Weak Measurements:

A New Type of Quantum Measurement and its Experimental Implications.

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The Problem.

A very brief history.

It goes back to the Bohr-Einstein discussions.

What is the implication of the quantum formalism for our view of reality?

Bohm

Quantum particles follow well defined trajectories.

[Bohm, Phys. Rev. **85** (1952) 166-179; and **85** (1952) 180-193.]

February 10,1954

Professor David Bohm
University of Sao Paulo
Sao Paulo,Brazil

Dear Bohm:

I was very impressed by your letter of February 3rd which was brought to me by the Kahlers. I was really very happy to hear that Rosen is trying to call you there and I have already written to him. I am,of course, very glad to do everything to facilitate the realization of this plan; so do not hesitate to write me as soon as you see a possibility that I could help in the matter.

Your picture of your idyllic environment made a vivid impression upon me. I believe your report was exhausting with nothing left out.

I am glad that you are deeply immersed seeking an objecting description of the phenomena and that you feel that the task is much more difficult as you felt hitherto. You should not be depressed by the enormity of the problem. If God has created the world his primary worry was certainly not to make its understanding easy for us. I feel it strongly since fifty years.

With kind regards and wishes,

yours,

A. Einstein.

Albert Einstein.

Tuesday, 12 June 2012

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Nano-technology.

What are individual particles up to?

Uncertainty principle gets in the way?

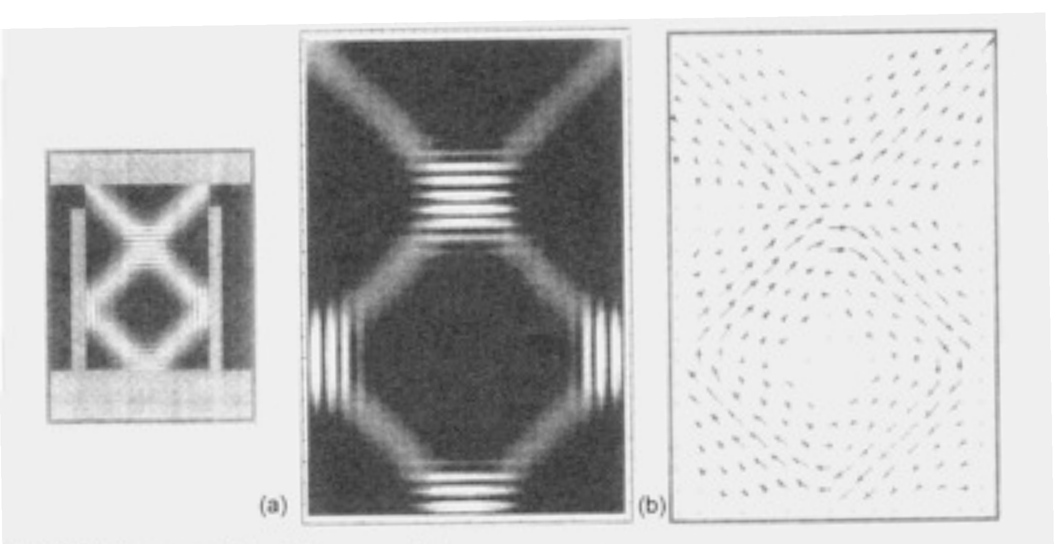


FIGURE 2 The computed flow of a long wave packet in an open rectangular dot. (a) The probability density; (b) the velocity field.

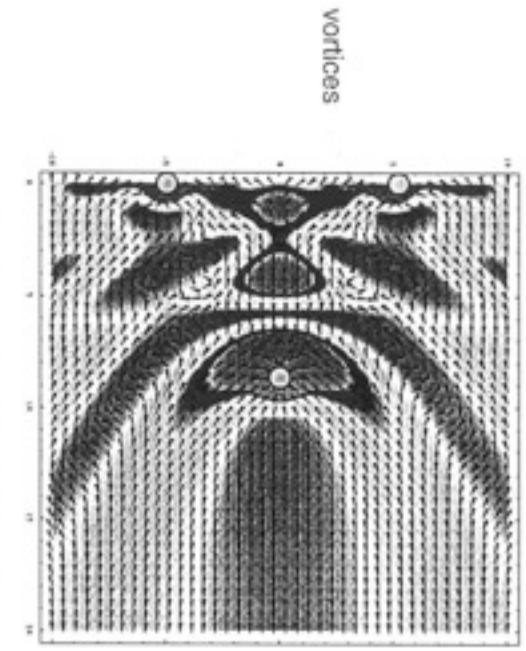


Figure 11: density landscape for potential scattering on three delta function impurities showing vortices.

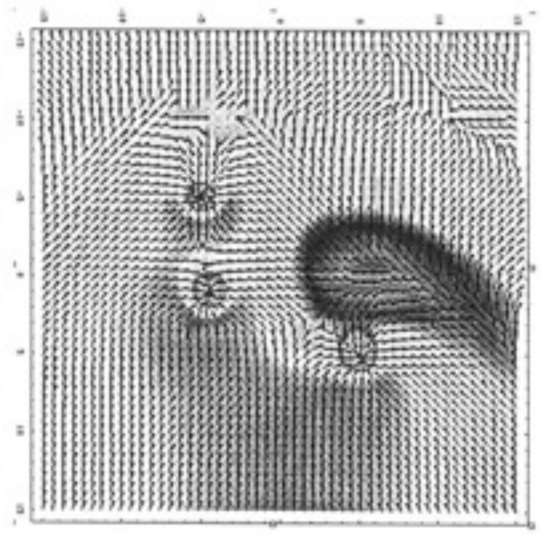


Figure 10: density landscape for potential scattering on three delta function impurities.

How to calculate quantum 'trajectories'.

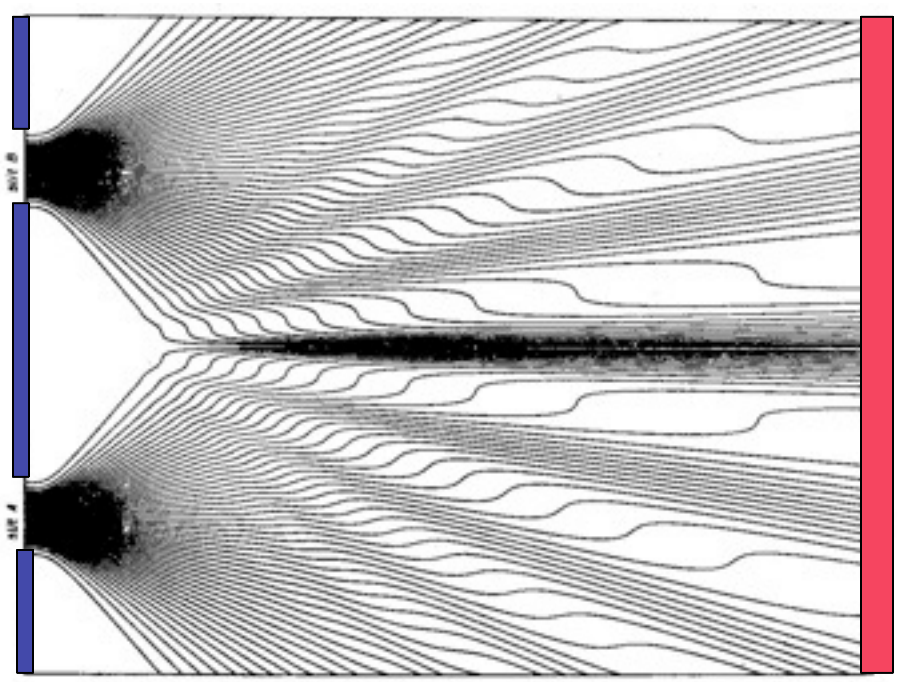
Insert the wave function $\psi(x,t) = R(x,t)e^{-iS(x,t)}$ into the Schrödinger equation.

Real part gives: $$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + Q + V = 0$$ Quantum Hamilton-Jacobi.

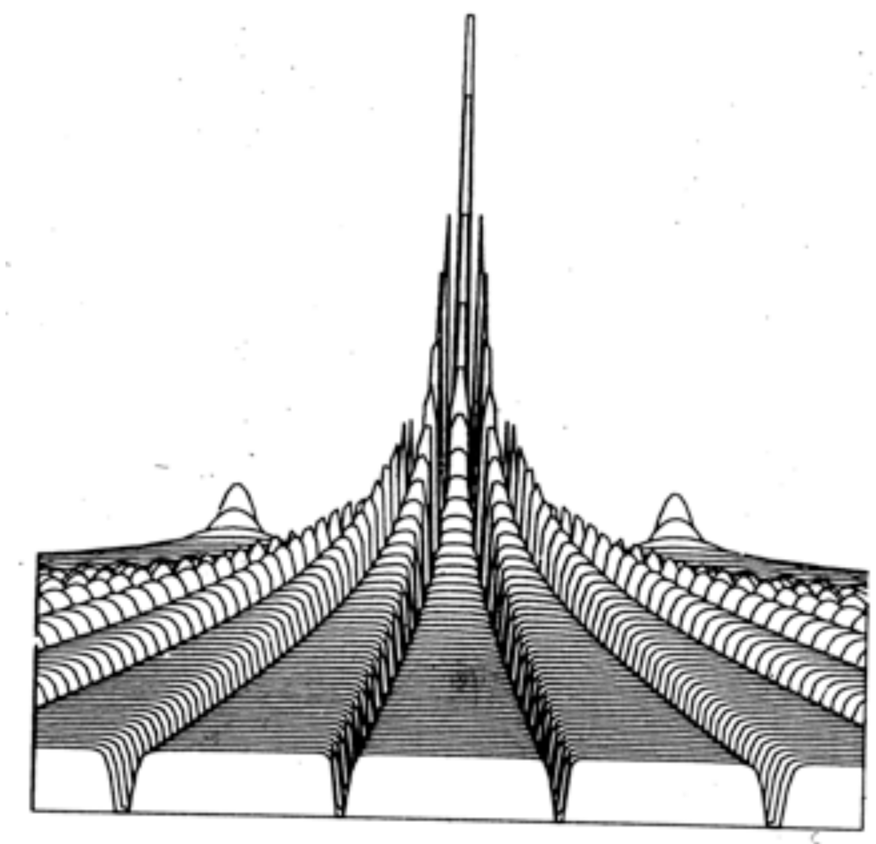
$$E_B = -\frac{\partial S}{\partial t} \qquad P_B = \nabla S$$

Quantum Potential = $Q = -\dfrac{\hbar^2}{2m}\dfrac{\nabla^2 R}{R}$

[Bohm & Hiley, The Undivided Universe, 1993]



Schrödinger trajectories



Quantum Potential

[Philippidis, Dewdney and Hiley, Nuovo Cimento **52B**, 15-28 (1979)]

Reactions!

Heisenberg:

"….especially some strange quantum potentials introduced ad hoc by Bohm."

Bohm is "trying to make $2 + 2 = 5$". [Heisenberg, Physics and Philosophy, p. 114, 1958.]

Pauli:

Bohm reports: "..they [Pauli] had made fun of it by calling it the "Kindergarten Theory" that the particle was like a little child who had to be guided."

[Bohm-Wilkins Tapes, AIP Arch.1986]

Rosenfeld:

"...that could protect them from the confusion created by Bohm, Landé, and other dilettantes…"

[Rosenfeld letter to Bohr, 14 Jan 1957, Arch. Hist. Q.P.,Am. Phil/. Soc.]

Bohm:

"I then went to talk to Einstein and he said, "Okay, it's alright," but he didn't like it because it seemed it did not go deep enough."

[Bohm-Wilkins Tapes, AIP Arch.1986]

Fermi: Replace wave function by geometric structure in phase space

$$g_F(x,p,t) = [p - \hbar\nabla\theta(x,t)]^2 + \hbar^2 \frac{\nabla^2\rho(x,t)}{\rho(x,t)} = 0$$

[Fermi, Nuo. Cim., **7**, (1930) 361-66]

Feynman:

In discussing superfluidity, he writes down the equation

$$\hbar\frac{\partial\theta}{\partial t} = -\frac{m}{2}v^2 + q\phi - \frac{\hbar^2}{2m}\left\{\frac{1}{\sqrt{\rho}}\nabla^2(\sqrt{\rho})\right\}$$

"...we could call a "quantum mechanical energy", but inside superconductor ρ is nearly constant so it can be neglected."

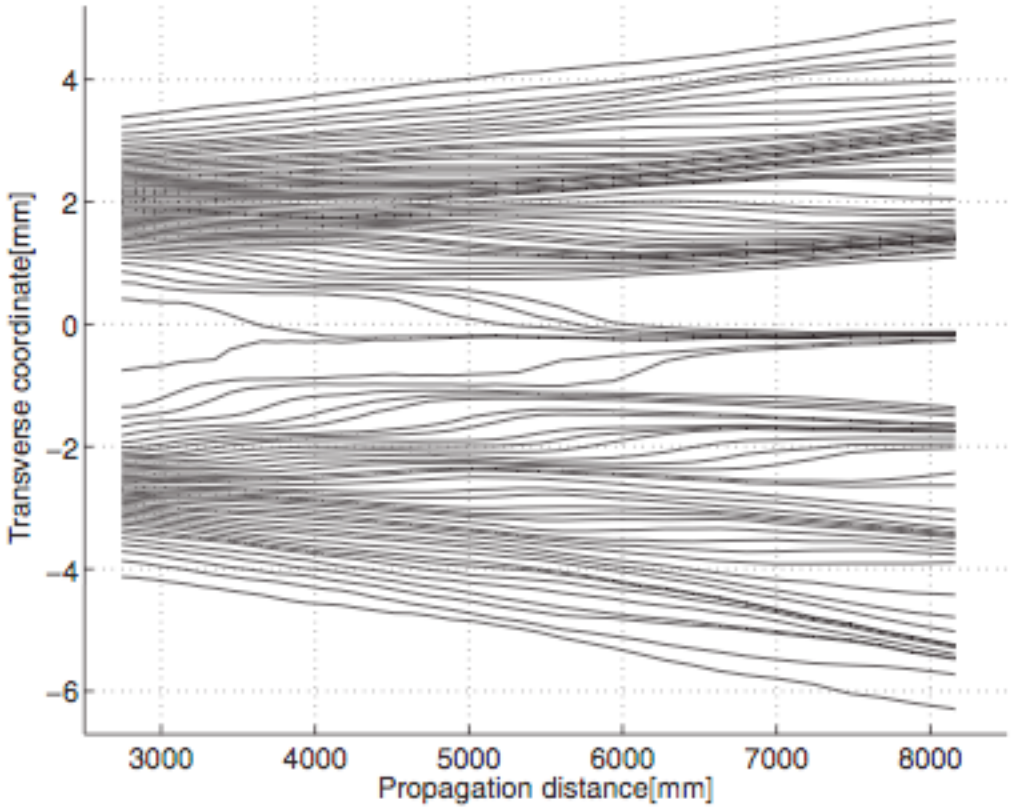
[The Feynman Lectures on Physics, III]

Messiah:

"…..the controversy has finally reached a point where it can no longer be decided by any further experimental observations; it henceforth belongs to the philosophy of science rather than to the domain of physical science proper."

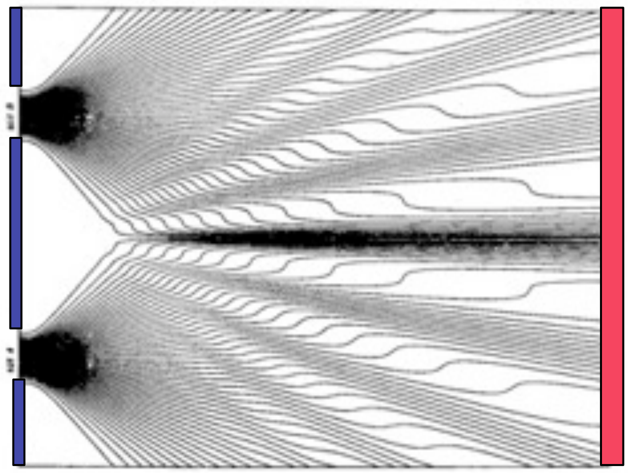
[Messiah, Quantum mechanics, vol I, p. 48, 1964]

Photon ‘trajectories’.



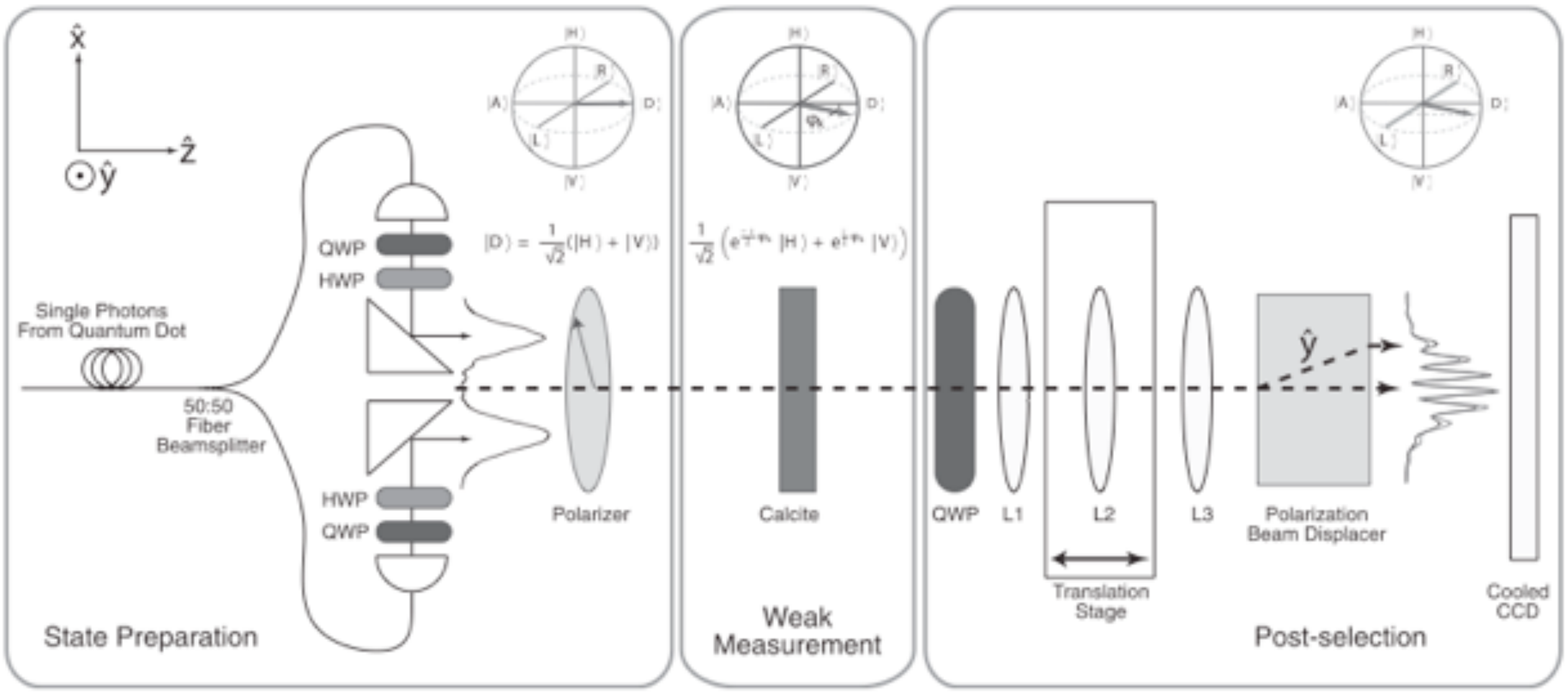
[Kocsis, Braverman,Ravets,Stevens,Mirin, Shalm, Steinberg, *Science* **332**, 1170 (2011)]

[Prosser, IJTP , 15, (1976) 169]



Experimental--Photons.

Actual experimental arrangement for photon trajectories.



Weak Measurement.

Measures **weak values**

[Aharonov and Vaidman, Phys. Rev. **41**, (1990) 11-19]

What is a weak value? $A_W = \dfrac{\langle\phi|A|\psi\rangle}{\langle\phi|\psi\rangle}$ N.B. $A_W \in \mathbb{C}$

How do they appear in the formalism?

[Hiley quant-ph/1111.6536]

$$\langle\psi|A|\psi\rangle = \sum \langle\psi|\phi_j\rangle\langle\phi_j|A|\psi\rangle$$

where $|\phi_j\rangle$ form a complete orthonormal set.

Then

Post select

$$\langle\psi|A|\psi\rangle = \sum \langle\psi|\phi_j\rangle \left(\frac{\langle\phi_j|\psi\rangle}{\langle\phi_j|\psi\rangle}\right) \langle\phi_j|A|\psi\rangle = \sum \rho_j \frac{\langle\phi_j|A|\psi\rangle}{\langle\phi_j|\psi\rangle}$$

Weak value.

Special case:

If $A|\phi_j\rangle = a_j|\phi_j\rangle$

$$\langle\psi|A|\psi\rangle = \sum \rho_j a_j$$

Well known result!

Eigenvalue!

Remember $\dfrac{\langle\phi|A|\psi\rangle}{\langle\phi|\psi\rangle}$ is a complex number. But what does it mean in general?

It is clearly a transition probability amplitude.

Weak Measurement: the principle.

$$|\mathrm{initial}\rangle = |\psi_s\rangle|\Psi_d\rangle \quad \rightarrow \quad H_I = \lambda(t)\hat{A}_s\hat{B}_d \quad \rightarrow \quad |\mathrm{final}\rangle = e^{i\hat{A}_s b}|\psi_s\rangle|\Psi_d\rangle$$

Where $\hat{B}|\Psi_d\rangle = b|\Psi_d\rangle$, $\int \lambda(t)dt = 1$

To find the weak value we post select with $\langle x, \phi_s|$ to form $T_{\phi-\psi} = \langle x, \phi_s|\mathrm{final}\rangle$

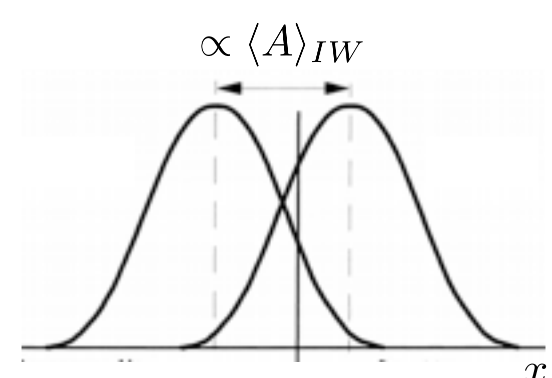
$$T_{\phi-\psi} = \langle\phi_s|\psi_s\rangle\left[e^{ix\langle A\rangle_W} + \underbrace{\sum_{n=2}\frac{(ix)^n}{n!}[\langle A^n\rangle_W - \langle A\rangle_W^n]}_{\text{small}}\right]\Psi_d(x)$$

[Duck, Stevenson and Sudarshan Phys. Rev, 40 (1989) 2112-7]

$$T_{\phi-\psi} = \langle\phi_s|\psi_s\rangle e^{ix\langle A\rangle_W}\Psi_d(x)$$

Choose $\Psi_d(x) = \exp\left[\frac{-x^2}{4(\Delta x)^2}\right]$ and take the imaginary part of $\langle A\rangle_W$ we find

$$T_{\phi-\psi} \propto e^{-x\langle A\rangle_{IW}}\exp\left[\frac{-x^2}{4(\Delta x)^2}\right] \propto \exp\frac{-\left[x + 2(\Delta x)^2\langle A\rangle_{IW}\right]^2}{4(\Delta x)^2}$$



Centre of Gaussian in x-space shifted by amount $\propto \langle A\rangle_{IW}$

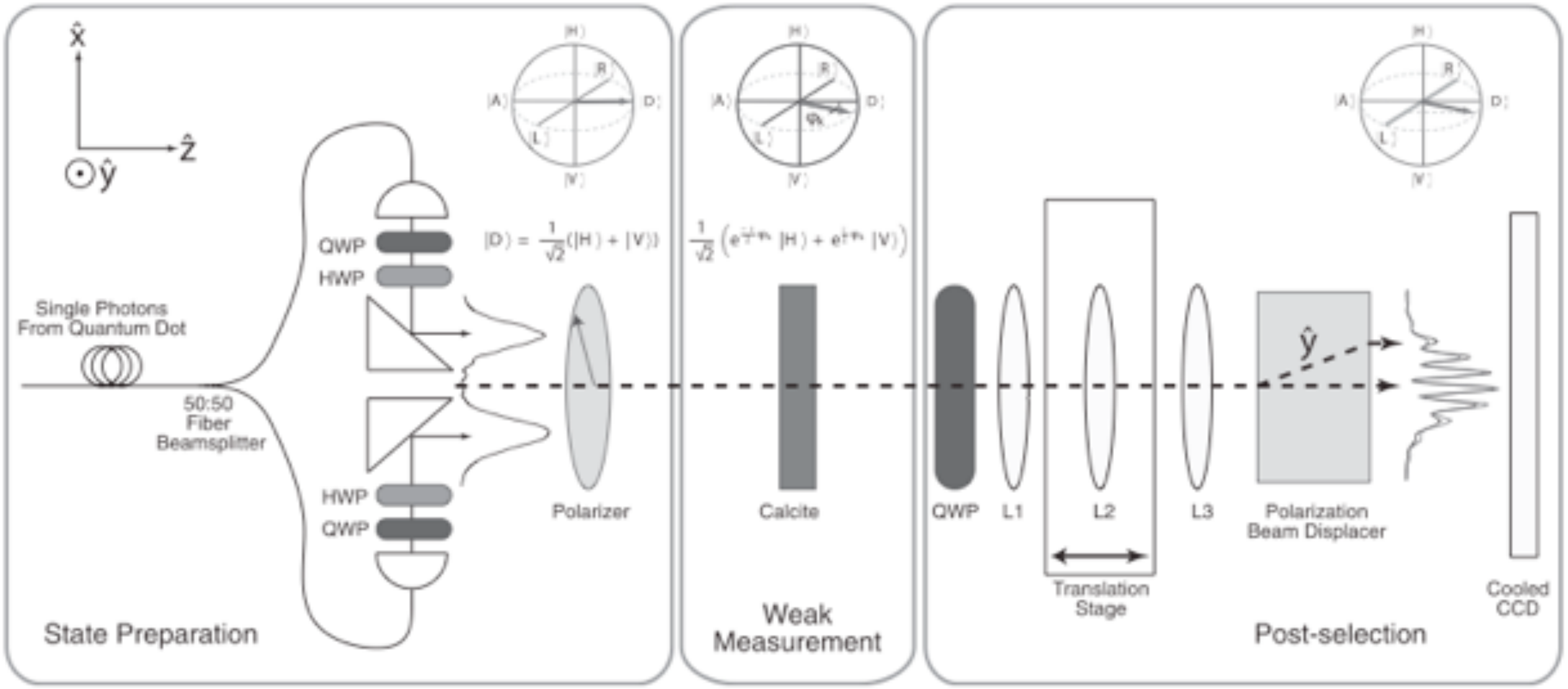
Centre of Gaussian in p-space shifted by amount $\propto \langle A\rangle_{RW}$

Statistical measurement.

Weak Experiment : the practice.

Actual experimental arrangement for photon trajectories.

[Kocsis, Braverman,Ravets,Stevens,Mirin, Shalm, Steinberg, *Science* **332**, 1170 (2011)]



$$|\text{initial}\rangle = |\psi_s\rangle|\Psi_d\rangle \qquad H = \lambda(t)\hat{P}_x\hat{S}_1 \qquad \hat{S}_1 = (|H\rangle\langle H| - |V\rangle\langle V|)/2$$

photon   polarisation

$$|\Psi'\rangle = e^{-i\hat{P}_x\hat{S}_1}|\Psi\rangle[\,|H\rangle + |V\rangle\,]$$

$$T_{x-\Psi'} \approx \left(e^{-i\langle P_x\rangle_W}|H\rangle + e^{i\langle P_x\rangle_W}|V\rangle\right)\Psi_d(x)$$

$$\hat{S}_3 = |R\rangle\langle R| - |L\rangle\langle L|$$

$$\langle\hat{S}_3\rangle = \sin[\xi\langle P_x\rangle_W] = \frac{I_R - I_L}{I_R + I_L}$$

Weak values when $\hat{P}$ is involved.

Form: $$\langle \boldsymbol{x}|\hat{\boldsymbol{P}}|\psi(t)\rangle = \int \langle \boldsymbol{x}|\hat{\boldsymbol{P}}|\boldsymbol{x}'\rangle\langle \boldsymbol{x}'|\psi(t)\rangle d\boldsymbol{x}' = -i\boldsymbol{\nabla}\psi(\boldsymbol{x},t)$$

Weak value $$\langle \boldsymbol{P}\rangle_W = \frac{\langle \boldsymbol{x}|\boldsymbol{P}|\psi(t)\rangle}{\langle \boldsymbol{x}|\psi(t)\rangle}$$

Write $\psi(\boldsymbol{x},t) = R(\boldsymbol{x},t)e^{iS(\boldsymbol{x},t)}$ then

$$\langle \boldsymbol{P}\rangle_W = \boldsymbol{\nabla}S(\boldsymbol{x},t) - i\boldsymbol{\nabla}\rho(\boldsymbol{x},t)/2\rho(\boldsymbol{x},t)$$ with $\rho(\boldsymbol{x},t) = |\psi(\boldsymbol{x},t)|^2$

Bohm momentum. osmotic momentum.

Real part of weak value:

$$\Re[i\rho\langle \boldsymbol{P}\rangle_W] = [\boldsymbol{\nabla}\psi^*(\boldsymbol{x})]\psi(\boldsymbol{x}) - \psi^*(\boldsymbol{x})[\boldsymbol{\nabla}\psi(x)] = \psi^*(\boldsymbol{x})\overleftrightarrow{\boldsymbol{\nabla}}\psi(\boldsymbol{x}) = \rho\boldsymbol{P}_B$$ **Bohm momentum**

Imaginary part of weak value: $$\Im[-i\rho\langle \boldsymbol{P}\rangle_W] = [\boldsymbol{\nabla}\psi^*(\boldsymbol{x})]\psi(\boldsymbol{x}) + \psi^*(\boldsymbol{x})[\boldsymbol{\nabla}\psi(\boldsymbol{x})] = \boldsymbol{\nabla}[\rho(\boldsymbol{x})].$$

The Bohm kinetic energy.

$$\Re[\langle \boldsymbol{P}^2\rangle_W] = (\boldsymbol{\nabla}S(\boldsymbol{x}))^2 - \frac{\boldsymbol{\nabla}^2R(\boldsymbol{x})}{R(\boldsymbol{x})} = P_B^2 + Q.$$

$$\Im[\langle \boldsymbol{P}^2\rangle_W] = \boldsymbol{\nabla}^2S(\boldsymbol{x}) + \left(\frac{\boldsymbol{\nabla}\rho(\boldsymbol{x})}{\rho(\boldsymbol{x})}\right)\boldsymbol{\nabla}S(\boldsymbol{x}).$$

[Leavens, Found. Phys., 35 (2005) 469-91]

[Wiseman, New J. Phys., 9 (2007) 165-77.]

[Hiley, J. Phys Conf. Series, **361**, (2012), 012014]

Remark: Relation to Energy-Momentum Tensor.

$$T^{\mu\nu} = -\left\{ \frac{\partial \mathcal{L}}{\partial(\partial^\mu \psi)} \partial^\nu \psi + \frac{\partial \mathcal{L}}{\partial(\partial^\mu \psi^*)} \partial^\nu \psi^* \right\}$$

Take the Schrödinger Lagrangian: $$\mathcal{L} = -\frac{1}{2m} \nabla \psi^* \cdot \nabla \psi + \frac{i}{2} [\psi^* (\partial_t \psi) - (\partial_t \psi^*) \psi] - V \psi^* \psi.$$

and find

$$T^{0\mu} = -\frac{i}{2} [(\partial^\mu \psi^*) \psi - \psi^* (\partial^\mu \psi)] = \frac{i}{2} [\psi^* \overleftrightarrow{\partial}^\mu \psi] = -\rho \partial^\mu S$$

Recalling that

$$P_B = \nabla S \qquad \text{and} \qquad E_B = -\partial_t S$$

Then we find

$$\rho P_{jB} = \rho \partial_j S = -T^{0j} = \Re[i\rho \langle P_j \rangle_W]$$ **Bohm momentum.**

$$\rho E_B = -\rho \partial_t S = -T^{00} = \Re[i\rho \langle P_t \rangle_W]$$ **Bohm energy.**

Bohm Approach and Pauli spin.

Take the wave function as $\Psi = \begin{pmatrix} R_1 e^{iS_1} \\ R_2 e^{iS_2} \end{pmatrix}$

The Bohm momentum and energy

$$\rho P_B(x) = \rho_1(x)\nabla_x S_1(x) + \rho_2(x)\nabla_x S_2(x) = \Re[i\rho\langle P_j\rangle_W]$$ **Bohm Momentum**

$$\rho E_B(x) = \rho_1(x)\partial_t S_1(x) + \rho_2(x)\partial_t S_2(x) = \Re[i\rho\langle P_t\rangle_W]$$ **Bohm Energy**

The Bohm kinetic energy is

$$\Re[\langle P^2\rangle_W] = P_B^2(x) + [2(\nabla_x W(x)\cdot S(x)) + W^2(x)] = P_B^2 + Q.$$

Spin of particle $S = i(\phi_L e_3 \widetilde{\phi}_L)$ and $\rho W = \nabla_x(\rho S)$

[Hiley, and Callaghan, Found. Phys. **42**, (2012) 192 and Maths-ph: 1011.4031]

This generalises to the Dirac particle

[Hiley and Callaghan, Fond. Phys **42** (2012) 192-208, math-ph:1011.4031 and 1011.4033]