PHAS0072
Particle Physics
Exam 2020-RESIT

[Prof. R. Saakyan]

Answer ALL THREE questions.

The numbers in square brackets show the provisional allocation of maximum marks per question or part of question.

The following data may be used if required:

The convention: $\hbar = c = 1$ will be used throughout this paper unless specified otherwise.

Meaning	Value
Masses of u,d,s,c,b,t quarks	2.2 MeV, 4.7 MeV, 96 MeV, 1.3 GeV, 4.2 GeV, 173 GeV
Masses of e,μ,τ leptons	0.5 MeV, 106 MeV, 1.8 GeV
Mass of all neutrinos	$\sim$0
Mass of the $\pi^{\pm}$	140 MeV
Mass of Z boson (M_Z)	91 GeV
Mass of W boson (M_W)	80 GeV
Reduced Planck constant, $\hbar$	1.06×10^{-34} J·s
Speed of light	3×10^{8} m/s
Elementary Electric Charge	1.6×10^{-19} C
Lifetime of $\mu^{\pm}$, $\tau^{\pm}$, $\pi^{\pm}$	2.2×10^{-6}s , 2.9×10^{-13}s, 2.6×10^{-8}s

CKM Matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.974 & 0.227 & 0.004 \\ 0.227 & 0.973 & 0.042 \\ 0.008 & 0.042 & 0.999 \end{pmatrix}$$

where V_{ij} is the factor for interactions involving quarks i and j.

Dirac Matrices

The Dirac γ matrices satisfying $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$ (for $\mu, \nu = 0,1,2,3$) can be written as:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma^{i=1,2,3} = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

The Pauli spin matrices, σ_i, are:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

which satisfy: $(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{c}) = \vec{a} \cdot \vec{c} + i\vec{\sigma} \cdot (\vec{a} \times \vec{c})$ for 3 component vectors $\vec{a}$, $\vec{c}$.

[Part marks]

1. (a) The Dirac Lagrangian for a free spin-half particle is given by:

$$\mathcal{L} = i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\overline{\psi}\psi$$

Show that this Lagrangian is *not* invariant under a phase transformation: $\psi(x) \rightarrow \psi'(x) = e^{iq\chi(x)}\psi(x)$ only if χ is a function of a coordinate $x \equiv x^{\mu}$, and *not* a constant. **[3]**

(b) The required gauge invariance can be restored by replacing the derivative ∂_{μ} with the covariant derivative D_{μ}, $\partial_{\mu} \rightarrow D_{\mu} = \partial_{\mu} + iqA_{\mu}$, where A_{μ} is a new field. What transformation properties must A_{μ} have to make this work? What is the physical interpretation of A_{μ}? **[3]**

(c) A Lagrangian describing a massive scalar Higgs field h and a *hypothetical* massive gauge boson B associated with a U(1) local gauge symmetry can be written as follows,

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}h)(\partial^{\mu}h) - \lambda v^2 h^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2 v^2 B_{\mu}B^{\mu}$$

$$+g^2 v B_{\mu}B^{\mu}h + \frac{1}{2}g^2 B_{\mu}B^{\mu}h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4$$

Identify the terms of the Lagrangian responsible for the massive h scalar, massive gauge boson B, $h - B$ interactions, and h self-interactions. **[4]**

QUESTION 1 CONTINUES ON THE NEXT PAGE

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(d) Draw a lowest-order Feynman diagram showing the production of a Higgs boson at the LHC and its decay to two photons. **[3]**

(e) For the decay $h \to \gamma\gamma$ with photon energies E_1 and E_2 and the opening angle between them θ, show that the invariant mass, m_h, of the Higgs boson is given by:

$$m_h^2 = 4E_1E_2\sin^2\left(\frac{\theta}{2}\right).$$

[4]

(f) What would be the experimental signature of the Higgs di-photon decay in detectors such as ATLAS or CMS at the LHC? **[3]**

[Part marks]

2. (a) Draw the lowest order Feynman diagrams for the charged current (CC) and neutral current (NC) $\nu_e e^- \to \nu_e e^-$ scattering. **[2]**

(b) The non-zero matrix elements for the $\nu_e e^- \to \nu_e e^-$ process expressed in terms of left and right electron chiral couplings, c_L and c_R, are:

$$M_{LL}^{CC} = \frac{g_W^2 s}{m_W^2} \quad M_{LL}^{NC} = c_L \frac{g_Z^2 s}{m_Z^2} \quad M_{LR}^{NC} = c_R \frac{g_Z^2 s}{m_Z^2} \frac{1}{2} (1 + \cos\theta^*)$$

where θ^* is the angle between the incoming and scattered neutrino in the centre-of-mass frame, and s is the Mandelstam variable. Show that the spin-averaged matrix element for the mixed NC and CC interaction can be written as:

$$\langle |M|^2_{NC+CC} \rangle = \frac{1}{2} \frac{g_W^4 s^2}{m_W^4} \left[(1 + c_L)^2 + \frac{1}{4} c_R^2 (1 + \cos\theta^*)^2 \right]$$

You can use the relation $g_Z / m_Z = g_W / m_W$. **[5]**

(c) Draw the lowest order Feynman diagram for the decay $\pi^0 \to \nu_e \bar{\nu}_e$ and explain why this process is effectively forbidden. **[3]**

QUESTION 2 CONTINUES ON THE NEXT PAGE

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(d) The long-baseline neutrino beams are normally produced from the pion decay in flight, $\pi^+ \rightarrow \mu^+\nu_\mu$. Find the neutrino energy (in MeV) in the pion rest frame. **[4]**

(e) Explain briefly how CP-violation in the neutrino sector can be searched for using long-baseline neutrino beams. **[3]**

(f) What experimental observation could be used to show that the neutrino is its own anti-particle? Explain why this observation would not occur for Dirac and/or massless neutrino. **[3]**

[Part marks]

3. (a) Draw the lowest order Feynman diagrams for the decays

$$B^+ \to \pi^0 e^+ \nu_e \qquad B^+ \to \bar{D}^0 e^+ \nu_e.$$

(The quark content of the B^+ meson is $u\bar{b}$, that of the $\bar{D}^0$ meson is $u\bar{c}$.) **[3]**

(b) For the above diagrams, write down the factors at each interaction vertex in terms of the coupling constant, Dirac gamma matrices and elements of the CKM matrix. **[3]**

(c) The right-handed anti-particle chiral state v_R is defined by the projection operator $P_L v = v_R$, where $P_L = \frac{1}{2}\left(1 - \gamma^5\right)$. Show explicitly that

$$\bar{v}\gamma^\mu \frac{1}{2}\left(1 - \gamma^5\right) v = \bar{v}_R \gamma^\mu v_R$$

and interpret the significance of this result for the quark vertices in the B-meson decay diagrams above. **[5]**

QUESTION 3 CONTINUES ON THE NEXT PAGE

(d) Consider ep collisions occurring along the z-axis. Let the energy of the final state jet be E_j, and the z-component of its momentum be p_j^z. Show that the y variable defined by $y \equiv (P_p \cdot q)/(P_p \cdot P_e)$, can be written as:

$$y = \frac{E_j - p_j^z}{2E_e},$$

where P_p and P_e are the four-momenta of the colliding proton and electron respectively, q is the four-momentum transferred in the interaction and E_e is the total energy of the colliding electron. You can assume that electron and proton masses can be neglected. **[5]**

(e) What is the expected value of

$$\int_0^1 [u(x) - \bar{u}(x)]\, dx$$

for the proton, where $u(x)$ and $\bar{u}(x)$ are the u-quark and $\bar{u}$-quark parton distribution functions respectively? **[4]**