Brunel University
Queen Mary, University of London
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Intercollegiate post-graduate course in High Energy Physics

Paper 1 : The Standard Model

Friday, 29 January 2010

Time allowed for Examination : 3 hours

Answer 6 from 8 questions

Books and notes may be consulted

Question 1 (20 marks)

For a four-momentum, p_μ, show that $p_\mu p^\mu$ is a Lorentz invariant, by considering a Lorentz transformation along a spatial axis of your choice. **[5]**

At a collider, two high energy particles, A and B with energies E_A and E_B, which are much greater than their rest masses, collide head on. Derive the expression for the centre-of-mass energy. Using this expression, what would be the centre-of-mass energy of a proposed future facility ("LHeC") which is supposed to collide 7 TeV protons with 70 GeV electrons ? Now consider particle B (the proton) to be at rest. Derive the formula for the centre-of-mass energy of such a fixed-target experiment. What electron beam energy would be required in the fixed-target experiment in order to achieve the same centre-of-mass energy as in the proposed LHeC facility ?

[5]

Obtain the relation for the centre-of-mass energy in electron–neutrino scattering

$$s = m_e(2E_\nu + m_e)\,.$$

[3]

A particle of mass M decays into two particles with masses m_1 and m_2. Determine the energies of the decay products in the rest frame of the parent particle. **[5]**

Hence write down, in terms of masses and the centre-of-mass energy, the energy in the rest frame of particle A in a scattering, $AB \to CD$. **[2]**

Question 2 (20 marks)

Given the cross section for the scattering of an electron from a fixed Coulomb potential of point charge Ze :

$$\frac{d\sigma}{d\Omega} = \frac{2(Z\alpha)^2 m^2}{|\mathbf{q}|^4} \mathrm{Tr}\left[\gamma_0 \frac{\not{p}_i + m}{2m} \gamma_0 \frac{\not{p}_f + m}{2m}\right] ,$$

where p_i and p_f are the initial and final momenta and $\mathbf{q} = \mathbf{p}_f - \mathbf{p}_i$, determine the Mott cross section :

$$\frac{d\sigma}{d\Omega} = \frac{(Z\alpha)^2}{4(\gamma\beta^2)^2 (mc^2)^2 \sin^4 \theta/2} \left(1 - \beta^2 \sin^2 \frac{\theta}{2}\right) .$$

Trace theorems used should be explicitly stated. **[15]**

Show that in the non-relativistic limit

$$\frac{d\sigma}{d\Omega} = \frac{(Z\alpha)^2}{16E^2 \sin^4 \frac{\theta}{2}}$$

and in the extreme-relativistic limit

$$\frac{d\sigma}{d\Omega} = \frac{(Z\alpha)^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}} .$$

[5]

Question 3 (20 marks)

Evaluate, in terms of the four-vectors (you do not have to convert to the Mandelstam variable),

$$\mathrm{Tr}\left[\gamma^\mu \not{k} \gamma^\nu (\not{p} + m)\gamma_\nu \not{k}' \gamma_\mu (\not{p}' + m)\right]$$

and

$$\mathrm{Tr}\left[\gamma^\mu \not{k} \gamma^\nu (\not{p} + m)\gamma_\mu \not{k}' \gamma_\nu (\not{p}' + m)\right] .$$

that occur in the calculation of electron-photon scattering. Trace theorems and identities for γ matrices need not be derived, but should be quoted. **[10]**

In a massless limit, the terms in final squared transition amplitude for Compton scattering are

$$\text{(a)}\ \ 2e^4\left(-\frac{u}{s}\right) \qquad \text{(b)}\ \ 2e^4\left(-\frac{s}{u}\right) \qquad \text{(c)}\ \ 2e^4\frac{t}{us}\left(s+u+t\right) .$$

Identify the Feynman diagram(s) which contribute to each term. Hence write down the final squared transition amplitudes when the incoming photon is real and when it is virtual. **[5]**

Given the Compton condition

$$\lambda' = \lambda + \frac{2\pi}{m}(1 - \cos\theta)$$

and the Klein-Nishina formula for Compton scattering

$$\frac{d\sigma}{d\Omega}(\lambda, \lambda') = \frac{\alpha^2}{4m^2}\left(\frac{\omega'}{\omega}\right)^2\left[\frac{\omega'}{\omega} + \frac{\omega}{\omega'} + 4(\epsilon'^* \cdot \epsilon)^2 - 2\right]$$

determine the cross section in the low-energy limit, i.e. $\omega \to 0$, in terms of the fine structure constant, α, the mass of the electron, m, and the polarisation vectors of the photon, ϵ and ϵ'^*. **[5]**

Question 4 (20 marks)

What property of the EM interaction means that photons do not self-couple ? **[2]**

Explain the four terms in the Lagrangian of QED :

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi + e\bar{\psi}\gamma_\mu A^\mu \psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

[4]

Briefly explain the concept of a "running" coupling constant in QED where the variation is with the scale of the process, Q^2. Draw two Feynman diagrams, one for QED and and for QCD, to illustrate the effect. And draw a diagram which leads to the QCD coupling having a different dependence. **[5]**

State what are meant by global and local gauge transformations. **[2]**

Given the phase transformations of the wave function and the electromagnetic field :

$$\phi(x) \rightarrow \phi'(x) = \exp(iq\alpha)\phi(x) \qquad A^\mu(x) \rightarrow A'^\mu(x) = A^\mu(x) - \partial^\mu \alpha(x)$$

and the gauge-covariant derivative,

$$\partial^\mu \rightarrow D^\mu = \partial^\mu + iqA^\mu$$

show that the Klein-Gordon equation is invariant under these transformations. **[7]**

Question 5 (20 marks)

In the decay of a π^- at rest, $\pi^- \to e^- + \bar{\nu}_e$, show that

$$\frac{1}{2}\left(1 - \frac{v_e}{c}\right) = \frac{m_e^2}{m_\pi^2 + m_e^2} \, .$$

where v_e is the velocity of the electron. **[5]**

To lowest order, the partial decays rate for pions are :

$$\frac{1}{\tau(\pi \to e\bar{\nu}_e)} = \frac{\alpha_\pi^2}{4\pi}\left(1 - \frac{v_e}{c}\right) p_e^2 E_e \, , \qquad \frac{1}{\tau(\pi \to \mu\bar{\nu}_\mu)} = \frac{\alpha_\pi^2}{4\pi}\left(1 - \frac{v_\mu}{c}\right) p_\mu^2 E_\mu \, .$$

where α_π is an effective coupling constant and E_e, E_μ and p_e, p_μ are the charged lepton's energy and momentum. Hence show :

$$\frac{\tau(\pi \to \mu\bar{\nu}_\mu)}{\tau(\pi \to e\bar{\nu}_e)} = \frac{m_e^2(m_\pi^2 - m_e^2)^2}{m_\mu^2(m_\pi^2 - m_\mu^2)^2} \, .$$

[5]

Use the analogue of the above equation for the decay of the K^- to estimate the ratio

$$\frac{\tau(K \to \mu\bar{\nu}_\mu)}{\tau(K \to e\bar{\nu}_e)}$$

and compare with the observed value $(2.4 \pm 0.1) \times 10^{-5}$.
Given the lifetimes $\tau(K \to \mu\bar{\nu}_\mu) = 1.948 \times 10^{-8}\,\mathrm{s}$ and $\tau(\pi \to \mu\bar{\nu}_\mu) = 2.603 \times 10^{-8}\,\mathrm{s}$, estimate α_K/α_π .
($m_K = 493.67\,\mathrm{MeV}$, $m_\pi = 139.57\,\mathrm{MeV}$, $m_\mu = 105.66\,\mathrm{MeV}$, $m_e = 0.511\,\mathrm{MeV}$.) **[5]**

Draw quark model diagrams for the decays $\pi^- \to \mu^- + \bar{\nu}_\mu$ and $K^- \to \mu^- + \bar{\nu}_\mu$, stating which element of the CKM matrix is involved in each.
Neglecting masses, the ratio of the CKM elements is equal to α_K/α_π. Hence estimate $\sin\theta_{12}$.
[5]

Question 6 (20 marks)

In deep inelastic scattering at HERA, the four-momenta of the incoming and scattered electron are $(E, \mathbf{p})$ and $(E', \mathbf{p}')$, respectively. Show that the square of the four-momentum transfer is given by

$$Q^2 = 4EE' \sin^2 \frac{\theta}{2}$$

where θ is the angle of the scattered electron. And show that the mass of the proton is related to the energies by

$$M(E - E') - 2EE' \sin^2 \frac{\theta}{2} = 0\,.$$

[6]

Give a physical description of the kinematic variables, x, y and Q^2, which describe deep inelastic scattering. **[4]**

The electron-quark cross section is :

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 \hat{s}} 2e^4 q_i^2 \left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \right] .$$

Show that this can be written in a more useful form as :

$$\frac{d\sigma}{dy} = \frac{2\pi\alpha^2}{Q^4} q_i^2 s \left[1 + (1-y)^2 \right] .$$

[6]

The cross section for the QCD Compton and Boson-gluon fusion processes are :

$$\frac{d\sigma}{d\Omega} \sim -\frac{\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} + \frac{2\hat{u}Q^2}{\hat{s}\hat{t}}$$
$$\frac{d\sigma}{d\Omega} \sim \frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - \frac{2\hat{s}Q^2}{\hat{t}\hat{u}} ,$$

respectively. State where (e.g. with reference to Feynman diagrams or the Mandelstam channel) each term comes from. **[4]**

Question 7 (20 marks)

A possible decay of the W boson with associated four-momenta is

$$W(q) \to \mu(p) + \nu_\mu(k) \,.$$

The transition amplitude squared is :

$$|T_{\text{fi}}|^2 = \frac{g_W^2}{3}\left[p \cdot k + \frac{2}{M_W^2}(q \cdot k)(q \cdot p)\right] .$$

In the W rest frame, show that :

$$q \cdot k = \frac{M_W^2}{2}\left(1 - \frac{m_\mu^2}{M_W^2}\right) \quad q \cdot p = \frac{M_W^2}{2}\left(1 + \frac{m_\mu^2}{M_W^2}\right) \quad p \cdot k = \frac{M_W^2}{2}\left(1 - \frac{m_\mu^2}{M_W^2}\right) .$$

[10]

Then using the relationship :

$$\Gamma = \frac{1}{16\pi M_W}|T_{\text{fi}}|^2$$

derive the partial width in terms of the masses of the W and μ and show that this can be simplified to

$$\Gamma = \frac{G_F}{\sqrt{2}}\frac{M_W^3}{6\pi} .$$

($m_\mu = 105.66\,\text{MeV},\ M_W = 80.4\,\text{GeV}$) **[10]**

Question 8 (20 marks)

In the Weinberg-Salam electroweak theory, the vacuum potential, boson masses, couplings, and mixing angle are related by :

$$M_Z = \frac{M_W}{\cos\theta_W}\,, \qquad \frac{g'}{g_W} = \tan\theta_W\,, \qquad M_W = \frac{v g_W}{2}$$

Hence show that

$$M_Z = \frac{1}{2} v \sqrt{g_W^2 + g'^{\,2}}\,.$$

[4]

From the Lagrangian

$$\frac{1}{8}\left[g_W^2 (v+h)^2 (W_\mu^1 - iW_\mu^2)(W_\mu^1 + iW_\mu^2) - (v+h)^2 (g' B_\mu - g_W W_\mu^3)(g' B^\mu - g_W W_3^\mu)\right]$$

derive the WWH and $WWHH$ couplings and the ZZH and $ZZHH$ couplings. (Simplify your answer to remove dependencies on both v and g'.) **[9]**

Given Fermi's constant, $G_F = 1.16637 \times 10^{-5}\,\mathrm{GeV}^{-2}$, calculate the value of v. **[4]**

The Higgs Boson was searched for in e^+e^- collisions at LEP. Draw the Feynman diagram of the process for Higgs production. Masses of up to about 114 GeV were ruled out; hence give the approximate maximum centre-of-mass energy of LEP. **[3]**

[Total Marks = 120]

END OF PAPER