Proposal for an improved “M_X method”

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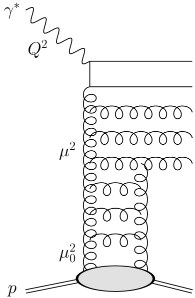
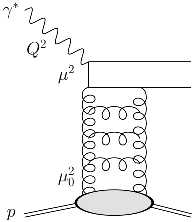
1. Collinear factorisation in DDIS
2. Motivation for an improved “M_X method”
3. Reminder of the “M_X method”
4. Regge theory of DDIS
5. The “$(M_X^2 + Q^2)$ method”
6. The “$x_{\mathbb{P}}$ method”
7. Summary

Collinear factorisation in DDIS

Resolved $\mathbb{P}$:



Direct $\mathbb{P}$:



- Pomeron structure is analogous to photon structure: both resolved and direct contributions.
- $Q^2 \gg \ldots \gg \mu^2 \gg \ldots \gg \mu_0^2 \sim 1\ \text{GeV}^2$
- Direct Pomeron contribution responsible for **exclusive** diffractive processes, but also contributes to **inclusive** diffraction.

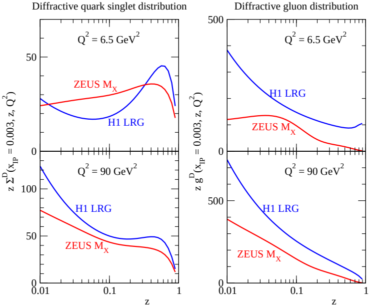
DDIS structure function: $$F_2^{\mathrm{D}(3)}(x_{\mathbb{P}}, \beta, Q^2) = \underbrace{\sum_{a=q,g} C_{2,a} \otimes a^{\mathrm{D}}}_{\text{Resolved Pomeron}} + \underbrace{C_{2,\mathbb{P}}}_{\text{Direct Pomeron}}$$

DPDF evolution: $$\frac{\partial a^{\mathrm{D}}(x_{\mathbb{P}}, z, Q^2)}{\partial \ln Q^2} = \underbrace{\sum_{a'=q,g} P_{aa'} \otimes a'^{\mathrm{D}}}_{\text{DGLAP term}} + \underbrace{P_{a\mathbb{P}}(z)\, f_{\mathbb{P}}(x_{\mathbb{P}}; Q^2)}_{\text{Inhomogeneous term}}$$

pQCD Pomeron flux factor: $$f_{\mathbb{P}}(x_{\mathbb{P}}; Q^2) = \frac{1}{x_{\mathbb{P}} B_D} \left[R_g \frac{\alpha_S(Q^2)}{Q}\, x_{\mathbb{P}} g(x_{\mathbb{P}}, Q^2) \right]^2$$

Motivation for an improved “M_X method”

- Diffractive gluon distribution sensitive to Q^2 dependence of DDIS data:



- See HERA-LHC proceedings.
- H1 LRG and ZEUS M_X data give very different DPDFs due to different Q^2 dependence of data sets.
- From results shown at last collaboration meeting:
  - New ZEUS LRG data also seem to have different Q^2 dependence than published ZEUS M_X data.
  - New ZEUS M_X (high Q^2) data seem to be compatible with published ZEUS M_X data.

Possible (tentative) explanations:

1. Amount of proton dissociation is Q^2 dependent?
   But would be difficult to explain theoretically.
2. Significant non-diffractive contribution to LRG events (even at small $x_{\mathbb{P}}$)?
   But should be exponentially suppressed compared to diffractive contribution.
3. Unjustified approximations made in “M_X method” used to subtract non-diffractive events?

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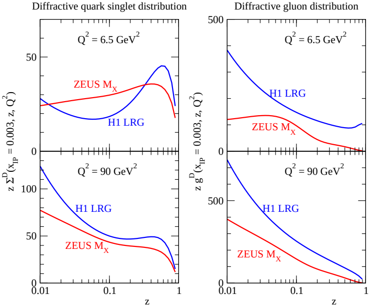

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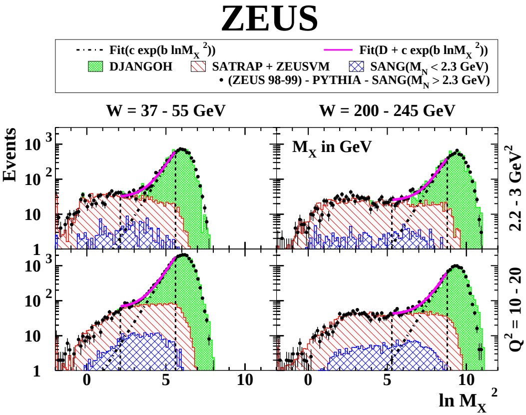
3. Unjustified approximations made in "M_X method" used to subtract non-diffractive events?

Reminder of the "M_X method"

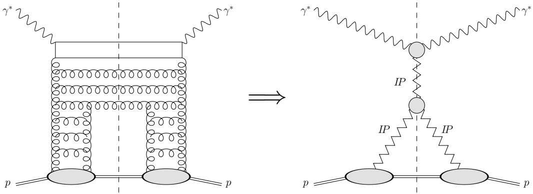
- Used in three ZEUS papers [Z. Phys. C **70** (1996) 391; Eur. Phys. J. C **6** (1999) 43; Nucl. Phys. B **713** (2005) 3] + analysis in progress.
- Subtract non-diffractive events in each (W, Q^2) bin by fitting (in a limited range of $\ln M_X^2$):

$$\frac{dN}{d\ln M_X^2} = D + \underbrace{c\exp(b\ln M_X^2)}_{\text{non-diffractive}}$$

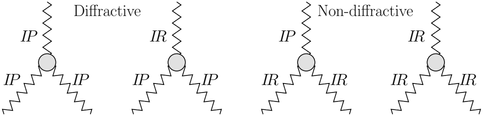


Regge theory of DDIS

- Replace pQCD ladders by "effective" Regge trajectories, e.g.



- But don't expect the "effective" trajectories to be universal, e.g. "effective" $\alpha_{\mathbb{P}}(0)$ greater than value for "soft" Pomeron and depends on Q^2 [NPB **713** (2005) 3].
- For $W^2 \gg M_X^2$ and $Q^2 \gg t$, consider triple Regge diagrams [see Barone & Predazzi, *High-energy particle diffraction*, Chap. 10.5]:



$$\frac{\mathrm{d}\sigma_{\gamma^* p}}{\mathrm{d}\ln x_{\mathbb{P}}} = \frac{|g_{\mathbb{P}}(\bar{t})|^2}{16\pi^2} x_{\mathbb{P}}^{2-2\alpha_{\mathbb{P}}(\bar{t})} \left[\mathcal{A}_{\mathbb{PPP}}(Q^2)\beta^{1-\alpha_{\mathbb{P}}(0)} + \mathcal{A}_{\mathbb{PPR}}(Q^2)\beta^{1-\alpha_{\mathbb{R}}(0)}\right]$$
$$+ \frac{|g_{\mathbb{R}}(\bar{t})|^2}{16\pi^2} x_{\mathbb{P}}^{2-2\alpha_{\mathbb{R}}(\bar{t})} \left[\mathcal{A}_{\mathbb{RRP}}(Q^2)\beta^{1-\alpha_{\mathbb{P}}(0)} + \mathcal{A}_{\mathbb{RRR}}(Q^2)\beta^{1-\alpha_{\mathbb{R}}(0)}\right],$$

where $\alpha_{\mathbb{P}}(0) \approx 1.1$–$1.2$, $\alpha_{\mathbb{R}}(0) \lesssim 0.5$, and $\bar{t}$ is some average value of t.

The "$(M_X^2 + Q^2)$ method"

- Since $x_{\mathbb{P}} = (M_X^2 + Q^2)/(W^2 + Q^2)$ and $\beta = Q^2/(M_X^2 + Q^2)$, rewrite as

$$\frac{\mathrm{d}\sigma_{\gamma^* p}}{\mathrm{d}\ln(M_X^2 + Q^2)} = A_{\mathbb{PPP}}(\bar{t})(W^2)^{2\alpha_{\mathbb{P}}(\bar{t})-2}(M_X^2 + Q^2)^{1+\alpha_{\mathbb{P}}(0)-2\alpha_{\mathbb{P}}(\bar{t})}$$
$$+A_{\mathbb{PPR}}(\bar{t})(W^2)^{2\alpha_{\mathbb{P}}(\bar{t})-2}(M_X^2 + Q^2)^{1+\alpha_{\mathbb{R}}(0)-2\alpha_{\mathbb{P}}(\bar{t})}$$
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- The "M_X method" assumes that $M_X^2 \gg Q^2$ ($\Rightarrow \beta \ll 1$, so $\mathbb{PPR}$ and $\mathbb{RRR}$ contributions are negligible), and $\alpha_{\mathbb{P}}(0) \approx \alpha_{\mathbb{P}}(\bar{t}) \approx 1$. Then

$$\frac{\mathrm{d}\sigma_{\gamma^* p}}{\mathrm{d}\ln M_X^2} = D + c(M_X^2)^b = D + c\exp(b\ln M_X^2),$$

where $b = 1 + \alpha_{\mathbb{P}}(0) - 2\alpha_{\mathbb{R}}(\bar{t})$.

- More generally, look at $(M_X^2 + Q^2)$ distribution:

$$\boxed{\frac{\mathrm{d}\sigma_{\gamma^* p}}{\mathrm{d}\ln(M_X^2 + Q^2)} = \underbrace{c_{\mathbb{P}}(M_X^2 + Q^2)^{-b_{\mathbb{P}}}}_{\text{diffractive}} + \underbrace{c_{\mathbb{R}}(M_X^2 + Q^2)^{b_{\mathbb{R}}}}_{\text{non-diffractive}},}$$

where $-b_{\mathbb{P}} \leq 1 + \alpha_{\mathbb{P}}(0) - 2\alpha_{\mathbb{P}}(\bar{t}) \approx -(0.1\text{–}0.2)$ and $b_{\mathbb{R}} \geq 1 + \alpha_{\mathbb{R}}(0) - 2\alpha_{\mathbb{R}}(\bar{t}) \gtrsim 0.5$.

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$b_{\mathbb{R}} \geq 1+\alpha_{\mathbb{R}}(0)-2\alpha_{\mathbb{R}}(\bar{t}) \gtrsim 0.5$.

The "$x_{\mathbb{P}}$ method"

- The "$(M_X^2 + Q^2)$ method" is better than the "M_X method", but it uses a combination of two powers of $(M_X^2 + Q^2)$ to approximate a combination of four powers.
- Much more direct way of subtracting the non-diffractive contribution is to look at the $x_{\mathbb{P}}$ distribution:

$$\frac{\mathrm{d}\sigma_{\gamma^* p}}{\mathrm{d}\ln x_{\mathbb{P}}} = \frac{\left|g_{\mathbb{P}}(\bar{t})\right|^2}{16\pi^2} x_{\mathbb{P}}^{2-2\alpha_{\mathbb{P}}(\bar{t})} \left[\mathcal{A}_{\mathbb{PPP}}(Q^2)\beta^{1-\alpha_{\mathbb{P}}(0)} + \mathcal{A}_{\mathbb{PPR}}(Q^2)\beta^{1-\alpha_{\mathbb{R}}(0)}\right] + \frac{\left|g_{\mathbb{R}}(\bar{t})\right|^2}{16\pi^2} x_{\mathbb{P}}^{2-2\alpha_{\mathbb{R}}(\bar{t})} \left[\mathcal{A}_{\mathbb{RRP}}(Q^2)\beta^{1-\alpha_{\mathbb{P}}(0)} + \mathcal{A}_{\mathbb{RRR}}(Q^2)\beta^{1-\alpha_{\mathbb{R}}(0)}\right]$$

$$\Rightarrow \boxed{\frac{\mathrm{d}\sigma_{\gamma^* p}}{\mathrm{d}\ln x_{\mathbb{P}}} = \underbrace{c_{\mathbb{P}}(x_{\mathbb{P}})^{-b_{\mathbb{P}}}}_{\text{diffractive}} + \underbrace{c_{\mathbb{R}}(x_{\mathbb{P}})^{b_{\mathbb{R}}}}_{\text{non-diffractive}},}$$

where $-b_{\mathbb{P}} = 2 - 2\alpha_{\mathbb{P}}(\bar{t}) \approx -(0.2\text{–}0.4)$ and $b_{\mathbb{R}} = 2 - 2\alpha_{\mathbb{R}}(\bar{t}) \gtrsim 1$.

- In most general case, fit four parameters $c_{\{\mathbb{P},\mathbb{R}\}} \geq 0$ and $b_{\{\mathbb{P},\mathbb{R}\}} \geq 0$ separately in each (β, Q^2) bin, in some limited range of $x_{\mathbb{P}}$.

Summary

- "M_X method" justified if $M_X^2 \gg Q^2$ and $\alpha_{\mathbb{P}}(0) = 1$:

$$\frac{\mathrm{d}N}{\mathrm{d}\ln M_X^2} = \underbrace{D}_{\text{diffractive}} + \underbrace{c(M_X^2)^b}_{\text{non-diffractive}}$$

- "$(M_X^2 + Q^2)$ method" more general:

$$\boxed{\frac{\mathrm{d}N}{\mathrm{d}\ln(M_X^2 + Q^2)} = \underbrace{c_{\mathbb{P}}(M_X^2 + Q^2)^{-b_{\mathbb{P}}}}_{\text{diffractive}} + \underbrace{c_{\mathbb{R}}(M_X^2 + Q^2)^{b_{\mathbb{R}}}}_{\text{non-diffractive}}}$$

- "$x_{\mathbb{P}}$ method" even better:

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- More details given in write-up uploaded to ZEMS.