Intercollegiate post-graduate course in High Energy Physics

Paper 1: The Standard Model

Monday, 28 January 2008

Time allowed for Examination: 3 hours

Answer ALL questions

Books and notes may be consulted
The Standard Model

Question 1 (4 marks)

At a collider, two high energy particles, A and B with energies \(E_A\) and \(E_B\), which are much greater than their rest masses, collide head on. Derive the expression for the centre-of-mass energy.

Using this expression, what would be the centre-of-mass energy of a proposed future facility (“LHeC”) which will collide 7 TeV protons with 70 GeV electrons?

Now consider particle B (the proton) to be at rest. Derive the formula for the centre-of-mass energy of such a fixed-target experiment.

What electron beam energy would be required in the fixed-target experiment in order to achieve the same centre-of-mass energy as in the proposed LHeC facility?

Question 2 (12 marks)

Consider the Compton scattering of a photon, \(k = (\omega, \vec{k})\), off a stationary electron, \(p = (m, \vec{0})\). The photon is scattered through an angle \(\theta\) and the four momenta of the final state particles are \(k' = (\omega', \vec{k'})\) and \(p' = (E', \vec{p'})\) for the photon and electron respectively. Derive the Compton shift relation

\[
\lambda' - \lambda = 2\lambda_c \sin^2(\theta/2)
\]

where \(\lambda = 2\pi/\omega\), \(\lambda' = 2\pi/\omega'\) and \(\lambda_c = 2\pi/m\).

Draw the leading order Feynman diagrams for Compton scattering and state whether they are s, t or u channel.

Part of the trace calculation for evaluating the cross section involves

\[
A = \frac{1}{4} Tr \left[ \epsilon' \epsilon \left( p + m \right) \gamma^\mu \gamma^\nu \left( p' + m \right) \right],
\]

where \(\epsilon\) and \(\epsilon'\) are the photon’s initial and final polarization four vectors. In a gauge for which \(p \cdot \epsilon = p \cdot \epsilon' = 0\), show that

\[
A = 2\epsilon^2 k \cdot p \left[ 2 (\epsilon' \cdot k)^2 - \epsilon'^2 (k' \cdot p) \right].
\]

Trace theorems for \(\gamma\) matrices need not be derived, but should be quoted. Note that \(\phi \phi^\dagger = 2a \cdot b - a^2 b\).
Question 3 (10 marks)

Draw the leading order Feynman diagram for electron-muon scattering, $e^-(k) + \mu^-(p) \to e^-(k') + \mu^-(p')$, where the four momenta are indicated in the reaction. [1]

Simplify the expression for the transition amplitude:

$$|T_{fi}|^2 = \frac{e^4}{4q^4} \text{Tr} \left[ (\not{k}' + m)(\not{k} + m)\gamma_\mu \right] \cdot \text{Tr} \left[ (\not{p}' + M)(\not{p} + M)\gamma^\nu \right]$$

such that the Traces are removed, assuming the high-$s$ limit of zero masses. Trace theorems for $\gamma$ matrices need not be derived, but should be quoted. [4]

Use the cross-section definition (in the centre-of-mass system):

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{p_f}{p_i} |T_{fi}|^2,$$

where $p_f$ and $p_i$ are the final and initial three-momenta, to derive the cross section.

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \left( 1 + \cos^2 \theta \right),$$

where $\alpha$ is the fine structure constant and $\theta$ is the angle between the $e^-$ and $\mu^-$. [5]

Question 4 (4 marks)

The branching ratios for $D^+ \to K^0\pi^+$ and $D^+ \to K^+\pi^0$ are very different, viz. 1.47% and 2.37E-4. Assuming the simple spectator model, draw diagrams for the two decays. [2]

Give a reasoning for some of the difference in rate. [2]

Question 5 (6 marks)

State what is meant by local and global gauge transformations. [2]

From the Lagrangian

$$\frac{1}{8} \left[ g_W(v+h)^2(W^1_{\mu} - iW^2_{\mu})(W^1_{\mu} + iW^2_{\mu}) - (v+h)^2(g'B_{\mu} - gwW^3_{\mu})(g'B^\mu - gwW^3_{\mu}) \right]$$

derive the ZZH and ZZHH couplings. (Simplify your answer to remove any dependency on $v$.) [4]
Question 6 (6 marks)

For $\sqrt{s} = 35$ GeV, what would you expect the value of

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

to be when considering only the EM coupling? At what higher energy would you expect the value to change? \[3\]

Draw a higher-order diagram (i.e. consideration of the strong force) which would affect this value. \[1\]

Briefly describe how such higher-order diagrams led to the discovery of the gluon. \[2\]

Question 7 (7 marks)

The amplitude for the decay $\pi^-(q) \rightarrow \mu^-(p) + \bar{\nu}_\mu(k)$ is given by:

$$|T_{fi}|^2 = \frac{G_F^2}{2} f_{\pi}^2 \cos^2 \theta_c m_\mu^2 \text{Tr} \left[ (\not{p} + m_\mu)(1 - \gamma^5)\not{k}(1 + \gamma^5) \right]$$

Use Trace theorems to show this simplifies to

$$|T_{fi}|^2 = 4G_F^2 f_{\pi}^2 \cos^2 \theta_c m_\mu^2 (p \cdot k)$$ \[4\]

The ratio of decay rates:

$$R = \frac{\Gamma(K^- \rightarrow e^- + \bar{\nu}_e)}{\Gamma(K^- \rightarrow \mu^- + \bar{\nu}_\mu)}$$

can be written in terms of the particle masses. Use this relation to give the value to 2 decimal places showing that the rate is close to that measured from experiment, $\sim 2.44 \times 10^{-5}$. \(m_e = 0.511 \text{ MeV}, m_\mu = 105.7 \text{ MeV}, m_K = 493.7 \text{ MeV}\) \[3\]
Question 8 (12 marks)

Draw the Feynman diagrams of the two leading order (in $\alpha_s$) processes in deep inelastic $ep$ scattering. [2]

The photon emitted from the electron can also be sometimes considered to have a structure, by fluctuating into a pair of quarks. Draw an example Feynman diagram of these so-called “resolved” photon processes. [2]

In this way, an $ep$ collider can also be thought of as a hadron collider. Draw all Feynman representations, including the initial hadrons and their products, for the hard scatters, \( qq' \to qq' \) and \( qq \to qq \). [4]

Write down the forms of the (partonic) cross sections for \( qq' \to qq' \) and \( qq \to qq \) in terms of the Mandelstam variables, \( s \), \( t \) and \( u \), associating each term with the relevant Feynman diagram. [4]

Question 9 (5 marks)

Contrast the advantages and disadvantages of $e^+e^-$ and $pp$ colliders. Use two headline measurements or major discoveries to justify your answer. [5]

Question 10 (6 marks)

What property of the EM interaction means that photons do not self-couple? [1]

Draw a Feynman diagram of a process at the LHC in which three gluons couple at one vertex. [2]

Explain briefly why the QCD coupling, $\alpha_s$, has a different behaviour with the scale, $Q^2$, compared to that of the QED coupling, $\alpha$. [3]

Question 11 (6 marks)

Draw a Feynman diagrams for each of neutral current and charge current deep inelastic scattering at HERA. [2]

Draw a sketch of how their cross sections vary with $Q^2$ and explain the features. [2]

The neutral current cross section is sensitive to all quarks in the proton. Which quarks are the charge current cross section for (a) $e^+p$ and (b) $e^-p$ sensitive to? [2]