

# Diffractive, generalised and unintegrated PDFs

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# Introduction

- Most effort has been concentrated on determining “standard” PDFs for use in **fixed-order** (usually NLO,  $\overline{\text{MS}}$ ) **collinear factorisation** calculations of **inclusive processes**.
- (This is the cleanest theoretical situation, but is not necessarily consistent with calculations in MC generators.)
- **This talk:** discuss some “**non-standard**” PDFs used for specific types of processes: **diffractive**, **generalised** and **unintegrated**.
- (Won't discuss polarised, nuclear, multiparton, . . . , PDFs.)
- See also talk by K. Kutak for a different approach to unintegrated PDFs.

# Introduction to diffraction

- **Diffractive** processes characterised by a **Large Rapidity Gap**.
- Exchange of **vacuum quantum numbers** at **high energies** ( $\equiv$  “**Pomeron**” exchange). What is the **Pomeron** ( $\mathbb{P}$ )?

## “Soft” processes ( $\sim 1$ fm)

- Use Regge theory.
- $\mathbb{P}$  is a Regge trajectory:  

$$\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}} t.$$
- $\sigma_{\text{tot}} \sim s^{\alpha_{\mathbb{P}}(0)-1}$
- $d\sigma_{\text{el}}/dt \sim s^{2[\alpha_{\mathbb{P}}(t)-1]}$
- Donnachie–Landshoff fit [hep-ph/9209205]:  

$$\alpha_{\mathbb{P}}(0) = 1.08, \alpha'_{\mathbb{P}} = 0.25 \text{ GeV}^{-2}$$
- *Effective*: includes multi- $\mathbb{P}$ .

## “Hard” processes ( $\gtrsim 1$ GeV)

- Use perturbative QCD.
- $\mathbb{P} \sim$  DGLAP/BFKL ladder  
 $\sim$  usual gluon density.
- Appears in cross section for inclusive processes.
- Appears in **amplitude** for diffractive processes.  
 $\Rightarrow$  Appears **squared** in the diffractive cross section.

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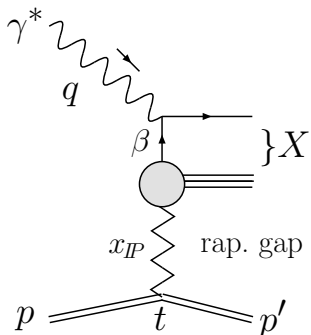
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# Inclusive diffractive DIS (DDIS) kinematics



- $q^2 \equiv -Q^2$  (photon virtuality)
- $W^2 \equiv (q + p)^2 \simeq -Q^2 + 2 p \cdot q$   
 $\Rightarrow x_{\text{Bj}} \equiv \frac{Q^2}{2 p \cdot q} \simeq \frac{Q^2}{Q^2 + W^2}$  (at LO, fraction of proton's momentum carried by struck quark)
- $t \equiv (p - p')^2 \simeq 0$ ,  $(p - p') \simeq x_{\mathbb{P}} p$

- $M_X^2 \equiv (q + p - p')^2 \simeq -Q^2 + x_{\mathbb{P}}(Q^2 + W^2)$   
 $\Rightarrow x_{\mathbb{P}} \simeq \frac{Q^2 + M_X^2}{Q^2 + W^2} \ll 1$   
 (fraction of proton's momentum carried by Pomeron)
- $\beta \equiv \frac{x_{\text{Bj}}}{x_{\mathbb{P}}} \simeq \frac{Q^2}{Q^2 + M_X^2}$  (at LO, fraction of Pomeron's momentum carried by struck quark)

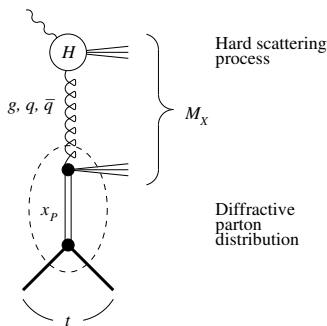
# Leading-twist collinear factorisation in DDIS

Diffractive structure function (integrated over  $t$ ):

$$F_2^{\text{D}(3)}(x_{\mathbb{P}}, \beta, Q^2) = \sum_{a=q,g} \beta \int_{\beta}^1 \frac{dz}{z} C_{2,a} \left( \frac{\beta}{z} \right) f_{a/p}^{\text{D}}(x_{\mathbb{P}}, z, \mu_F^2)$$

$$= \sum_q e_q^2 \beta f_{q/p}^{\text{D}}(x_{\mathbb{P}}, \beta, \mu_F^2) \quad \text{at LO.}$$

- $C_{2,a}$  are the **same** coefficient functions as in inclusive DIS.
- Diffractive PDFs  $f_{a/p}^{\text{D}}$  satisfy DGLAP evolution.
- Proven by J. Collins [hep-ph/9709499] to hold up to power-suppressed corrections.



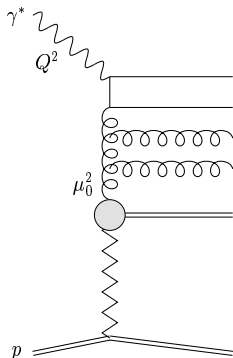
# Regge factorisation in DDIS

Ingelman, Schlein [PLB 152 (1985) 256].

Assume Regge factorisation:

$$f_{a/p}^D(x_{\mathbb{P}}, z, \mu_F^2) = f_{\mathbb{P}/p}(x_{\mathbb{P}}) f_{a/\mathbb{P}}(z, \mu_F^2)$$

- $f_{\mathbb{P}/p}(x_{\mathbb{P}})$  is the **Pomeron flux factor**.  
Take from Regge phenomenology.
- $f_{a/\mathbb{P}}(z, \mu_F^2)$  are the **Pomeron PDFs**  
DGLAP-evolved from an input scale  $\mu_0^2$   
up to the factorisation scale  $\mu_F^2$  of the  
hard scattering process.



**Paradox:** **hard** scattering, but assumption of **soft** Pomeron exchange due to DGLAP strong-ordering in virtualities:

$$\mu_F^2 \gg \dots \gg \mu^2 \gg \dots \gg \mu_0^2 \sim 1 \text{ GeV}^2.$$

# H1 2006/7 analysis of diffractive PDFs

## Pomeron flux factor

$$f_{\mathbb{P}/p}(x_{\mathbb{P}}) = \int_{t_{\text{cut}}}^{t_{\text{min}}} dt e^{B_{\mathbb{P}} t} x_{\mathbb{P}}^{1-2\alpha_{\mathbb{P}}(t)},$$

- Fix  $\alpha'_{\mathbb{P}} = 0.06_{-0.06}^{+0.19} \text{ GeV}^{-2}$  and  $B_{\mathbb{P}} = 5.5_{-2.0}^{+0.7} \text{ GeV}^{-2}$  from H1 leading-proton data [hep-ex/0606003].
- Treat  $\alpha_{\mathbb{P}}(0)$  as a free parameter.

## Input Pomeron PDFs

$$z f_{a/\mathbb{P}}(z, \mu_0^2) = A_a z^{B_a} (1-z)^{C_a}$$

- Evolve from  $\mu_0^2 \simeq 2 \text{ GeV}^2$  with usual NLO DGLAP evolution.
- No momentum sum rule: Pomeron is not a particle.
- $u = d = s = \bar{u} = \bar{d} = \bar{s}$ , so only singlet and gluon.



# H1 2006/7 extraction of diffractive PDFs

- Fit to inclusive DDIS data points with  $M_X \geq 2$  GeV and  $Q^2 \geq 8.5$  GeV<sup>2</sup> [hep-ex/0606004].
- Also simultaneous fit to inclusive DDIS and DDIS dijets [0708.3217].

	$\alpha_{\mathbb{P}}(0)$	$z f_{g/\mathbb{P}}(z, \mu_0^2)$	$\chi_{\text{DDIS}}^2/N_{\text{pts.}}$	$\chi_{\text{dijet}}^2/N_{\text{pts.}}$
<b>H1 2006 Fit A</b>	$1.118 \pm 0.007$	$A_g (1-z)^{C_g}$	158/190	—
<b>H1 2006 Fit B</b>	$1.111 \pm 0.007$	$A_g$	164/190	—
<b>H1 2007 Jets Fit</b>	$1.104 \pm 0.007$	$A_g z^{B_g} (1-z)^{C_g}$	169/190	27/36

- Comments:

- ① Cut on  $Q^2 \geq 8.5$  GeV<sup>2</sup> required to achieve stability.
- ②  $\alpha_{\mathbb{P}}(0)$  values larger than values for soft Pomeron.  
No reason to expect universality of these values.
- ③ Fit B would be excluded if  $\Delta\chi^2 = 1$  is taken literally.
- ④  $\chi^2$  of inclusive DDIS data deteriorates when dijet data are added: suggests some inconsistency within this framework.

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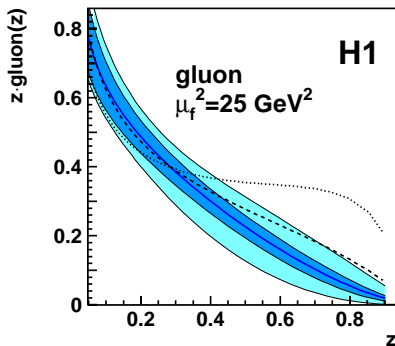
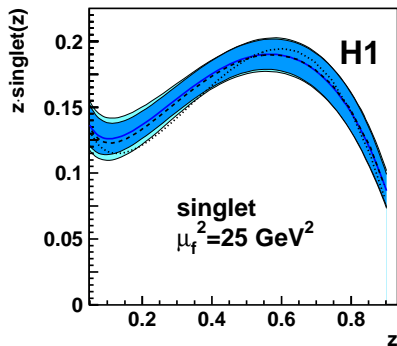
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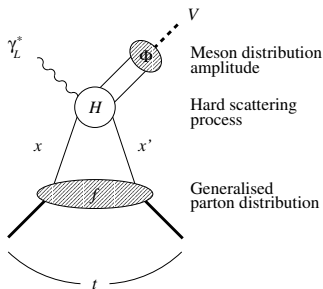
- H1 2007 Jets DPDF
- exp. uncertainty
- exp. + theo. uncertainty
- H1 2006 DPDF fit A
- H1 2006 DPDF fit B

- Gluon determined **indirectly** from scaling violations of inclusive DDIS data (Fit A) **larger at high z** compared to gluon determined **directly** from dijet data.

# Collinear factorisation for exclusive hard processes

Collins, Frankfurt, Strikman [hep-ph/9611433]:

$$\begin{aligned}
 \mathcal{A}(\gamma_L^* p \rightarrow V p) &= \sum_{i,j} \int_0^1 dz \int dx \overbrace{f_{i/p}(x, x', t, \mu^2)}^{\text{Generalised PDF}} \overbrace{H_{ij}(x, x', z, Q^2, \mu^2)}^{\text{Hard scattering}} \overbrace{\Phi_j^V(z, \mu^2)}^{\text{Meson dist. amp.}} \\
 &+ \text{power-suppressed corrections.}
 \end{aligned}$$



- Valid also for large  $x$ , not just in the diffractive region  $x \ll 1$ .
- Generalised (a.k.a. skewed, off-diagonal) PDFs reduce to usual PDFs in limit of  $x = x'$  and  $t = 0$ .
- Fourier transform from  $\Delta$  (where  $t = -|\Delta|^2$ ) to impact parameter  $\mathbf{b} \Rightarrow$  spatial distribution of partons in transverse plane.

## Progress in determination of generalised PDFs

- **NLO corrections** for  $\gamma_L^* p \rightarrow \rho p$  are **huge at small-x** [Diehl, Kugler, 0708.1121]. Small-x resummation needed. Important **power corrections** due to parton transverse momenta.
- Situation better for  $\gamma^* p \rightarrow \gamma p$  (deeply virtual Compton scattering): known to NNLO [Kumerički, Müller, Passek-Kumerički, hep-ph/0703179].
- “**Global**” analysis of **generalised PDFs** still a long way off. Generally rely on models/approximations to relate generalised PDFs to usual PDFs, e.g. for small  $|t|$  and  $x' \ll x \ll 1$ :

$$f_{g/p}(x, x', t, \mu^2) = \exp(B_D t/2) R_g(x, \mu^2) xg(x, \mu^2),$$

with  $t$ -slope  $B_D$  taken from HERA data and

$$R_g(x, \mu^2) = \frac{2^{2\lambda+3}}{\sqrt{\pi}} \frac{\Gamma(\lambda + 5/2)}{\Gamma(\lambda + 4)},$$

assuming  $xg(x, \mu^2) \sim x^{-\lambda}$  [Shuvaev *et al.*, hep-ph/9902410].

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# Perturbative Pomeron contribution to inclusive DDIS

Martin, Ryskin, G.W. [hep-ph/0406224, hep-ph/0504132, hep-ph/0609273]

- **Hard** Pomeron responsible for **exclusive** diffractive processes at HERA also contributes to **inclusive** DDIS.

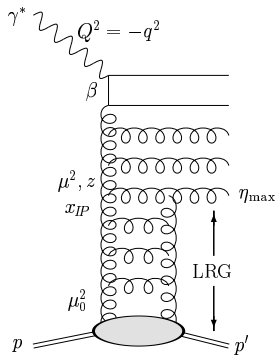
- Contribution to **diffractive** PDFs is **calculable** in terms of the **generalised** PDFs:

$$f_{a/p}^D(x_{\mathbb{P}}, z, \mu_F^2) = \int_{\mu_0^2}^{\mu_F^2} \frac{d\mu^2}{\mu^2} f_{\mathbb{P}/p}(x_{\mathbb{P}}; \mu^2) f_{a/\mathbb{P}}(z, \mu_F^2; \mu^2)$$

- Perturbative Pomeron flux factor:

$$f_{\mathbb{P}/p}(x_{\mathbb{P}}; \mu^2) = \frac{1}{x_{\mathbb{P}} B_D} \left[ R_g \frac{\alpha_S(\mu^2)}{\mu} x_{\mathbb{P}} g(x_{\mathbb{P}}, \mu^2) \right]^2$$

- Pomeron PDFs  $f_{a/\mathbb{P}}(z, \mu_F^2; \mu^2)$  DGLAP-evolved up to  $\mu_F^2$  from input Pomeron-to-parton splitting functions  $P_{a/\mathbb{P}}(z) \equiv f_{a/\mathbb{P}}(z, \mu^2; \mu^2)$ .



# Evolution equation for diffractive PDFs

- Treat contribution from  $\mu^2 < \mu_0^2 \sim 1 \text{ GeV}^2$  as in usual Regge factorisation approach (need to fit to data):

$$\Rightarrow f_{a/p}^{\text{D}}(x_{\mathbb{P}}, z, \mu_F^2) = f_{\mathbb{P}/p}(x_{\mathbb{P}}) f_{a/\mathbb{P}}(z, \mu_F^2) + \int_{\mu_0^2}^{\mu_F^2} \frac{d\mu^2}{\mu^2} f_{\mathbb{P}/p}(x_{\mathbb{P}}; \mu^2) f_{a/\mathbb{P}}(z, \mu_F^2; \mu^2)$$

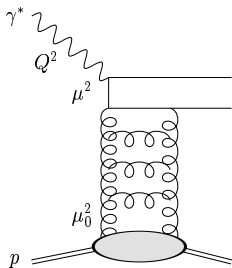
- Differentiate with respect to  $\ln \mu_F^2$ :

$$\frac{\partial f_{a/p}^{\text{D}}(x_{\mathbb{P}}, z, \mu^2)}{\partial \ln \mu^2} = \sum_{a'=q,g} P_{aa'} \otimes f_{a'/p}^{\text{D}} + P_{a\mathbb{P}}(z) f_{\mathbb{P}/p}(x_{\mathbb{P}}; \mu^2)$$

- Extra **inhomogeneous term** in the evolution equation due to the perturbative **Pomeron-to-parton splitting**.
- cf. the evolution equation for the **photon** PDFs also has an inhomogeneous term from the photon-to-parton splitting.



# Direct Pomeron contribution to DDIS

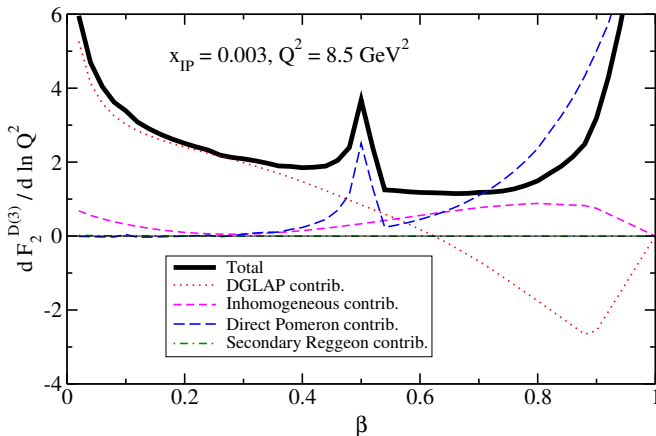


- Perturbative Pomeron can participate **directly** in the hard scattering process.
- “**Direct Pomeron**” contribution is analogous to the “**direct photon**” component of the photon structure function.

$$F_2^{D(3)}(x_{\mathbb{P}}, \beta, Q^2) = \underbrace{\sum_{a=q,g} C_{2,a} \otimes f_{a/p}^D}_{\text{Resolved Pomeron}} + \underbrace{C_{2,\mathbb{P}}}_{\text{Direct Pomeron}}$$

$$\text{where } \frac{\partial f_{a/p}^D(x_{\mathbb{P}}, z, Q^2)}{\partial \ln Q^2} = \underbrace{\sum_{a'=q,g} P_{aa'} \otimes f_{a'/p}^D}_{\text{DGLAP term}} + \underbrace{P_{a\mathbb{P}}(z) f_{\mathbb{P}/p}(x_{\mathbb{P}}; Q^2)}_{\text{Inhomogeneous term}}$$

# Effect of perturbative Pomeron terms on $Q^2$ slope

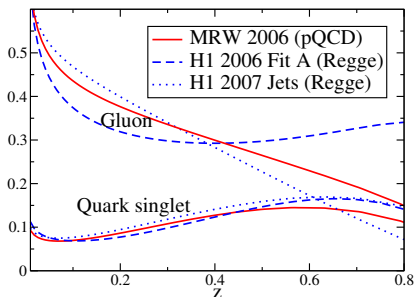


- Peak due to threshold for  $\gamma^* \mathbb{P} \rightarrow c \bar{c}$  at  $\beta = Q^2 / (Q^2 + 4m_c^2)$ .
- Additional contributions to scaling violations apart from DGLAP contribution, important for  $\beta \gtrsim 0.3$ .

# MRW 2006 analysis of diffractive PDFs [hep-ph/0609273]

- Repeat the H1 2006 analysis with the additional LO perturbative Pomeron terms included (using MRST NLO PDFs in the perturbative Pomeron flux factor).

$$x_{\text{IP}} = 0.003, Q^2 = 10 \text{ GeV}^2$$

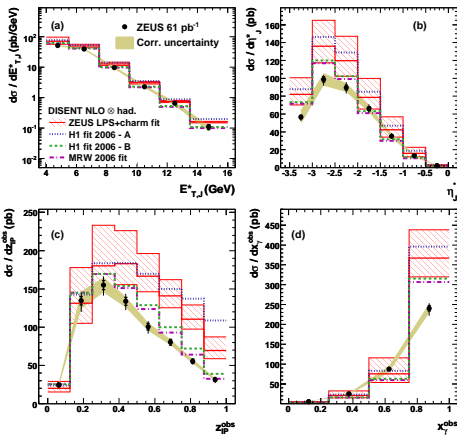


- Smaller gluon at high  $z$  compared to H1 2006 Fit A.
- $\Rightarrow$  Tension between inclusive DDIS and dijet data in Regge factorisation approach partly alleviated by inclusion of perturbative Pomeron terms.

- MRW 2006 diffractive PDFs available from <http://durpdg.dur.ac.uk/hepdata/mrw.html>

# Comparison to ZEUS data on DDIS dijets [0708.1415]

## ZEUS

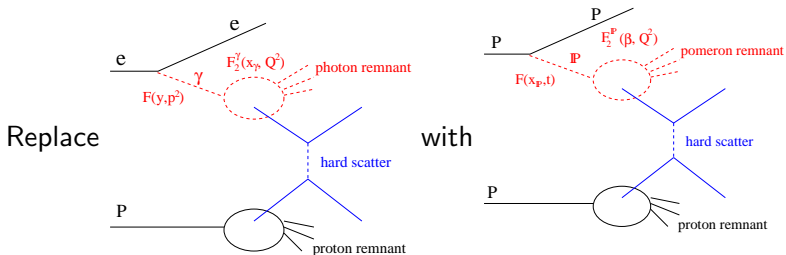


- Only the “direct photon” and “resolved Pomeron” component.
- MRW 2006 diffractive PDFs give the best description of data.

# Diffractive PDFs in Monte Carlo event generators

Not used in the “standard” MCs (PYTHIA, HERWIG, SHERPA).

- RAPGAP for  $ep$  collisions (H. Jung).
- POMWIG (B. Cox and J. Forshaw). Philosophy:



- POMPYT (P. Bruni, A. Edin and G. Ingelman): not current.

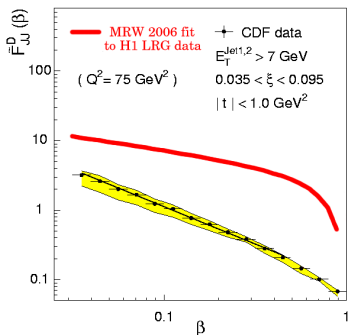
RAPGAP and POMWIG have the recent H1 2006 diffractive PDFs.

# Factorisation breaking in diffractive $pp$ and $p\bar{p}$ collisions

- Consider diffractive dijet production at Tevatron.  
Diffractive structure function of the antiproton:

$$\tilde{F}_{JJ}^D(\beta) = \frac{1}{\xi_{\max} - \xi_{\min}} \int_{\xi_{\min}}^{\xi_{\max}} d\xi \left[ \beta g^D(\xi, \beta, Q^2) + \frac{4}{9} \beta \Sigma^D(\xi, \beta, Q^2) \right],$$

measured by CDF [PRL **84** (2000) 5043].

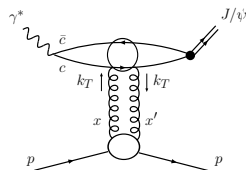


- Factorisation broken** in diffractive hadron–hadron collisions by (soft) interaction between spectator partons.
- Suppression factor  $S^2$  calculable** from eikonal models with parameters fitted to soft hadron–hadron data.

# Unintegrated, generalised PDFs

Martin, Ryskin, Teubner [hep-ph/9912551]:

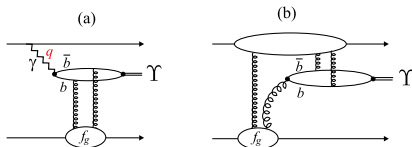
- Exclusive  $J/\psi$  production at HERA calculated within  $k_t$ -factorisation using a generalised, unintegrated gluon density ( $T_g =$  Sudakov form factor):



$$f_g(x, x', t, k_t^2, \mu^2) = \exp(B_D t/2) R_g \frac{\partial [T_g(k_t^2, \mu^2) x g(x, k_t^2)]}{\partial \ln k_t^2}$$

Khoze, Martin, Ryskin [0802.0177]:

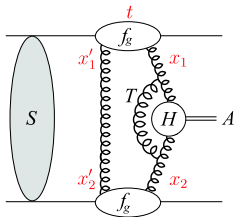
- Probe  $f_g$  by measuring exclusive  $\Upsilon$  production at LHC via photon/odderon exchange.



# Relevance for central exclusive production at LHC

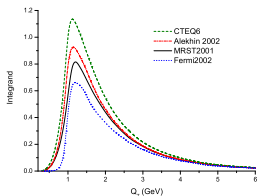
Khoze, Martin, Ryskin [0802.0177, and references therein]:

$$\sigma(pp \rightarrow p+A+p) \sim \frac{S^2}{B_D^2} \left| \frac{\pi}{8} \int \frac{dQ_T^2}{Q_T^4} f_g(x_1, x'_1, Q_T^2, \mu^2) f_g(x_2, x'_2, Q_T^2, \mu^2) \right|^2 \hat{\sigma}(gg \rightarrow A)$$



$$f_g(x, x', Q_T^2, \mu^2) = R_g \frac{\partial}{\partial \ln Q_T^2} \left[ \sqrt{T_g(Q_T^2, \mu^2)} xg(x, Q_T^2) \right]$$

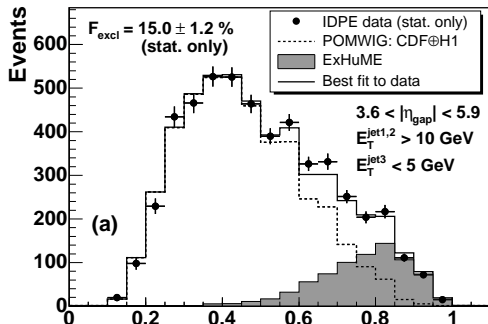
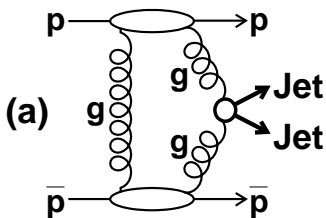
- Sudakov factor,  $T_g(Q_T^2, \mu^2)$ , suppresses the infrared  $Q_T$  region.
- Plot by J. Forshaw [hep-ph/0508274].
- Integrand dominated by  $Q_T \sim 1-2$  GeV  $\Rightarrow$  pQCD applicable (just).
- Calculations implemented in ExHuME event generator (J. Monk and A. Pilkington).





# Exclusive diffractive dijet production at the Tevatron

- Distribution of dijet mass fraction  $R_{jj} = M_{jj}/M_X$  measured by CDF [0712.0604].



- 1 POMWIG = generator for resolved Pomeron contribution.
- 2 ExHuME = generator for direct Pomeron contribution.

Resolved Pomeron contribution is a **background** to the exclusive diffractive signal (shown shaded).

# Doubly-unintegrated PDFs

G.W., Martin, Ryskin [[hep-ph/0306169](#), [hep-ph/0309096](#)]

Type of PDF	PDF integrated over	Incoming momentum
Integrated	$k^-, \mathbf{k}_t$	$k = (k^+, 0, \mathbf{0})$
Unintegrated	$k^-$	$k = (k^+, 0, \mathbf{k}_t)$
<b>Doubly-unintegrated</b>	<b>Nothing!</b>	<b><math>k = (k^+, k^-, \mathbf{k}_t)</math></b>

- Doubly-unintegrated PDFs in terms of integrated PDFs from last step of LO DGLAP evolution:

$$\begin{aligned}
 f_a(k, \mu^2) &\equiv f_a(x, z, k_t^2, \mu^2) \\
 &= T_a(k_t^2, \mu^2) \frac{\alpha_S(k_t^2)}{2\pi} \sum_{b=q,g} P_{ab}(z) \frac{x}{z} b\left(\frac{x}{z}, k_t^2\right) \\
 &\quad + \text{angular-ordering constraints}
 \end{aligned}$$

- Take input integrated PDFs  $b(x/z, k_t^2)$  from MRST 2001 LO.

# Doubly-unintegrated PDFs

G.W., Martin, Ryskin [hep-ph/0306169, hep-ph/0309096]

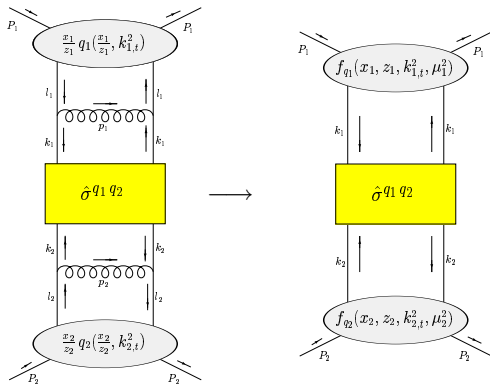
Type of PDF	PDF integrated over	Incoming momentum
Integrated	$k^-, \mathbf{k}_t$	$k = (k^+, 0, \mathbf{0})$
Unintegrated	$k^-$	$k = (k^+, 0, \mathbf{k}_t)$
<b>Doubly-unintegrated</b>	<b>Nothing!</b>	<b><math>k = (k^+, k^-, \mathbf{k}_t)</math></b>

- Doubly-unintegrated PDFs in terms of integrated PDFs from last step of LO DGLAP evolution:

$$\begin{aligned}
 f_a(k, \mu^2) &\equiv f_a(x, z, k_t^2, \mu^2) \\
 &= T_a(k_t^2, \mu^2) \frac{\alpha_S(k_t^2)}{2\pi} \sum_{b=q,g} P_{ab}(z) \frac{x}{z} b\left(\frac{x}{z}, k_t^2\right) \\
 &\quad + \text{angular-ordering constraints}
 \end{aligned}$$

- Take input integrated PDFs  $b(x/z, k_t^2)$  from MRST 2001 LO.

# Factorisation with doubly-unintegrated PDFs



[hep-ph/0309096]

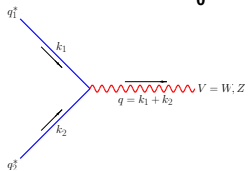
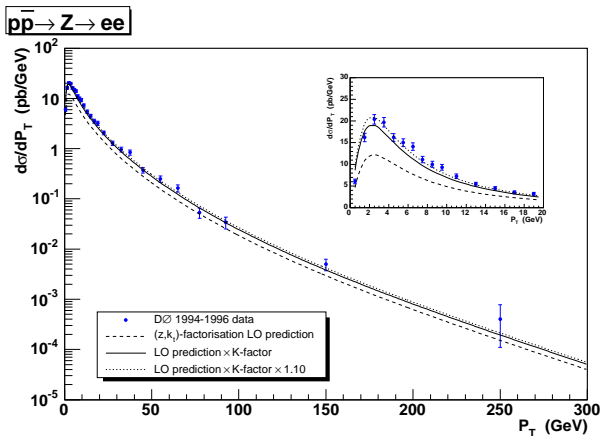
$$\hat{d}\hat{\sigma} = d\Phi |\mathcal{M}|^2 / F$$

- Evaluate  $|\mathcal{M}|^2 / F$  in collinear approximation:  
 $k_i = (k_i^+, 0, \mathbf{0})$ .
- Evaluate  $d\Phi$  with **full kinematics**:  
 $k_i = (k_i^+, k_i^-, \mathbf{k}_t)$ .
- No small- $x$  approximations are made.

$$\sigma = \int d^4 k_1 \int d^4 k_2 f_{q_1}(k_1, \mu_1^2) f_{q_2}(k_2, \mu_2^2) \hat{\sigma}^{q_1 q_2}$$

See also: Collins, Zu [hep-ph/0411332]; Collins, Rogers, Staśto [0708.2833]; Höche, Krauss, Teubner [0705.4577].

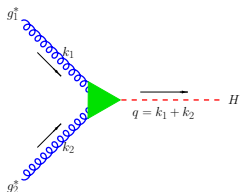
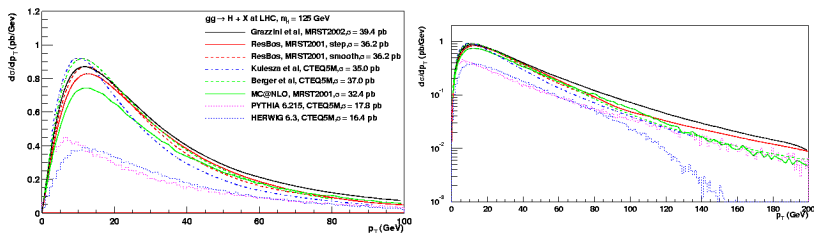
# $Z P_T$ distribution at Tevatron Run I



- $K\text{-factor} = \exp\left(\pi^2 C_F \alpha_S / 2\pi\right)$
- Good description of data over entire  $P_T$  range from only LO  $q\bar{q} \rightarrow Z$  subprocess: **explicit matrix-element corrections not needed at large  $P_T$ .**

# Higgs $P_T$ distribution at LHC

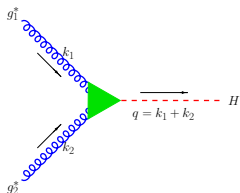
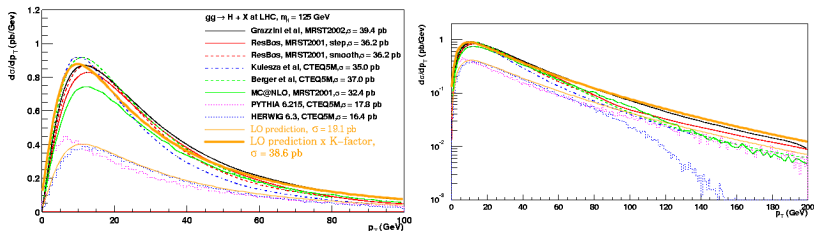
C. Balazs, M. Grazzini, J. Huston, A. Kulesza and I. Puljak [hep-ph/0403052]:



- Overlay predictions using doubly-integrated PDFs (orange lines).
- $K\text{-factor} = \exp(\pi^2 C_A \alpha_S / 2\pi)$
- Only LO  $gg \rightarrow H$  subprocess: explicit matrix-element corrections not needed at large  $P_T$ , cf. HERWIG.
- Code: <http://www.hep.ucl.ac.uk/~watt/>

# Higgs $P_T$ distribution at LHC

C. Balazs, M. Grazzini, J. Huston, A. Kulesza and I. Puljak [hep-ph/0403052]:



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- Code: <http://www.hep.ucl.ac.uk/~watt/>

## Diffractive PDFs

- Inclusive diffraction  $\Rightarrow$  resolved and direct Pomeron.
- Exclusive diffraction  $\Rightarrow$  only direct Pomeron.
- Diffractive PDFs relevant for resolved Pomeron contribution.
- Inhomogeneous evolution analogous to photon PDFs.
- POMWIG event generator for LHC.

## Generalised, unintegrated PDFs

- Relevant for exclusive diffractive processes.
- Can be approximately written in terms of usual PDFs.
- ExHUME event generator for LHC.

## Doubly-unintegrated PDFs

- Relevant for  $p_T$  distributions of final-state particles.
- Get the kinematics right at LO  
 $\Rightarrow$  less need for higher-order corrections.