PH4442 - Problem Sheet 5

(Answers should be returned on 14/03/2006)

1. (Relativistic kinematics) In the charged pion decay, $\pi^- \to \ell^- \bar{\nu}_\ell$, show that in the pion's rest frame

$$E_\ell = \frac{m_\pi^2 + m_\ell^2}{2m_\pi} \qquad p_\ell = \frac{m_\pi^2 - m_\ell^2}{2m_\pi} \qquad 1 - \beta_\ell = \frac{2m_\ell^2}{m_\pi^2 + m_\ell^2},$$

   where E_ℓ the energy, p_ℓ the magnitude of the momentum, and β_ℓ the velocity of the charged lepton.

2. (From the 2004 exams) The CKM unitary matrix gives the flavour-dependent relative couplings for the charged-current weak interactions for quarks, where V_{ij} is the factor for interactions involving quarks i and j. The numerical values of the magnitudes of the matrix elements can be taken to be

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.975 & 0.223 & 0.003 \\ 0.222 & 0.974 & 0.040 \\ 0.009 & 0.039 & 0.999 \end{pmatrix}.$$

   (a) The tau lepton has a mass of $1.78\,\text{GeV}$ and a lifetime of 0.29×10^{-12}s. It can decay semi-hadronically: to either a tau neutrino and one or more pions; or to a tau neutrino, a kaon and zero or more pions. Examples of these two types of decay are $\tau^- \to \nu_\tau \pi^-$ and $\tau^- \to \nu_\tau K^-$, respectively. Draw a quark level Feynman diagrams for each of these particular example decays. Ignoring mass effects, estimate their relative rates.

   (b) The only other decays of the tau are leptonic: $\tau^- \to \nu_\tau e^- \bar{\nu}_e$ or $\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu$. Ignoring mass effects, estimate the branching fractions for both of these decays and also for the two types of semi-hadronic decays described in part (a).

   (c) The charmed meson D^+ (quark content $c\bar{d}$) has a measured lifetime of 1.05×10^{-12}s. Draw a quark level Feynman diagram for the most common hadronic decay of this meson.

   (d) The total width of the tau is proportional to m_τ^5. Using this, and assuming asymptotic freedom holds for the D^+ decay ("spectator model"), estimate the D^+ lifetime and compare with the above value. The mass of the charm quark can be taken to be $1.4\,\text{GeV}$.