

# Parton distribution functions and $\alpha_S$

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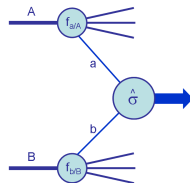
May 9, 2009

In collaboration with [A.D. Martin](#), [W.J. Stirling](#) and [R.S. Thorne](#)  
[[arXiv:0901.0002](#) (“**MSTW 2008**”, 154 pages) and [paper in preparation](#)]  
**MSTW** = **MRST** – R.G. Roberts (now retired) + G.W. (since 2006)

<http://projects.hepforge.org/mstwpdf/>

# Fixed-order collinear factorization at hadron colliders

$$\sigma_{AB} = \sum_{a,b=q,g} f_{a/A}(x_a, \mu_F^2) \otimes f_{b/B}(x_b, \mu_F^2) \otimes \hat{\sigma}_{ab}$$



- Expand  $\hat{\sigma}_{ab}$ ,  $P_{aa'}$  and  $\beta$  as perturbative series in  $\alpha_S$  ( $\overline{\text{MS}}$  scheme):

$$\hat{\sigma}_{ab} = \alpha_S^r \left[ \hat{\sigma}_{ab}^{\text{LO}} + \alpha_S \hat{\sigma}_{ab}^{\text{NLO}} + \alpha_S^2 \hat{\sigma}_{ab}^{\text{NNLO}} \dots \right] \quad (r \geq 0)$$

$$\frac{\partial f_{a/A}}{\partial \ln Q^2} = \alpha_S \sum_{a'=q,g} \left[ P_{aa'}^{\text{LO}} + \alpha_S P_{aa'}^{\text{NLO}} + \alpha_S^2 P_{aa'}^{\text{NNLO}} \dots \right] \otimes f_{a'/A}$$

$$\frac{\partial \alpha_S}{\partial \ln Q^2} = -\beta^{\text{LO}} \alpha_S^2 - \beta^{\text{NLO}} \alpha_S^3 - \beta^{\text{NNLO}} \alpha_S^4 - \dots$$

- Need to extract input values  $f_{a/A}(x, Q_0^2)$  and  $\alpha_S(M_Z^2)$  from data.

# Data sets fitted in MSTW 2008 NLO analysis [[arXiv:0901.0002](https://arxiv.org/abs/0901.0002)]

Data set	$\chi^2 / N_{\text{pts.}}$
H1 MB 99 $e^+p$ NC	9 / 8
H1 MB 97 $e^+p$ NC	42 / 64
H1 low $Q^2$ 96–97 $e^+p$ NC	44 / 80
H1 high $Q^2$ 98–99 $e^-p$ NC	122 / 126
H1 high $Q^2$ 99–00 $e^+p$ NC	131 / 147
ZEUS SVX 95 $e^+p$ NC	35 / 30
ZEUS 96–97 $e^+p$ NC	86 / 144
ZEUS 98–99 $e^-p$ NC	54 / 92
ZEUS 99–00 $e^+p$ NC	63 / 90
H1 99–00 $e^+p$ CC	29 / 28
ZEUS 99–00 $e^+p$ CC	38 / 30
H1/ZEUS $e^\pm p F_2^{\text{charm}}$	107 / 83
H1 99–00 $e^+p$ incl. jets	19 / 24
ZEUS 96–97 $e^+p$ incl. jets	30 / 30
ZEUS 98–00 $e^\pm p$ incl. jets	17 / 30
DØ II $p\bar{p}$ incl. jets	114 / 110
CDF II $p\bar{p}$ incl. jets	56 / 76
CDF II $W \rightarrow l\nu$ asym.	29 / 22
DØ II $W \rightarrow l\nu$ asym.	25 / 10
DØ II $Z$ rap.	19 / 28
CDF II $Z$ rap.	49 / 29

Data set	$\chi^2 / N_{\text{pts.}}$
BCDMS $\mu p F_2$	182 / 163
BCDMS $\mu d F_2$	190 / 151
NMC $\mu p F_2$	121 / 123
NMC $\mu d F_2$	102 / 123
NMC $\mu n / \mu p$	130 / 148
E665 $\mu p F_2$	57 / 53
E665 $\mu d F_2$	53 / 53
SLAC $ep F_2$	30 / 37
SLAC $ed F_2$	30 / 38
NMC/BCDMS/SLAC $F_L$	38 / 31
E866/NuSea $pp$ DY	228 / 184
E866/NuSea $pd/pp$ DY	14 / 15
NuTeV $\nu N F_2$	49 / 53
CHORUS $\nu N F_2$	26 / 42
NuTeV $\nu N xF_3$	40 / 45
CHORUS $\nu N xF_3$	31 / 33
CCFR $\nu N \rightarrow \mu\mu X$	66 / 86
NuTeV $\nu N \rightarrow \mu\mu X$	39 / 40
<b>All data sets</b>	<b>2543 / 2699</b>

- Red = New w.r.t. MRST 2006 fit.

# Input parametrization in MSTW 2008 NLO/NNLO fit

At input scale  $Q_0^2 = 1 \text{ GeV}^2$  (notation:  $f_{a/p} \equiv a$ ):

$$xu_v = A_u x^{\eta_1} (1-x)^{\eta_2} (1 + \epsilon_u \sqrt{x} + \gamma_u x)$$

$$xd_v = A_d x^{\eta_3} (1-x)^{\eta_4} (1 + \epsilon_d \sqrt{x} + \gamma_d x)$$

$$xS = A_S x^{\delta_S} (1-x)^{\eta_S} (1 + \epsilon_S \sqrt{x} + \gamma_S x)$$

$$x(\bar{d} - \bar{u}) = A_{\Delta} x^{\eta_{\Delta}} (1-x)^{\eta_S+2} (1 + \gamma_{\Delta} x + \delta_{\Delta} x^2)$$

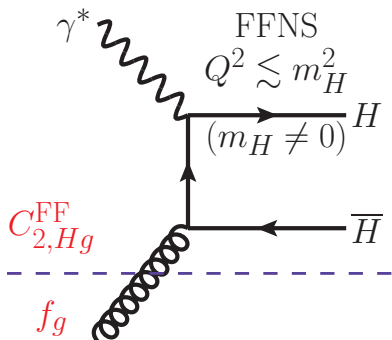
$$xg = A_g x^{\delta_g} (1-x)^{\eta_g} (1 + \epsilon_g \sqrt{x} + \gamma_g x) + A_{g'} x^{\delta_{g'}} (1-x)^{\eta_{g'}}$$

$$x(s + \bar{s}) = A_+ x^{\delta_S} (1-x)^{\eta_+} (1 + \epsilon_S \sqrt{x} + \gamma_S x)$$

$$x(s - \bar{s}) = A_- x^{\delta_-} (1-x)^{\eta_-} (1 - x/x_0)$$

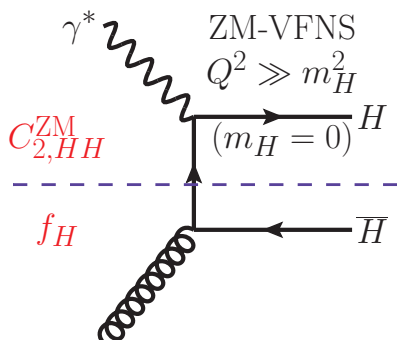
- $A_u$ ,  $A_d$ ,  $A_g$  and  $x_0$  are determined from sum rules.
- **20 parameters** allowed to go free for error propagation, cf. 15 for MRST error PDF sets (**1** more for  $g$ , **4** more for  $s$ ,  $\bar{s}$ ).

# Heavy quark contribution to DIS structure function $F_2$



Fixed flavor number scheme

- No heavy quark PDF.
- Includes  $\mathcal{O}(m_H^2/Q^2)$  terms.
- No resummation of  $\alpha_S \ln(Q^2/m_H^2)$  terms.



Zero-mass variable flavor number scheme

- Use heavy quark PDF.
- Mass dependence neglected.
- Resums  $\alpha_S \ln(Q^2/m_H^2)$  terms similar to light quarks.

# General-mass variable flavor number scheme (GM-VFNS)

Recent review by [R. Thorne](#) and [W.-K. Tung](#) [[arXiv:0809.0714](#)]

- Interpolate between two well-defined regions:  
FFNS for  $Q^2 \leq m_H^2$ , ZM-VFNS for  $Q^2 \gg m_H^2$ .
- Define by demanding equivalence of the  $n_f = n$  (FFNS) and  $n_f = n + 1$  (VFNS) flavor descriptions above transition point:

$$\begin{aligned}
 F_i(x, Q^2) &= \sum_k C_{i,k}^{\text{FF},n}(Q^2/m_H^2) \otimes f_k^n(Q^2) \\
 &= \sum_j C_{i,j}^{\text{VF},n+1}(Q^2/m_H^2) \otimes f_j^{n+1}(Q^2) \\
 &\equiv \sum_{j,k} C_{i,j}^{\text{VF},n+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2) \otimes f_k^n(Q^2) \\
 \Rightarrow C_{i,k}^{\text{FF},n}(Q^2/m_H^2) &= \sum_j C_{i,j}^{\text{VF},n+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2)
 \end{aligned}$$

- But  $C_{i,j}^{\text{VF},n_f,(m)}$  only uniquely defined in massless limit  $\Rightarrow$  ambiguous up to  $\mathcal{O}(m_H^2/Q^2)$  terms (can redistribute between orders).

# Choice of GM-VFNS by MRST/MSTW

- MRST 1998–2004 used the **Thorne–Roberts (TR)** scheme [[hep-ph/9709442](#)]: demand  $\partial F_2^H / \partial \ln Q^2$  continuous.
- PDFs are discontinuous at NNLO in a VFNS [[Buza et al. '96](#)]:

$$f_{j/p}^{n_f+1}(x, m_H^2) = f_{j/p}^{n_f}(x, m_H^2) + \alpha_S^2 \sum_k A_{jk}(x) \otimes f_{k/p}^{n_f}(x, m_H^2),$$

but neglected in MRST 2001–2004 NNLO analyses.

- Structure functions at NNLO are then discontinuous in ZM-VFNS, but should be continuous in GM-VFNS. Original **TR** scheme technically difficult to implement at NNLO.
- Instead, **R. Thorne** [[hep-ph/0601245](#)] redefined simpler GM-VFNS (denoted **TR'**) for use up to NNLO. Adopted elements of “**ACOT( $\chi$ )**” ( $\rightarrow$  talk by **F. Olness**):

$$C_{2,HH}^{\text{VF},n_f,(m)}(z, Q^2/m_H^2) = C_{2,HH}^{\text{ZM},n_f,(m)}(z/x_{\text{max}})$$

Ordering of the perturbative expansion for  $F_2^H$  (TR, TR')

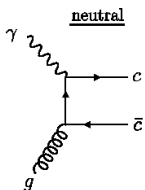
$n$ -flavor, $Q^2 < m_H^2$	
LO :	$\frac{\alpha_S}{4\pi} C_{2,Hg}^{\text{FF},n,(1)} \otimes g^n$
NLO :	$\left(\frac{\alpha_S}{4\pi}\right)^2 \left( C_{2,Hg}^{\text{FF},n,(2)} \otimes g^n + C_{2,Hq}^{\text{FF},n,(2)} \otimes \Sigma^n \right)$
NNLO :	$\left(\frac{\alpha_S}{4\pi}\right)^3 \sum_j C_{2,Hj}^{\text{FF},n,(3)} \otimes f_j^n$

$(n+1)$ -flavor, $Q^2 > m_H^2$	
LO :	$C_{2,HH}^{\text{VF},n+1,(0)} \otimes (H + \bar{H})$
NLO :	$\frac{\alpha_S}{4\pi} \left( C_{2,HH}^{\text{VF},n+1,(1)} \otimes (H + \bar{H}) + C_{2,Hg}^{\text{VF},n+1,(1)} \otimes g^{n+1} \right)$
NNLO :	$\left(\frac{\alpha_S}{4\pi}\right)^2 \sum_j C_{2,Hj}^{\text{VF},n+1,(2)} \otimes f_j^{n+1}$

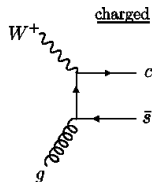
- Maintain continuity at  $Q^2 = m_H^2$  by freezing term with highest power of  $\alpha_S$  for  $Q^2 < m_H^2$  when moving above  $m_H^2$ .
- ACOT-type schemes instead use same order of  $\alpha_S$  for  $n$ - and  $(n+1)$ -flavors, e.g.  $F_2^H = 0$  at LO for  $Q^2 < m_H^2$ .



# Modeling of higher-order massive DIS coefficient functions

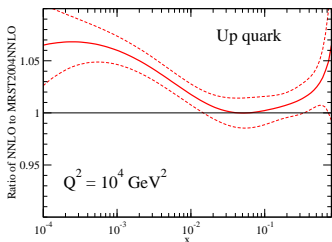
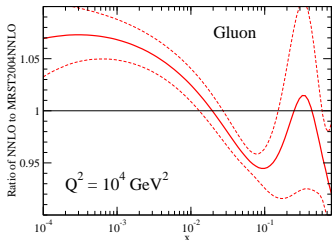


- Massive  $\mathcal{O}(\alpha_S^3)$  NC coefficient functions unknown, but needed for GM-VFNS at **NNLO**.
- Model [Thorne '06] using known **leading threshold logarithms** [Laenen, Moch '99] and **leading  $\ln(1/x)$  terms** [Catani, Ciafaloni, Hautmann '91]. Variation in free parameters does not lead to a large change.
- Massive  $\mathcal{O}(\alpha_S^2)$  CC coefficient functions unknown, but needed for GM-VFNS at **NLO** (important for  $s, \bar{s}$  determination from CCFR/NuTeV  $\nu N \rightarrow \mu^+ \mu^- X$  data).
- Model by modifying  $\mathcal{O}(\alpha_S^2)$  NC contributions for different threshold behaviour. More sophisticated modeling in MSTW 2008. No attempt made to model  $\mathcal{O}(\alpha_S^3)$  CC contribution needed at NNLO.



# Impact of consistent GM-VFNS at NNLO

Ratio 2006 to 2004:



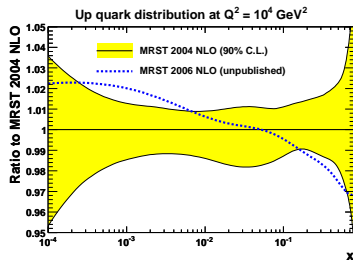
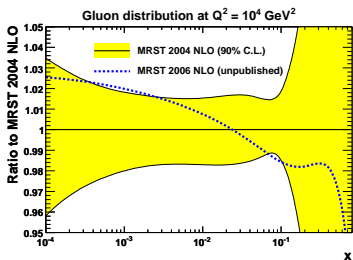
## MRST 2006 NNLO

[[arXiv:0706.0459](https://arxiv.org/abs/0706.0459)]

- First implementation of TR' scheme.
- Increase in low- $x$  PDFs when discontinuities included.
- $\sigma_{W,Z}$  at the LHC sensitive to light sea-quark PDFs at  $x \sim 0.006$  (for  $y = 0$ )  $\Rightarrow$   $\sigma_{W,Z}$  increase by 6%.  
A **correction**, not an *uncertainty*.
- From CTEQ6.1 NLO (ZM-VFNS) to CTEQ6.5 NLO (GM-VFNS):  
8% increase in  $\sigma_{W,Z}$  at LHC.  
A **correction**, not an *uncertainty*.

# Scheme dependence of GM-VFNS at NLO

- Change  $TR \rightarrow TR'$  allows study of scheme dependence at NLO.
- MRST 2004 (TR) and MRST 2006 (TR') fits use same data.



- Nearly 3% increase in  $\sigma_{W,Z}$  at the LHC at NLO. Genuine theory uncertainty: should decrease going to higher orders.
- An **uncertainty**, not a *correction* at NLO.

# Path to NNLO evolution in global PDF analysis

MRST 2001/2002 : approximate NNLO splitting [van Neerven, Vogt '00].

MRST 2004 : exact NNLO splitting [Moch, Vermaseren, Vogt '04].

MRST 2006 : added discontinuities at  $Q^2 = m_H^2$  [Buza *et al.* '96]:

$$f_{j/p}^{n_f+1}(x, m_H^2) = f_{j/p}^{n_f}(x, m_H^2) + \alpha_S^2 \sum_k A_{jk}(x) \otimes f_{k/p}^{n_f}(x, m_H^2),$$

with TR' GM-VFNS for structure functions [Thorne '06].

MSTW 2008 : minor refinements to NNLO evolution code:

- added perturbative NNLO generation of  $q \neq \bar{q}$  (very small).
- improved definition of  $\alpha_S$  (MRST form unconventional).
- evolution checked against public PEGASUS [Vogt '04] and HOPPET [Salam, Rojo '08] codes for fitted input PDFs.

# Treatment of jet data in MRST/MSTW analyses

## MRST 2001–2006

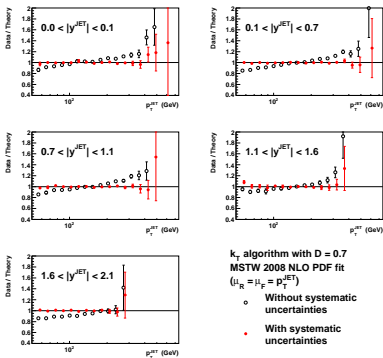
- Fit six “**pseudogluon**” points at  $Q^2 = 2000 \text{ GeV}^2$  inferred from **Tevatron Run I** inclusive jet data.
- Comparison to actual jet data, calculated at LO with a K-factor, only made *after* the fit.

## MSTW 2008

- Fit to **Tevatron Run II** and HERA DIS inclusive jet data.
- Complete treatment of correlated systematic errors.
- Use **FASTNLO** code [Kluge, Rabbertz, Wobisch '06] to calculate NLO cross sections exactly during the fit.
- Full NNLO  $\hat{\sigma}_{ab}$  not yet known: include 2-loop threshold corrections [Kidonakis, Owens '00] for Tevatron jet data at NNLO and exclude HERA DIS jet data.

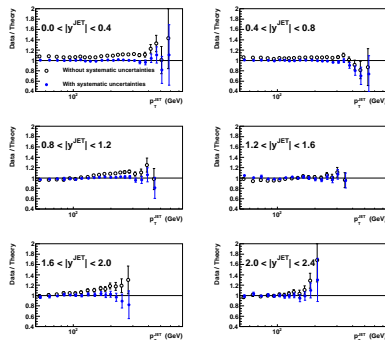
# Description of Tevatron Run II inclusive jet data

CDF Run II inclusive jet data,  $\chi^2 = 56$  for 76 pts.



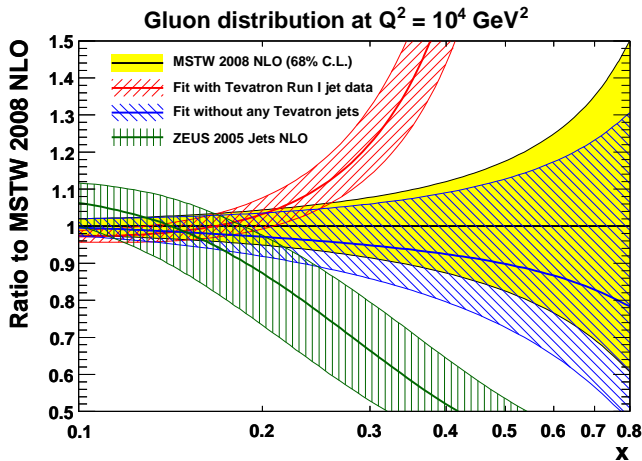
[Data: [hep-ex/0701051](https://arxiv.org/abs/hep-ex/0701051)]

$\Delta\emptyset$  Run II inclusive jet data (cone,  $R = 0.7$ )  
 MSTW 2008 NLO PDF fit ( $\mu_R = \mu_F = p_T^{\text{JET}}$ ),  $\chi^2 = 114$  for 110 pts.



[Data: [arXiv:0802.2400](https://arxiv.org/abs/0802.2400)]

# Impact of Run II jet data on high- $x$ gluon distribution



- Run II jet data prefer smaller gluon distribution at high  $x$ .

# Tension between Run I and Run II inclusive jet data

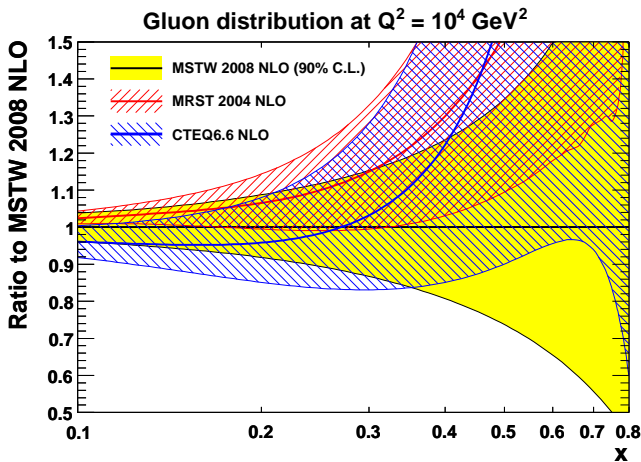
**Highlighted** numbers indicate  $\chi^2$  values for data sets explicitly included in various NLO global fits:

CDFI (33 pts.)	DØI (90 pts.)	CDFII( $k_T$ ) (76 pts.)	DØII (110 pts.)	$\Delta\chi^2_{\text{non-jet}}$ (2513 pts.)	$\alpha_S(M_Z^2)$
53	119	64	117	0	0.1197
<b>51</b>	<b>48</b>	132	180	9	0.1214
56	110	<b>56</b>	<b>114</b>	2	0.1202
<b>53</b>	<b>85</b>	<b>68</b>	<b>117</b>	1	0.1204

- Fit to Run I jets  $\Rightarrow$  description of Run II jets bad.
- Fit to Run II jets  $\Rightarrow$  description of Run I jets bad.
- Fit neither  $\Rightarrow$  similar description as fitting Run II only.
- **Summary:** Some inconsistency between Run I and Run II jets. Run II jets slightly more consistent with rest of data.



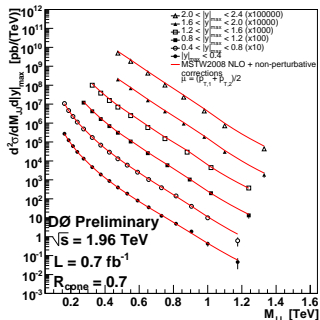
# New high- $x$ gluon distribution compared to previous sets



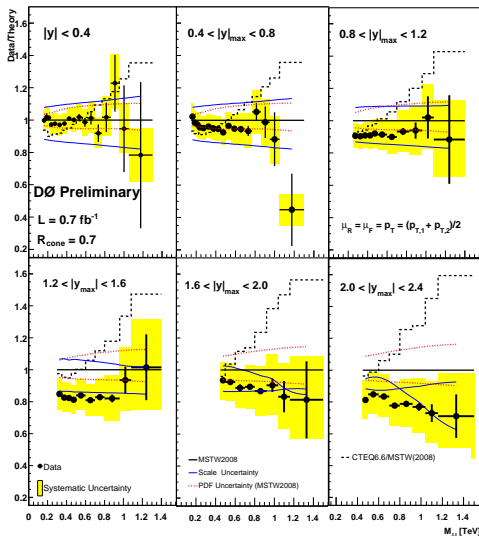
- Smaller high- $x$  gluon than previous MRST and CTEQ fits.

# Description of DØ dijet mass spectrum (April 29, 2009)

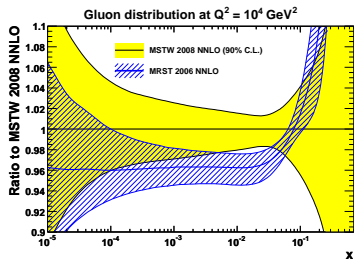
## [DØ Note 5919-CONF]



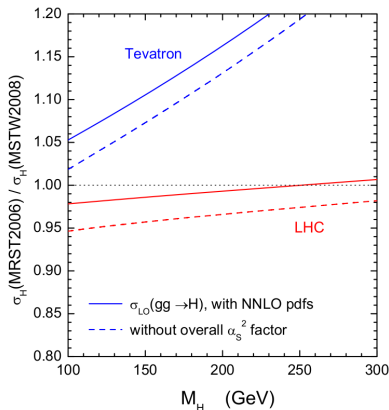
- Data favor less gluon at high  $x$  (MSTW 2008 over CTEQ6.6).



# Implications of new PDFs for Higgs cross sections



- NNLO trend similar to NLO: smaller 2008 gluon at high  $x$ , larger 2008 gluon at low  $x$  (momentum sum rule).
- $\alpha_S(M_Z^2) = 0.1191$  (2006)  $\rightarrow 0.1171$  (MSTW 2008)



- Higgs cross sections smaller at Tevatron with 2008 PDFs ( $\rightarrow$  talk by [D. de Florian](#)).

# Uncertainties in global PDF analysis

## “Theoretical” errors

- *Examples:* input parameterisation form, neglected higher-order and higher-twist QCD corrections, electroweak corrections, choice of cuts, nuclear corrections, heavy flavor treatment.
- Difficult to quantify *a priori*. Often correction only known after an improved treatment/calculation is available.

## “Experimental” errors

- If all the above sources of “theoretical” errors are fixed, how do we propagate the experimental uncertainties on the fitted data points through to the PDF uncertainties?
- Generally use the **Hessian** method [Pumplin *et al.* '01]: diagonalize covariance matrix from the fit and produce  $\pm$  **eigenvector PDF sets** displaced from best-fit PDF set.

Criteria for choice of tolerance  $T = \sqrt{\Delta\chi^2_{\text{global}}}$

### Parameter-fitting criterion

- $T^2 = 1$  for 68% (1- $\sigma$ ) C.L.,  $T^2 = 2.71$  for 90% C.L.
- **In practice:** minor inconsistencies between fitted data sets, and unknown experimental and theoretical uncertainties, so **not appropriate for global PDF analysis.**

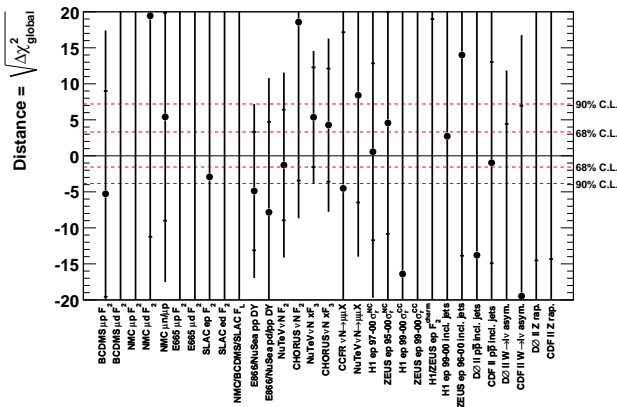
### Hypothesis-testing criterion (proposed by CTEQ)

- Much weaker: treat PDF sets obtained from eigenvectors of covariance matrix as **alternative hypotheses.**
- Determine  $T^2$  from the criterion that **each data set should be described within its 90% C.L. limit.** Very roughly, a “good” fit has  $\chi^2 \simeq N_{\text{pts.}} \pm \sqrt{2N_{\text{pts.}}}$  for each data set.
- **CTEQ:**  $T^2 = 100$  for 90% C.L. limit, **MRST:**  $T^2 = 50$ .

# Determination of tolerance for eigenvector number 13

Eigenvector number 13

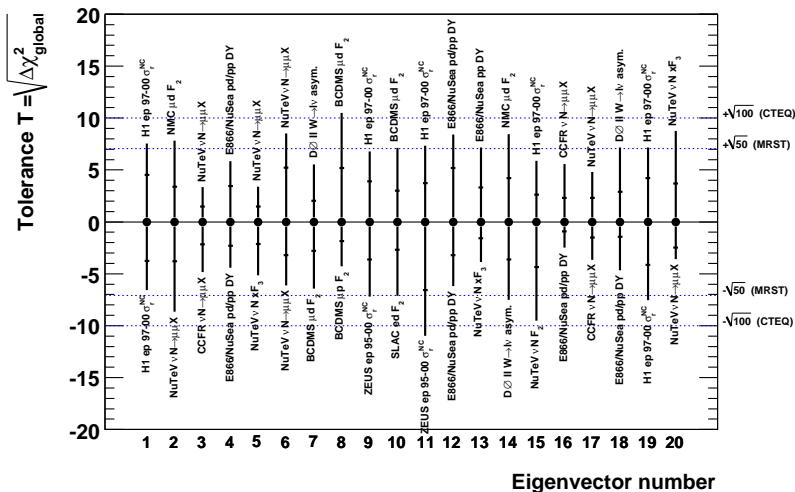
MSTW 2008 NLO PDF fit



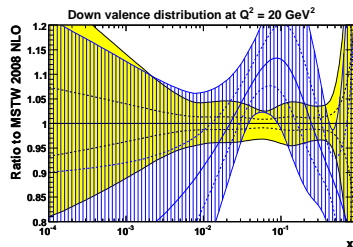
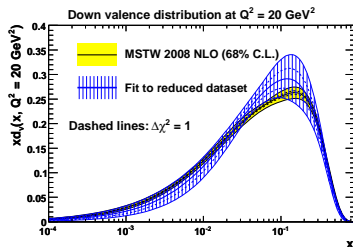
- Tolerance in “+” direction provided by E866/NuSea  $pp$  DY data.
- Tolerance in “-” direction provided by NuTeV  $\nu N$   $xF_3$  data.

# Dynamic tolerance: different for each eigenvector

## MSTW 2008 NLO PDF fit



# Test of dynamic tolerance: fit to reduced dataset



- Fit to **reduced dataset** comprising **589** DIS data points, cf. **2699** data points in **global** fit.
- Errors given by  $T^2 = 1$  don't overlap  $\Rightarrow$  inconsistent data sets included in global fit.
- **Dynamic tolerance**  $T^2 > 1$  **accommodates** mildly inconsistent data sets.



# Uncertainties on $\alpha_S$ in global PDF analysis

[MSTW, in preparation]

- PDFs (including uncertainty sets) are determined for a **fixed value of  $\alpha_S$**   $\Rightarrow$  use **same** value in cross section calculations.
- In MRST/MSTW analyses,  $\alpha_S$  is fitted, then **fixed** at the best-fit value for final error propagation.

## Problems:

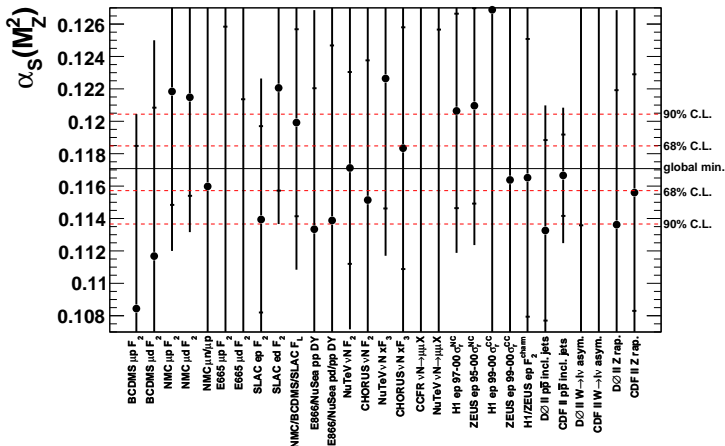
- ① What is the (experimental) error on  $\alpha_S$ ?
- ② How to include this error in cross section calculations?

## Solutions:

- ① Apply same method used to determine the tolerance for each eigenvector to determine the experimental error on  $\alpha_S(M_Z^2)$ .
- ② Then generate best-fit and eigenvector PDF sets for different fixed  $\alpha_S$  values for use in calculations of cross sections.

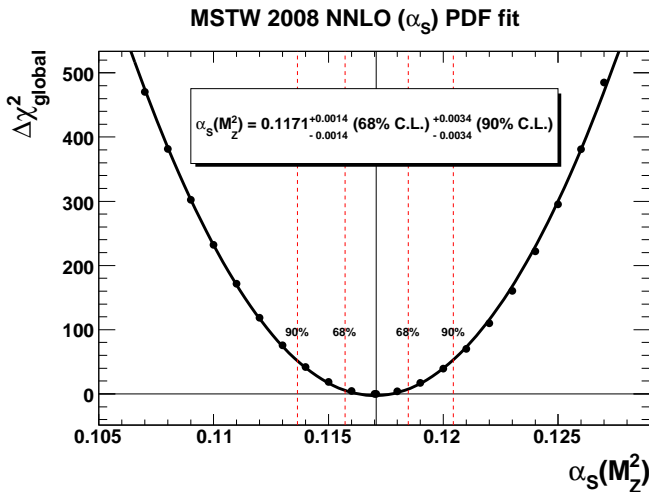
# Ranges of $\alpha_S(M_Z^2)$ for which data sets are well described

## MSTW 2008 NNLO ( $\alpha_S$ ) PDF fit



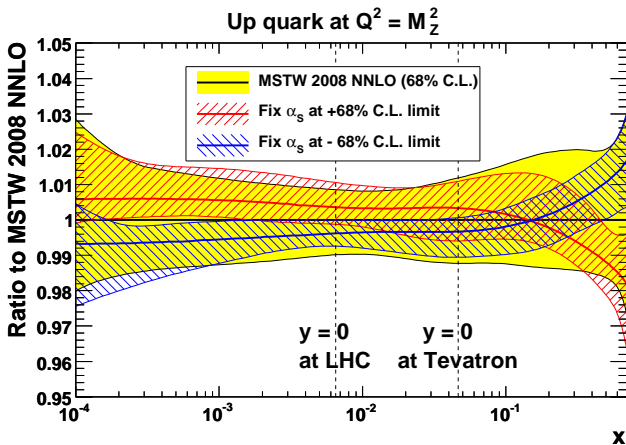
- **Upper** limit on  $\alpha_S(M_Z^2)$  provided by BCDMS  $F_2^{\mu p}$  data.
- **Lower** limit on  $\alpha_S(M_Z^2)$  provided by SLAC  $F_2^{ed}$  data.

# $\Delta\chi_{\text{global}}^2$ as a function of $\alpha_S(M_Z^2)$ for the NNLO global fit



- Additional theory uncertainty ( $\sim |\text{NNLO} - \text{NLO}| = \pm 0.003$ ).
- cf. PDG world average value of  $\alpha_S(M_Z^2) = 0.1176 \pm 0.002$ .

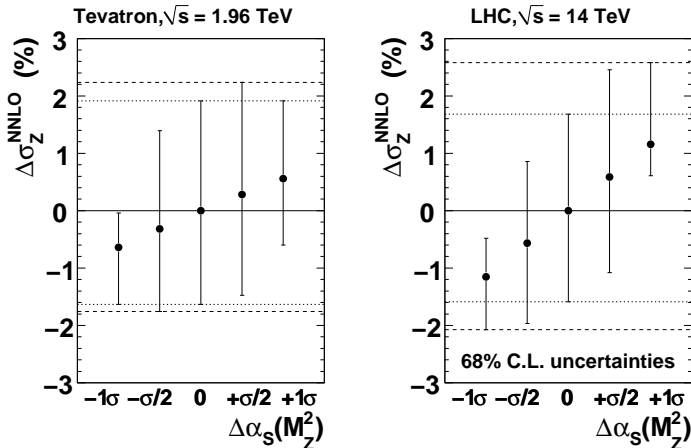
# Dependence of up quark distribution on $\alpha_S(M_Z^2)$



- **Correlation** at low  $x$ , **anticorrelation** at high  $x$ .
- PDF uncertainties **smaller** when  $\alpha_S$  shifted to limits.

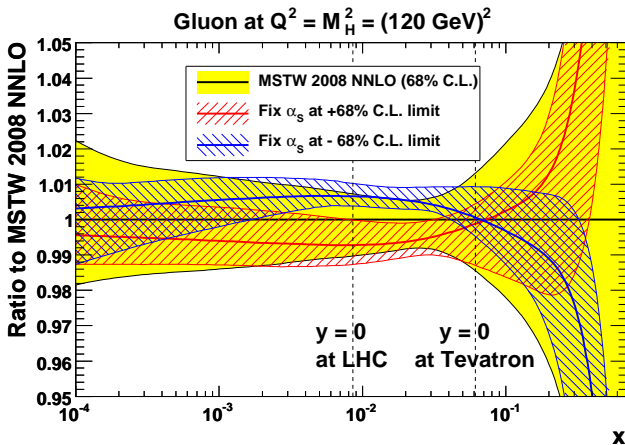
# Impact of $\alpha_S$ on $Z$ total cross section at NNLO

## $Z^0$ cross sections with MSTW 2008 NNLO PDFs



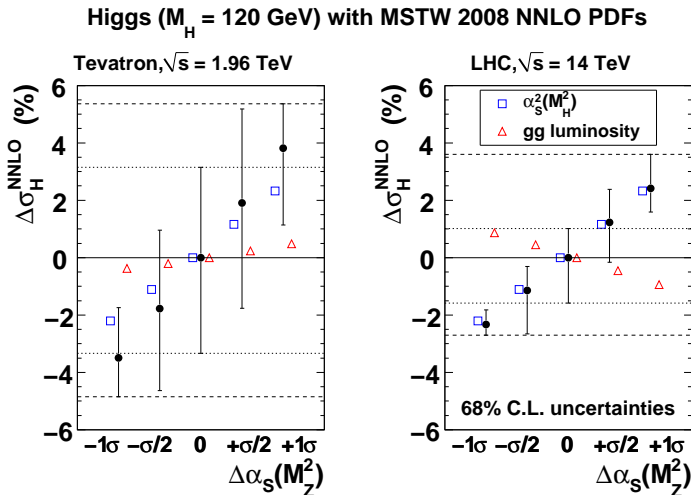
- Two effects: (i) correlation of quark distributions with  $\alpha_S$  at LHC (less correlation at Tevatron), (ii) higher-order corrections.

# Dependence of gluon distribution on $\alpha_S(M_Z^2)$



- **Anticorrelation** at low  $x$ : HERA  $\partial F_2 / \partial \ln Q^2 \sim \alpha_S g$ .
- **Correlation** at high  $x$  to maintain momentum sum rule.

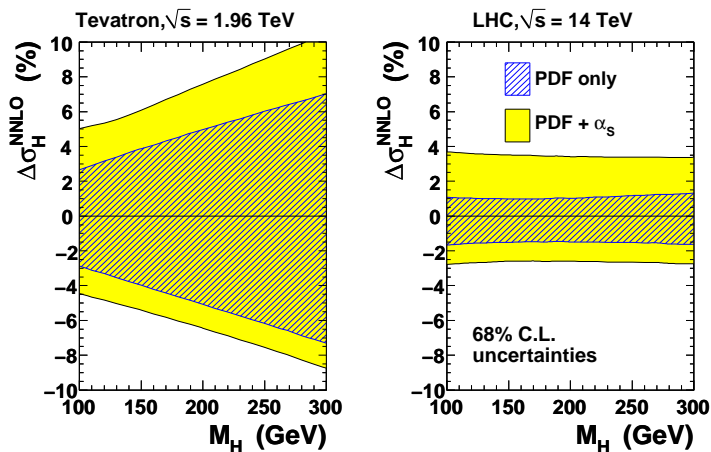
# Impact of $\alpha_S$ on SM Higgs total cross section at NNLO



- **Anticorrelation** of  $gg$  luminosity with  $\alpha_S$  at LHC cancels out (almost exactly) **correlation** due to higher-order corrections.

# Impact of $\alpha_S$ on SM Higgs uncertainty versus $M_H$

## Higgs cross sections with MSTW 2008 NNLO PDFs

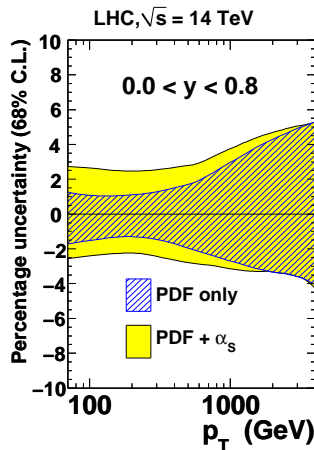
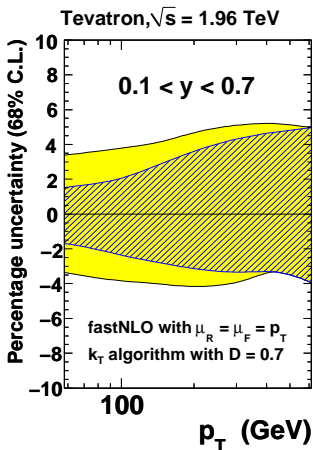


- Enhanced “PDF+ $\alpha_S$ ” uncertainty compared to “PDF only” uncertainty, particularly at the LHC.



# Impact of $\alpha_S$ on inclusive jet uncertainty versus $p_T$

## Inclusive jet cross sections with MSTW 2008 NLO PDFs

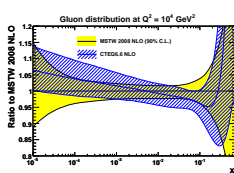
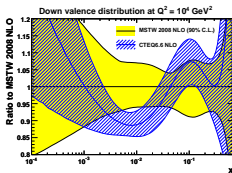
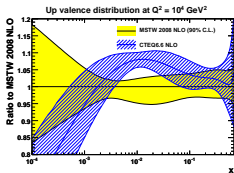
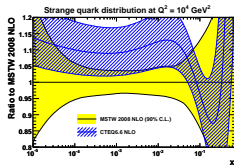
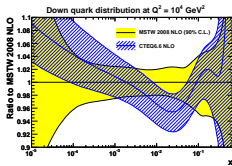
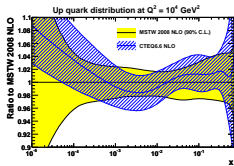


- Mostly gluon-initiated at low  $p_T \Rightarrow$  correlated with  $\alpha_S$ .
- Mostly quark-initiated at high  $p_T \Rightarrow$  anticorrelated with  $\alpha_S$ .

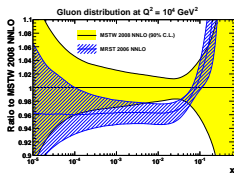
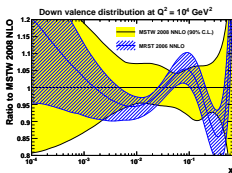
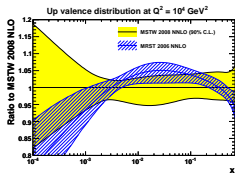
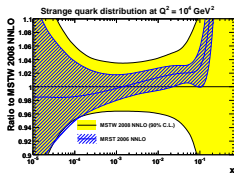
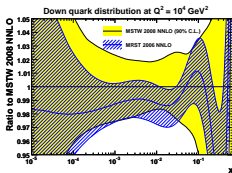
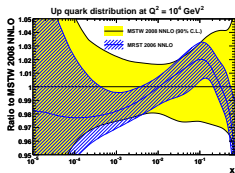
# Summary

- **MSTW 2008** (LO, NLO, NNLO) PDF fits [[arXiv:0901.0002](https://arxiv.org/abs/0901.0002)] are the most comprehensive to date: **supersede MRST sets**.
- Pre-2006 MRST NNLO PDF sets should be considered **obsolete** due to **incomplete heavy flavor treatment**.
- *Almost* all necessary NNLO processes are now known.  
*Exceptions:* inclusive jets, **massive**  $\mathcal{O}(\alpha_S^3)$  NC and  $\mathcal{O}(\alpha_S^2)$  CC DIS.
- **Tevatron Run II jets** prefer **smaller high- $x$  gluon** than Run I: impact on Higgs cross sections at Tevatron.
- **Improved “dynamic tolerance”** controlling propagation of experimental errors through to **PDF uncertainties**.
- Now possible to consistently calculate combined **“PDF+ $\alpha_S$ ”** uncertainty on cross sections: additional sets public soon.

# Comparison to CTEQ6.6 NLO



# Comparison to MRST 2006 NNLO



## Alternative approach: NNPDF Collaboration

**NNPDF Collaboration:** R. Ball, L. Del Debbio, S. Forte, A. Guffanti, J. Latorre, A. Piccione, J. Rojo, M. Ubiali

### MSTW approach [[arXiv:0901.0002](https://arxiv.org/abs/0901.0002)]

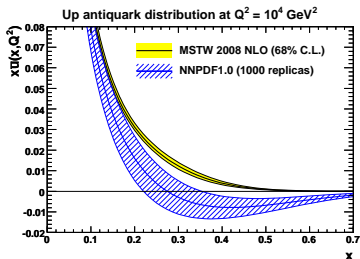
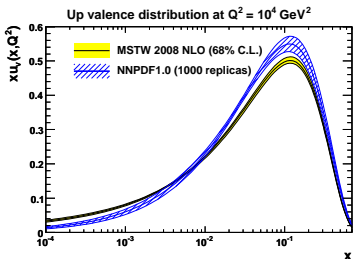
Parametrization	$xf_{a/p} \sim A_a x^{\Delta_a} (1-x)^{\eta_a} (1 + \epsilon_a \sqrt{x} + \gamma_a x)$
Minimisation	Non-linear least-squares (Marquardt method)
Error propagation	Hessian method with dynamical tolerance
Application	Use best-fit and 40 eigenvector PDF sets

### NNPDF approach [[arXiv:0808.1231](https://arxiv.org/abs/0808.1231)]

Parametrization	Neural network (37 free parameters per PDF)
Minimisation	Genetic algorithm (stop before overlearning)
Error propagation	Generate $N_{\text{rep}} \sim \mathcal{O}(1000)$ MC data replicas
Application	Calculate average and s.d. over $N_{\text{rep}}$ PDF sets

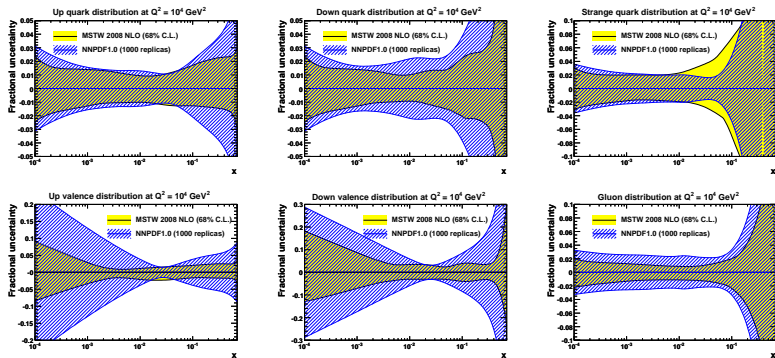
First results from NNPDF1.0 [[arXiv:0808.1231](https://arxiv.org/abs/0808.1231)]

- Fit restricted set of only DIS structure function data (SLAC, BCDMS, NMC, H1, ZEUS, CHORUS).
- Inadequate treatment of heavy quarks (ZM-VFNS).



- Up valence: relative data set normalizations fitted by MSTW.
- Up antiquark: NNPDF1.0 negative by  $\sim 2\text{-}\sigma$ , no Drell–Yan.

# Uncertainties for MSTW 2008 and NNPDF1.0



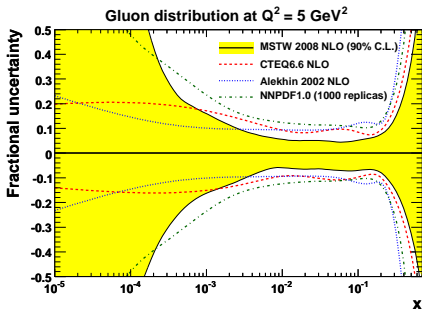
- NNPDF1.0 has fixed  $s = \bar{s} = (\bar{u} + \bar{d})/4$  at  $Q_0^2 = 2 \text{ GeV}^2$ .
- NNPDF1.1 [[arXiv:0811.2288](https://arxiv.org/abs/0811.2288)]: free strangeness but no  $\nu N$  dimuon data to constrain, so huge PDF uncertainties.
- **Conclusion:** NNPDF approach looks promising, but PDFs not yet directly comparable to those from standard approach.

# Parametrization dependence of low- $x$ gluon

- PDFs lose probabilistic interpretation beyond LO.
- Negative small- $x$  gluon distribution preferred at low scales.
- MRST/MSTW parametrize as:

$$xg(x, Q_0^2) = xg_1(x, Q_0^2) + xg_2(x, Q_0^2) \sim A_g x^{\delta_g} + A_{g'} x^{\delta_{g'}}$$

$$\Rightarrow \Delta g(x, Q_0^2) \sim \pm g_1(x, Q_0^2) \Delta\delta_g \ln(1/x) \pm g_2(x, Q_0^2) \Delta\delta_{g'} \ln(1/x)$$



- Other groups (CTEQ, Alekhin) parametrize with a valence-like  $xg(x, Q_0^2) \sim x^{\delta_g}$ : less freedom at small  $x$ .