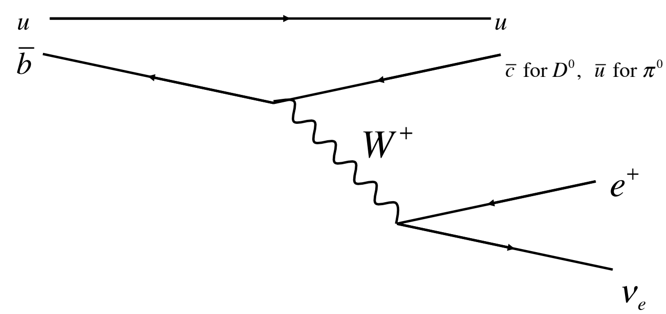
PHASM(G)442/2016 Model Answers

[Part marks]

1. (a) [**Unseen**].
B^+ decay



[4]

(b) [**Covered in lectures but unseen for this particular example**].
For quark vertices:
$-i\frac{g_W}{\sqrt{2}}V_{cb}^*\gamma^\mu\frac{1}{2}\left(1-\gamma^5\right)$, or V_{ub}^*.
(1 mark for getting the complex conjugate, $V_{cb}^*(V_{ub}^*)$ right). **[3]**
For lepton vertex:
$-i\frac{g_W}{\sqrt{2}}\gamma^\mu\frac{1}{2}\left(1-\gamma^5\right)$ **[1]**
(Missing $\frac{1}{\sqrt{2}}$ factors and the "minus" sign will not lose marks.)

(c) [**Unseen**].
Using the definition $P_L v = \frac{1}{2}\left(1-\gamma^5\right)v = v_R$ we have

$$\bar{v}\gamma^\mu\frac{1}{2}\left(1-\gamma^5\right)v = \bar{v}\gamma^\mu v_R$$

Also, $\bar{v} = \bar{v}_L + \bar{v}_R$. Therefore we need to evaluate $\bar{v}_L\gamma^\mu v_R$ and $\bar{v}_R\gamma^\mu v_R$. **[2]**
For $\bar{v}_L\gamma^\mu v_R$ we have:

$$\bar{v}_L\gamma^\mu v_R = v^\dagger\frac{1}{2}\left(1+\gamma^5\right)\gamma^0\gamma^\mu\frac{1}{2}\left(1-\gamma^5\right)v = \frac{1}{4}v^\dagger\gamma^0\left(1-\gamma^5\right)\gamma^\mu\left(1-\gamma^5\right)v =$$
$$= \frac{1}{4}\bar{v}\gamma^\mu\left(1+\gamma^5\right)\left(1-\gamma^5\right)v = \frac{1}{4}\bar{v}\gamma^\mu\left(1-\left(\gamma^5\right)^2\right)v = 0$$

where we used $\left(\gamma^5\right)^2 = 1$. **[3]**
Similar arguments lead to:

$$\bar{v}_R\gamma^\mu v_R = \frac{1}{4}\bar{v}\gamma^\mu\left(1-\gamma^5\right)^2 v \neq 0$$

Therefore,

$$\bar{v}\gamma^\mu\frac{1}{2}\left(1-\gamma^5\right)v = \bar{v}_R\gamma^\mu v_R$$

as required. **[1]**
The significance of this result is that it shows that only right chiral *a*nti-particle states participate in the weak interaction. **[1]**

(d) **[Unseen]**.
Most convenient to use natural units. From the Fermi Golden Rule:

$$\Gamma = |M_{fi}|^2 \times PS$$

where M_{fi} is the matrix element of the process and PS is the phase space. **[1]**
The energy scale of a decay is low compared the mass of the exchanged boson, M_W. Therefore $q^2 \ll M_W^2$ and $M_{fi} \propto g^2/M_W^2$, where g is a dimensionless coupling constant. Hence, $|M_{fi}|^2 \propto g^4 M_W^{-4} \propto [E]^{-4}$ **[3]**
Γ has units of $[E]$ and so PS must have units of $[E]^5$. Since the relevant energy scale is the mass of the B-meson, M_B, we have $\Gamma \propto M_B^5$. **[1]**

[Part marks]

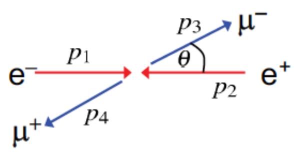
2. (a) **[Covered in lectures]**.
The spin-averaged matrix element is obtained by averaging over the initial spin states and summing over the final spin states:

$$\langle|M_{fi}|^2\rangle = \frac{1}{4}\left(|M_{RR}|^2 + |M_{RL}|^2 + |M_{LR}|^2 + |M_{LL}|^2\right) =$$

$$= \frac{1}{4}e^4\left(2\left(1+\cos\theta\right)^2 + 2\left(1-\cos\theta\right)^2\right) = e^4\left(1+\cos^2\theta\right) = \left(4\pi\alpha\right)^2\left(1+\cos^2\theta\right)$$

[3]

(b) **[Unseen]**.
We will work in the centre-of-mass frame and use the notation from the diagram below:



At $\sqrt{s} = 30GeV$ we can neglect the electron and muon masses. Hence,

$$p_1 = (E, 0, 0, E) \qquad p_2 = (E, 0, 0, -E)$$

$$p_3 = (E, E\sin\theta, 0, E\cos\theta) \qquad p_4 = (E, -E\sin\theta, 0, -E\cos\theta)$$

giving,

$$p_1 \cdot p_2 = 2E^2; \quad p_1 \cdot p_3 = E^2\left(1-\cos\theta\right); \quad p_1 \cdot p_4 = E^2\left(1+\cos\theta\right);$$

[2]

Therefore we can write:

$$\langle|M_{fi}|^2\rangle = 2e^4\frac{\left(p_1\cdot p_3\right)^2 + \left(p_1\cdot p_4\right)^2}{\left(p_1\cdot p_2\right)^2}$$

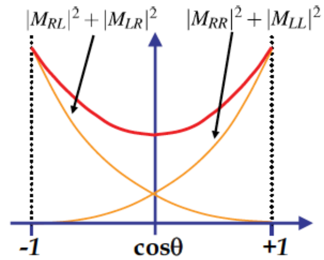
[2]

Using the definitions $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, $u = (p_1 - p_4)^2$, and neglecting the fermion masses, i.e. $p_i^2 \approx 0$, we can write:

$$\langle|M_{fi}|^2\rangle = 2e^4\left(\frac{t^2 + u^2}{s^2}\right)$$

[1]

(c) [**Covered in lectures**].
From the calculations in (a) the cross-section dependence on $\cos\theta$ should look as follows:



[2]

In reality the distribution is slightly asymmetric due to contribution of higher order QED and Z exchange. **[1]**

(d) [**Unseen**].
The QED calculation of the e^+e^- annihilation to muons and quarks is the same giving the total cross-section expression $\sigma = \frac{4\pi\alpha^2}{3s}Q^2$, where Q is the electric charge of the final state particles. The difference between the cross-sections with muons and hadrons in the final state comes from quark electric charges and three possible colour charges. Therefore:

$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q Q_q^2$$

[2]

At the centre-of-mass energy $\sqrt{s} = 30$ GeV all quark pairs can be produced apart from the top quark, i.e. $u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b}$. Hence,

$$R = 3 \times \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9}\right) = \frac{11}{3}$$

[2]

(e) [**Involves synthesising few ideas given in lectures**].
The main components of a typical collider detector are pixel detector, tracker, electromagnetic and hadronic calorimeters, muon chambers. Muons will leave tracks in all of the detector subsystems and hadrons will not go beyond the hadron calorimeter.
Hadrons will produce showers of secondary particles with characteristic lateral and longitudinal profiles. Muons only lose energy by ionisation and are the most penetrating charged particles. They will produce straight long tracks and, unlike hadrons, will get to the outermost muon detectors. **[3]**

(f) [**Briefly discussed in lectures**].
Hadrons containing b-quarks live sufficiently long to travel a few mm before decaying. The decay products of the b-quark can emerge at a relatively large angle to the original b-quark direction creating a $d$isplaced secondary vertex. Jets from b-quarks can be identified by resolving the secondary and primary vertices using high precision silicon microvertex detectors. **[2]**

[Part marks]

3. (a) [**Unseen**].

The hermitian conjugate of u_A is:

$$u_A^\dagger = \sqrt{|E|+m} \begin{pmatrix} \chi_A^\dagger & \chi_A^\dagger \left(\frac{\vec{\sigma}\cdot\vec{p}}{E+m}\right)^\dagger \end{pmatrix} = \sqrt{|E|+m} \begin{pmatrix} \chi_A^\dagger & \chi_A^\dagger \left(\frac{\vec{\sigma}\cdot\vec{p}}{E+m}\right) \end{pmatrix}$$

since $(\vec{\sigma}\cdot\vec{p})^\dagger = (\vec{\sigma}\cdot\vec{p})$ (Helicity is observable/hermitian). **[1]**

$$u_A^\dagger u_A = (E+m) \begin{pmatrix} \chi_A^\dagger & \chi_A^\dagger \left(\frac{\vec{\sigma}\cdot\vec{p}}{E+m}\right) \end{pmatrix} \begin{pmatrix} \chi_A \\ \left(\frac{\vec{\sigma}\cdot\vec{p}}{E+m}\right)\chi_A \end{pmatrix} =$$

$$= (E+m)\left(\chi_A^\dagger\chi_A + \chi_A^\dagger\chi_A\left(\frac{(\vec{\sigma}\cdot\vec{p})^2}{(E+m)^2}\right)\right)$$

Using formulae given in the cover sheet we have:

$$(\vec{\sigma}\cdot\vec{p})^2 = (\vec{p}\cdot\vec{p}) + i\vec{\sigma}\cdot(\vec{p}\times\vec{p}) = p^2$$

and therefore

$$\frac{(\vec{\sigma}\cdot\vec{p})^2}{(E+m)^2} = \frac{p^2}{(E+m)^2} = \frac{E^2-m^2}{(E+m)^2} = \frac{E-m}{E+m}$$

[3]

Finally, given that $\chi_A^\dagger\chi_A = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1$ we have

$$u_A^\dagger u_A = (E+m)\left(1 + \frac{E-m}{E+m}\right) = 2E$$

[1]

Normalisation is proportional to energy to take into account relativistic contraction, particle density increases by $\gamma = E/m$. **[1]**

(b) [**Unseen**].

In the massless limit $u_A = \begin{pmatrix} \chi_A \\ (\vec{\sigma}\cdot\hat{p})\chi_A \end{pmatrix}$ (omitting the normalisation). **[1]**

$$\hat{h} = \frac{1}{2}\begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix}\begin{pmatrix} \chi_A \\ (\vec{\sigma}\cdot\hat{p})\chi_A \end{pmatrix} = \frac{1}{2}\begin{pmatrix} (\vec{\sigma}\cdot\hat{p})\chi_A \\ (\vec{\sigma}\cdot\hat{p})^2\chi_A \end{pmatrix} = \frac{1}{2}\begin{pmatrix} (\vec{\sigma}\cdot\hat{p})\chi_A \\ \chi_A \end{pmatrix}$$

since $(\vec{\sigma}\cdot\hat{p})^2 = (\hat{p})^2 = 1$. **[1]**

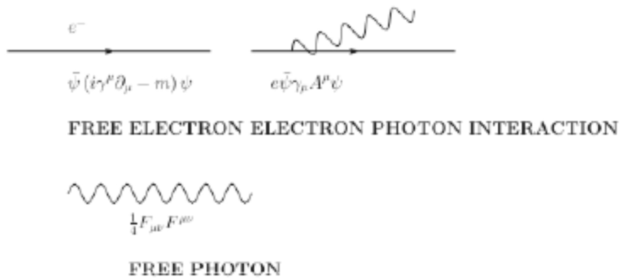
On the other hand we have:

$$\frac{1}{2}\gamma^5 u_A = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \chi_A \\ (\vec{\sigma}\cdot\hat{p})\chi_A \end{pmatrix} = \frac{1}{2}\begin{pmatrix} (\vec{\sigma}\cdot\hat{p})\chi_A \\ \chi_A \end{pmatrix}$$

and therefore $\hat{h}u_A = \frac{1}{2}\gamma^5 u_A$. **[1]**

The significance of this result is in the fact that it shows that in the massless limit the chirality operator, $\frac{1}{2}\gamma^5$, and the helicity operator, $\hat{h}$, are the same. **[1]**

(c) [**Covered in lectures**].



[3]

(d) [**Mostly covered in lectures**].
If ψ transforms as $\psi \to \psi e^{ie\theta}$ then $\bar{\psi}$ transforms as $\bar{\psi} \to \bar{\psi}e^{-ie\theta}$. Looking at each term in the Lagrangian:

$$i\bar{\psi}\gamma^\mu\partial_\mu\psi \to i\bar{\psi}e^{-ie\theta}\gamma^\mu\partial_\mu\left(\psi e^{ie\theta}\right) = i\bar{\psi}e^{-ie\theta}\gamma^\mu\left(e^{ie\theta}\partial_\mu\psi + ie\partial_\mu\theta e^{ie\theta}\psi\right) =$$

$$= i\bar{\psi}\gamma^\mu\partial_\mu\psi - e\bar{\psi}\gamma^\mu\partial_\mu\theta\psi$$

[3]

The second term is

$$eA_\mu\bar{\psi}\gamma^\mu\psi \to eA_\mu e^{-ie\theta}\bar{\psi}\gamma^\mu e^{ie\theta}\psi = eA_\mu\bar{\psi}\gamma^\mu\psi$$

is unchanged. It is obvious that $\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ is also unchanged. Hence, $\mathcal{L}_{\mathcal{QED}} \to \mathcal{L}_{\mathcal{QED}} - e\bar{\psi}\gamma^\mu\partial_\mu\theta\psi$ is not invariant. **[2]**

(e) [**Covered in lectures**].
Consider a transform of the form $A_\mu \to A_\mu + \partial_\mu\theta$.
Then:

$$eA_\mu\bar{\psi}\gamma^\mu\psi \to eA_\mu\bar{\psi}\gamma^\mu\psi + e\bar{\psi}\gamma^\mu\partial_\mu\theta\psi$$

and this cancels the gauge invariance violating term in (d). **[2]**

4. (a) **[Unseen]**.

The electron-neutron structure function expression can be obtained by replacing the proton PDFs with neutron PDFs and using the isospin symmetry, i.e. $u^p(x) = d^n(x)$:

$$F_2^{en}(x) = \frac{4}{9}x\left[d(x) + \bar{d}(x)\right] + \frac{1}{9}x\left[u(x) + \bar{u}(x) + s(x) + \bar{s}(x)\right]$$

where the strange quark content for the nucleons is the same since it comes from the sea. **[2]**

(b) **[Unseen]**.

Using the expressions for $F_2^{ep}(x)$ and $F_2^{en}(x)$ we have:

$$\int_0^1 \frac{[F_2^{ep}(x) - F_2^{en}(x)]}{x}dx = \frac{1}{3}\int_0^1 (u(x) - d(x) + \bar{u}(x) - \bar{d}(x))dx$$

[1]

Writing the quark PDFs in terms of valence and sea contributions and assuming that the quark and anti-quark sea contributions are the same, i.e. $u_S(x) = \bar{u}(x)$ and $d_S(x) = \bar{d}(x)$, the expression becomes:

$$\int_0^1 \frac{[F_2^{ep}(x) - F_2^{en}(x)]}{x}dx = \frac{1}{3}\int_0^1 (u_V(x) - d_V(x) + u_S(x) - d_S(x) + \bar{u}(x) - \bar{d}(x))dx =$$

$$= \frac{1}{3}\int_0^1 (u_V(x) - d_V(x) + 2\bar{u}(x) - 2\bar{d}(x))dx = \frac{1}{3} + \frac{2}{3}\int_0^1 (\bar{u}(x) - \bar{d}(x))dx$$

where we have used $\int_0^1 u_V(x) = 2$ and $\int_0^1 d_V(x) = 1$. **[3]**

The measured value can therefore be interpreted as

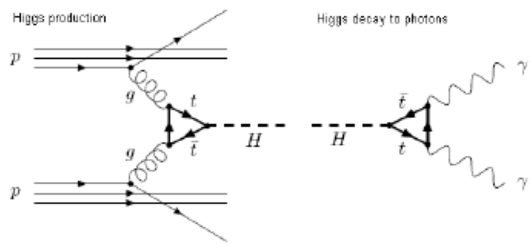
$$\int_0^1 (\bar{u}(x) - \bar{d}(x))dx = \frac{3}{2}\left[(0.24 - 0.33) \pm 0.03\right] = -0.14 \pm 0.05$$

demonstrating that there is a deficit of $\bar{u}$ quarks relative to $\bar{d}$ quarks in the proton. **[2]**

(c) **[Covered in lectures]**.

The production of the Higgs boson (and other "new" particles) comes from collisions between partons (the Higgs production in the LHC it is dominated by gluon-gluon fusion). The cross-section depends on partons PDFs. Uncertainty in PDF translates directly into uncertainty in the Higgs production cross-section. **[2]**

(d) [**Involves synthesising few ideas covered in lectures**].
Example diagrams are below. Other suitable diagrams will be accepted.



[3]

(e) [**Unseen**].
Consider the Higgs decay to two photon with four-momenta, $P_1 = (E_1, \vec{p}_1)$ and $P_2 = (E_2, \vec{p}_2)$. The Higgs invariant mass is $m_h^2 = (P_1 + P_2)^2$. **[1]**
Therefore,

$$m_h^2 = P_1^2 + P_2^2 + 2P_1P_2 = 0 + 0 + 2E_1E_2 - 2E_1E_2\cos\theta$$

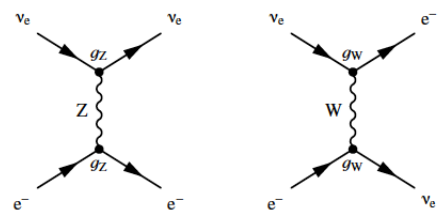
since $m_\gamma = 0$ and $E_\gamma = |\vec{p}_\gamma|$. **[2]**
Finally,

$$m_h^2 = 2E_1E_2(1 - \cos\theta) = 4E_1E_2\sin^2\left(\frac{\theta}{2}\right)$$

[1]

(f) [**Covered in lectures**].
Two photons would leave no tracks in the central tracking detector.
They will deposit energy in the electromagnetic calorimeter. The resulting shower profiles are consistent with their EM origin.
The reconstructed invariant mass is consistent with the mass of the Higgs boson. **[3]**

5. (a) **[Involves synthesising few ideas covered in lectures]**.
(No need to have coupling constants on the diagrams).



[2]

(b) **[Unseen]**.
Only left-handed chiral currents participate in the CC- interaction, while both left- and right-handed chiral currents contribute to the NC part of the interaction. **[1]**
Since the final states are the same the amplitudes of the left-handed chiral currents for the CC and NC diagrams are added up, $M_{LL}^{CC} + M_{LL}^{NC}$. **[1]**
The spin-averaged matrix element for the mixed NC + CC interaction must average over LL and LR contributions:

$$\langle |M|^2_{NC+CC} \rangle = \frac{1}{2}\left[\left(M_{LL}^{CC} + M_{LL}^{NC}\right)^2 + \left(M_{LR}^{NC}\right)^2\right]$$

[1]

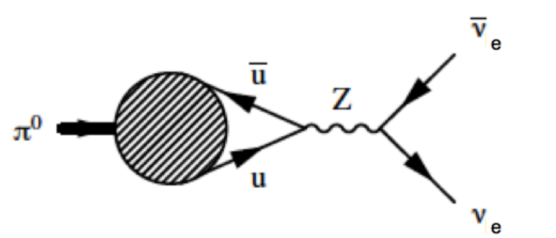
Finally, using $g_Z/m_Z = g_W/m_W$ and the expressions for individual matrix elements we have:

$$\langle |M|^2_{NC+CC} \rangle = \frac{1}{2}\left[\left(\frac{g_W^2 s}{m_W^2} + c_L\frac{g_Z^2 s}{m_Z^2}\right)^2 + \left(c_R\frac{g_Z^2 s}{m_Z^2}\frac{1}{2}\left(1+\cos\theta^*\right)\right)^2\right] =$$

$$= \frac{1}{2}\frac{g_W^4 s^2}{m_W^4}\left[\left(1+c_L\right)^2 + \frac{1}{4}c_R^2\left(1+\cos\theta^*\right)^2\right]$$

[2]

(c) **[Involves synthesising few ideas covered in lectures]**.



[1]

The neutrino is produced in a LH chiral state and the anti-neutrino is produced in a RH chiral state. Because neutrinos are almost massless the chiral states effectively correspond to helicity states. Therefore the decay will result in a $J = 1$ final state (students can illustrate that with a diagram). This violates conservation of angular momentum and therefore forbidden. **[2]**

(d) **[Unseen]**.

Using energy and momentum conservation in the pion rest frame we can write:

$$m_\pi = E_\mu + E_\nu \qquad 0 = \vec{p}_\mu + \vec{p}_\nu$$

[1]

In terms of four-vectors:

$$P_\pi^2 = (P_\mu + P_\nu)^2$$
$$m_\pi^2 = m_\mu^2 + 2E_\mu E_\nu - 2\vec{p}_\mu \vec{p}_\nu$$

[1]

since $P_\nu^2 = m_\nu^2 = 0$. Using the energy and momentum conservation relations above:

$$m_\pi^2 = m_\mu^2 + 2(m_\pi - E_\nu)E_\nu + 2\,|\vec{p}_\nu|^2 = m_\mu^2 + 2m_\pi E_\nu$$

[1]

Finally,

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} = \frac{140^2 - 106^2}{2 \cdot 140} \approx 30 \text{ MeV}$$

[1]

(e) **[Covered in lectures]**.

The CP-violating phase in the PMNS matrix is linked to the $\nu_\mu \leftrightarrow \nu_e$ oscillations via θ_{13} mixing angle. **[1]**

A long-baseline beam of ν_μ and $\bar{\nu}_\mu$ can be established with a far detector looking at the appearance of ν_e and $\bar{\nu}_e$. **[1]**

By comparing $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillation probabilities one can establish the phenomenon of CP-violation and measure the corresponding phase. **[1]**

(f) **[Covered in lectures]**.

The observation of neutrinoless double beta decay, $0\nu\beta\beta$. **[1]**

In order for this process to occur two conditions must be met: i) the emitted neutrino can turn into an anti-neutrino to take part in the second inverse β-decay; ii) the neutrino flips its helicity as it turns into an anti-neutrino.

The first condition cannot happen for Dirac particles and the second for massless particles. **[2]**