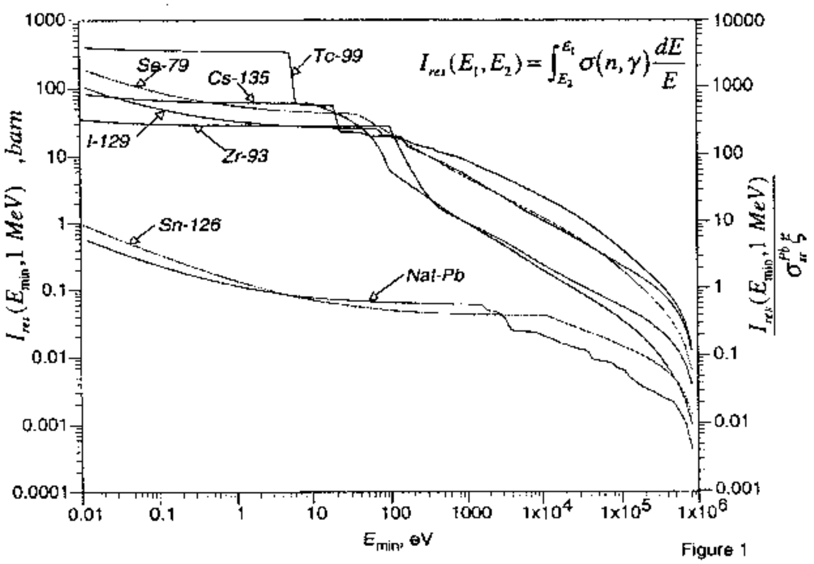

Figure 74: resonance intgeral for elements produced in reactors. A sudden drop in I_{res} indicates a resonant capture at a particular neutron energy.

This is obviosuly a great idea, but the equipment is expensive and so would require a certain capital investment before use could be made of a parasitic waste transmutor running off the reactor itself. Furthermore, the engineering is in a very early stage. This must represent a huge growth area in the next decade.

29 LECTURE 30

29.1 The Nuclear Shell Model

The basic premise of the shell model of nuclei is that a single nucleon moves in a common potential $V(r)$ which is a combination of the effect of all the other nucleons and that the interaction between nucleons can be viewed as a small perturbation. This allows for the possibility that nucleons act as if they are bound in angular momentum shells around a central potential, much like electrons are bound around the central nucleus in an atom. The central potential in the case of nuclei is not an identifiable object but just the effective potential of all the nucleons. The striking evidence which forced this view of the nucleus to the forefront was the existence of **magic numbers**. We have seen that nuclei with even numbers of nucleons are more stable than those with an odd number. It was found that special numbers of protons or neutrons form particularly stable configurations. These are when either Z or N is equal to:

2,8,14,20,28,50,82,126

and these numbers are called the magic numbers. There is a plethora of different evidence which backs up this claim that magic numbers do indeed exist, but it is not really relevant to the discussion. For more information about this see Williams.

The shell models' validity is very questionable and so it is rather surprising (and so far unexplained) that this model can be used successfully to explain the magic number phenomona and also more detailed properties such as the spins, magnetic moments and level spectra of many nuclei.

Start by assuming that the common potential $V(r)$ is an oscillator potential of the form $V=-V_0+ar^2$. The levels of this potential are

(1s),(2p),(3d)=(2s),(4f)=(3p),(5g)=(4d)=(3s)......

and each group of degenerate levels is a shell. Figure 75 shows the effective potential V_{eff}:

$$V_{eff}=-V_0+cr^2+\frac{l(l+1)}{2mr^2} \tag{190}$$

and the states below V_{eff}=0 are bound states. The protons and neutrons live *independently* from each other and so there are two separate shell structures, one for neutrons and one for protons. There is only this shell structure because protons and neutrons are spin $\frac{1}{2}$ and therefore obey Fermi-Dirac statistics.

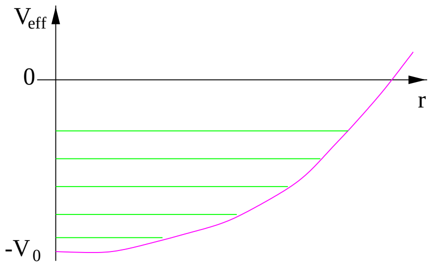

Figure 75: Effective potential for the nuclear shell model

Oscillator shells	l	l_j	from shell	to shell
I	0	$s_{\frac{1}{2}}$	2	2
II	1	$p_{\frac{3}{2}}, p_{\frac{1}{2}}$	6	8
IIa	2,0	$d_{\frac{5}{2}}$	6	14
III	2,0	$d_{\frac{3}{2}}, s_{\frac{1}{2}}$	6	20
IIIa	3,1	$f_{\frac{7}{2}}$	8	28
IV	3,1	$f_{\frac{5}{2}}, p_{\frac{3}{2}}, p_{\frac{1}{2}}, g_{\frac{9}{2}}$	22	50
V	4,2,0	$g_{\frac{7}{2}}, d_{\frac{5}{2}}, d_{\frac{3}{2}}, s_{\frac{1}{2}}, h_{\frac{11}{2}}$	32	82
VI	5,3,1	$h_{\frac{9}{2}}, f_{\frac{7}{2}}, f_{\frac{5}{2}}, p_{\frac{3}{2}}, p_{\frac{1}{2}}$	44	126

Table 9: : Oscillator Shells

In order to predict the correct magic numbers, it is neccessary to take into account the spin-orbit coupling. It is J which is conserved:

$$J^2 = (\bar{L} + \bar{S})^2 \quad (191)$$
$$= \bar{L}^2 + \bar{L}^2 + 2\bar{L} \cdot \bar{S} \quad (192)$$

29.2 References and Acknowledgements

In the course I have drawn heavily from a few books. I have tried to approach the particle physics from a Gauge theory perspective and have been guided by Halzen and Martin 'Quarks and Leptons' by Wiley. This book was rather advanced for this course however, and the theory had been substantially simplified by Perkins 'Introduction to High Energy Physics' which was heavily utilized for a few lectures. Finally, the compendium of all of nuclear physics of Blatt and Weisskopf 'Theoretical Nuclear Physics' by Dover, although much of it too detailed for this course, was useful but Cottingham and Greenwood 'An introduction to nuclear physics' by Cambridge University Press is an excellent summary of the nuclear physics useful in the modern day.