## Diffractive parton density functions

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In collaboration with A.D. Martin and M.G. Ryskin

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## Outline

- Diffractive deep-inelastic scattering (DDIS) is characterised by a large rapidity gap due to Pomeron (vacuum quantum number) exchange.
- How do we extract diffractive parton density functions (DPDFs) from DDIS data?

1. Demise of the 'Regge factorisation' approach currently
used by H1/ZEUS, where the exchanged Pomeron is
treated as a hadron-like object.
2. Rise of the 'perturbative QCD' approach, where the
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## Diffractive DIS kinematics



- $q^{2} \equiv-Q^{2}$
- $W^{2} \equiv(q+p)^{2}=-Q^{2}+2 p \cdot q$
$\Rightarrow \quad x_{B} \equiv \frac{Q^{2}}{2 p \cdot q}=\frac{Q^{2}}{Q^{2}+W^{2}}$ (fraction of proton's momentum carried by struck quark)
- $t \equiv\left(p-p^{\prime}\right)^{2} \approx 0,\left(p-p^{\prime}\right) \approx x_{\mathbb{P}} p$
- $M_{X}^{2} \equiv\left(q+p-p^{\prime}\right)^{2}=-Q^{2}+x_{\mathbb{P}}\left(Q^{2}+W^{2}\right)$
$\Rightarrow \quad x_{\mathbb{P}}=\frac{Q^{2}+M_{X}^{2}}{Q^{2}+W^{2}}$
(fraction of proton's momentum carried by Pomeron)
- $\beta \equiv \frac{x_{B}}{x_{\mathbb{P}}}=\frac{Q^{2}}{Q^{2}+M_{X}^{2}}$ (fraction of Pomeron's momentum carried by struck quark)


## Diffractive structure function $F_{2}^{\mathrm{D}(3)}$

- Diffractive cross section (integrated over $t$ ):

$$
\frac{\mathrm{d}^{3} \sigma^{\mathrm{D}}}{\mathrm{~d} x_{\mathbb{P}} \mathrm{d} \beta \mathrm{~d} Q^{2}}=\frac{2 \pi \alpha_{\mathrm{em}}^{2}}{\beta Q^{4}}\left[1+(1-y)^{2}\right] \sigma_{r}^{\mathrm{D}(3)}\left(x_{\mathbb{P}}, \beta, Q^{2}\right),
$$

where $y=Q^{2} /\left(x_{B} s\right), s=4 E_{e} E_{p}$, and

$$
\sigma_{r}^{\mathrm{D}(3)}=F_{2}^{\mathrm{D}(3)}-\frac{y^{2}}{1+(1-y)^{2}} F_{L}^{\mathrm{D}(3)} \approx F_{2}^{\mathrm{D}(3)}\left(x_{\mathbb{P}}, \beta, Q^{2}\right),
$$

for small $y$ or assuming that $F_{L}^{\mathrm{D}(3)} \ll F_{2}^{\mathrm{D}(3)}$

- Measurements of $F_{2}^{\mathrm{D}(3)} \Rightarrow$ diffractive parton distribution functions (DPDFs)

$$
a^{\mathrm{D}}\left(x_{\mathbb{P}}, z, Q^{2}\right)=z q^{\mathrm{D}}\left(x_{\mathbb{P}}, z, Q^{2}\right) \text { or } z g^{\mathrm{D}}\left(x_{\mathbb{P}}, z, Q^{2}\right),
$$

where $\beta \leq z \leq 1$, cf. $x_{B} \leq x \leq 1$ in DIS.

## Collinear factorisation in DDIS

$$
\begin{equation*}
F_{2}^{\mathrm{D}(3)}=\sum_{a=q, g} C_{2, a} \otimes a^{\mathrm{D}}+\mathcal{O}(1 / Q) \tag{1}
\end{equation*}
$$

where $C_{2, a}$ are the same coefficient functions as in inclusive DIS and where $a^{\mathrm{D}}=z q^{\mathrm{D}}$ or $z g^{\mathrm{D}}$ satisfy DGLAP evolution in $Q^{2}$ :

$$
\begin{equation*}
\frac{\partial a^{\mathrm{D}}}{\partial \ln Q^{2}}=\sum_{a^{\prime}=q, g} P_{a a^{\prime}} \otimes a^{\prime \mathrm{D}} \tag{2}
\end{equation*}
$$

"The factorisation theorem applies when $Q$ is made large while $x_{B}, x_{\mathbb{P}}$, and $t$ are held fixed." [Collins,'98]

- Says nothing about the mechanism for diffraction: what is the colourless exchange ('Pomeron') which causes the large rapidity gap. Assuming a 'QCD Pomeron' we need to modify both (1) and (2).
- Factorisation is broken in hadron-hadron collisions, but hope that same formalism can be applied with extra suppression factor calculable from eikonal models.
- LO diffractive dijet photoproduction: resolved photon contribution should be suppressed. Complications at NLO $\rightarrow$ talk by M. Klasen.


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## H1 extraction of DPDFs (ZEUS similar)

- Assume Regge factorisation [Ingelman-Schlein,'85]:

$$
a^{\mathrm{D}}\left(x_{\mathbb{P}}, z, Q^{2}\right)=f_{\mathbb{P}}\left(x_{\mathbb{P}}\right) a^{\mathbb{P}}\left(z, Q^{2}\right)
$$

- Pomeron flux factor from Regge phenomenology:

$\left(\alpha_{\mathbb{P}}(t)=\alpha_{\mathbb{P}}(0)+\alpha_{\mathbb{P}}^{\prime} t\right)$
"Regge factorisation relates the power of $x_{\mathbb{P}}$ measured in DDIS to the power of s measured in hadron-hadron elastic scattering." [Collins,'98]
$\Rightarrow$ Fit to $\mathrm{H}_{1} F_{2}^{\mathrm{D}(3)}$ data gives $\alpha_{\mathbb{P}}(0)=1.17>1.08$, the value of the 'soft Pomeron' [Donnachie-Landshoff,'92]. By Collins' definition, Regge factorisation is broken. H1/ZEUS meaning of 'Regge factorisation' is that the $x_{P}$ dependence factorises as a power law, with the power independent of $\beta$ and $Q^{2}$ (also broken, see tater),
$\Rightarrow$ Pomeron PDFs $a^{\mathbb{P}}\left(z, Q^{2}\right)=z \Sigma^{\mathbb{P}}\left(z, Q^{2}\right)$ or $z g^{\mathbb{P}}\left(z, Q^{2}\right)$ are DGLAP-evolved from inputs at $Q_{0}^{2}=3 \mathrm{GeV}^{2}$ :

$$
a^{W}\left(z, Q_{0}^{2}\right)=\left[A_{a}+B_{a}(2 z-1)+C_{a}\left(2(2 z-1)^{2}-1\right)\right]^{2} \exp (-0.01 /(1-z))
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f_{\mathbb{P}}\left(x_{\mathbb{P}}\right)=\int_{t_{\text {cut }}}^{t_{\text {min }}} \mathrm{d} t \mathrm{e}^{\mathbb{P}_{\mathbb{P}} t} x_{\mathbb{P}}^{1-2 \alpha_{\mathbb{P}}(t)} \quad\left(\alpha_{\mathbb{P}}(t)=\alpha_{\mathbb{P}}(0)+\alpha_{\mathbb{P}}^{\prime} t\right)
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## Recent measurements of DDIS $\rightarrow$ talk by L. Favart

1. Detect leading proton. No proton dissociation background, but low statistics. Both $\mathbb{P}$ and $\mathbb{R}$ contributions. [ZEUS: Eur. Phys. J. C 38 (2004) 43, H1prelim-01-112]
2. Look for large rapidity gap (LRG). (Non-diffractive contribution is exponentially suppressed as a function of the gap size.) Proton dissociation background. Both $\mathbb{P}$ and $\mathbb{R}$ contributions. [H1prelim-02-012, H1prelim-02-112, H1prelim-03-011]
3. Use " $M_{X}$ method". Subtract non-diffractive contribution in each $\left(W, Q^{2}\right)$ bin by fitting:


Motivated by Regge theory assuming $t=0, \alpha_{\mathbb{P}}(0) \equiv 1, Q^{2} \ll M_{X}^{2}$. (Validity in pQCD?) Proton dissociation background. Only $\mathbb{P}$ contribution. [ZEUS: Nucl. Phys. B 713 (2005) 3]

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## H1 vs. ZEUS $M_{X}$ DPDFs

NLO QCD fits to H1 and ZEUS data


Fits and plot by F.-P. Schilling (H1)

- Same procedure used to fit H1 LRG and ZEUS $M_{X}$ data. (ZEUS $M_{X}$ data scaled by a constant factor to account for different amount of proton dissociation.)
- Gluon from ZEUS $M_{X}$ fit ~ factor two smaller than gluon from H1 LRG data, due to different $Q^{2}$ dependence of the data sets. H1 2002 fit gives good agreement with (LRG) DDIS dijet and $D^{*}$ production data.
- N.B. 2-loop $\alpha_{S}$ fixed by $\Lambda_{\mathrm{QCD}}=200 \mathrm{MeV}$ for 4 flavours. Gives $\alpha_{S}$ values much smaller than world average $\Rightarrow \mathrm{H} 12002$ gluon artificially enhanced. Will be corrected for H 1 publication.


## H1 vs. ZEUS $M_{X}$ vs. ZEUS LPS DPDFs

Diffractive PDFs ( $\mathrm{x}_{\mathrm{IP}}=\mathbf{0 . 0 1}$ )


Plot by T. Tawara (ZEUS)

- No correction made for different amounts of proton dissociation.
- GLP = Groys-Levy-Proskuryakov (ZEUS) fit to ZEUS $M_{X}$ data, gives much too low prediction for ZEUS (LRG) DDIS dijets.
- ZEUS LPS fit describes dijets well, but:
"The shape of the fitted PDFs changes significantly depending on the functional form of the initial parameterisation, a consequence of the relatively large statistical uncertainties of the present sample. Therefore, these data cannot constrain the shapes of the PDFs."
[ZEUS: Eur. Phys. J. C 38 (2004) 43]


## $Q^{2}$ dependence of effective Pomeron intercept

H1 Diffractive Effective $\alpha_{1 P}(0)$


ZEUS


- Recall that 'Regge factorisation' fits assume that $\alpha_{\mathbb{P}}(0)$ is independent of $\beta$ and $Q^{2}$.
- $\alpha_{\mathbb{P}}(0)$ clearly rises with $Q^{2}$, but is smaller than in inclusive DIS, indicating that the $x_{\mathbb{P}}$ dependence is controlled by some scale $\mu^{2}<Q^{2}$.
$-\alpha_{\mathbb{P}}(0)>1.08$ [Donnachie-Landshoff,'92] indicating that the Pomeron in DDIS is not the 'soft' Pomeron exchanged in hadron-hadron collisions $\Rightarrow$ should use pQCD instead of Regge phenomenology. In pQCD, Pomeron exchange can be described by two-gluon exchange.


## How to reconcile two-gluon exchange with DPDFs?



Two-gluon exchange
calculations are the basis for the colour dipole model description of DDIS ( $\rightarrow$ talk by G. Shaw).

ZEUS 1994

- Right: $x_{\mathbb{P}} F_{2}^{\mathrm{D}(3)}$ for $x_{\mathbb{P}}=0.0042$ as a function of $\beta$
[Golec-Biernat-Wüsthoff,'99].
- dotted lines: $\gamma_{T}^{*} \rightarrow q \bar{q} g$,
- dashed lines: $\gamma_{T}^{*} \rightarrow q \bar{q}$,
- dot-dashed lines: $\gamma_{L}^{*} \rightarrow q \bar{q}$,
important at low, medium, and high $\beta$ respectively.
- $\gamma_{L}^{*} \rightarrow q \bar{q}$ is higher-twist, but DPDFs only include leading-twist contributions, therefore H1/ZEUS DPDFs are artificially large at high $z$.



## The QCD Pomeron is a parton ladder

- Generalise $\gamma^{*} \rightarrow q \bar{q}$ and $\gamma^{*} \rightarrow q \bar{q} g$ to arbitrary number of parton emissions [Ryskin,'90; Levin-Wüsthoff,'94].
- Work in Leading Logarithmic Approximation (LLA) $\Rightarrow$ transverse momenta are strongly ordered.

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$$
\begin{aligned}
F_{2}^{\mathrm{D}(3)} & =\int_{\mu_{0}^{2}}^{Q^{2}} \frac{\mathrm{~d} \mu^{2}}{\mu^{2}} f_{\mathbb{P}}\left(x_{\mathbb{P}} ; \mu^{2}\right) F_{2}^{\mathbb{P}}\left(\beta, Q^{2} ; \mu^{2}\right) \\
f_{\mathbb{P}}\left(x_{\mathbb{P}} ; \mu^{2}\right) & =\frac{1}{x_{\mathbb{P}} B_{D}}\left[R_{g} \frac{\alpha_{S}\left(\mu^{2}\right)}{\mu} x_{\mathbb{P}} g\left(x_{\mathbb{P}}, \mu^{2}\right)\right]^{2} \\
F_{2}^{\mathbb{P}}\left(\beta, Q^{2} ; \mu^{2}\right) & =\sum_{a=q, g} C_{2, a} \otimes a^{\mathbb{P}}
\end{aligned}
$$

$\mu_{0}^{2} \sim 1 \mathrm{GeV}^{2}, B_{D}$ from $t$-integration, $R_{g}$ from skewedness [Shuvaev et al.,'99]

- Pomeron PDFs $a^{\mathbb{P}}\left(z, Q^{2} ; \mu^{2}\right)$ DGLAP-evolved from an input scale $\mu^{2}$ up to $Q^{2}$.


## Gluonic and sea-quark Pomeron



- At low scales, sea-quark density of the proton dominates over gluon density at small $x \Rightarrow$ need to account for sea-quark density in perturbative Pomeron fux factor.

- Pomeron structure function $F_{2}^{\mathbb{P}}\left(\beta, Q^{2} ; \mu^{2}\right)$ calculated from quark singlet $\Sigma^{\mathbb{P}}\left(z, Q^{2} ; \mu^{2}\right)$ and gluon $g^{\mathbb{P}}\left(z, Q^{2} ; \mu^{2}\right)$ DGLAP-evolved from an input scale $\mu^{2}$ up to $Q^{2}$.
- Input Pomeron PDFs $\Sigma^{\mathbb{P}}\left(z, \mu^{2} ; \mu^{2}\right)$ and $g^{\mathbb{P}}\left(z, \mu^{2} ; \mu^{2}\right)$ are Pomeron-to-parton splitting functions.


## LO Pomeron-to-parton splitting functions



- LO Pomeron-to-parton splitting functions calculated in Eur. Phys. J. C 44 (2005) 69.
- Notation: ${ }^{‘} \mathbb{P}=G$ ' means gluonic Pomeron, ${ }^{\prime} \mathbb{P}=S$ ' means sea-quark Pomeron, ' $\mathbb{P}=G S$ ' means interference between these.

$$
\begin{aligned}
& z \Sigma^{\mathbb{P}=G}\left(z, \mu^{2} ; \mu^{2}\right)=P_{q, \mathbb{P}=G}(z)=z^{3}(1-z), \\
& z g^{\mathbb{P}=G}\left(z, \mu^{2} ; \mu^{2}\right)=P_{g, \mathbb{P}=G}(z)=\frac{9}{16}(1+z)^{2}(1-z)^{2}, \\
& z \Sigma^{\mathbb{P}=S}\left(z, \mu^{2} ; \mu^{2}\right)=P_{q, \mathbb{P}=S}(z)=\frac{4}{81} z(1-z), \\
& z g^{\mathbb{P}=S}\left(z, \mu^{2} ; \mu^{2}\right)=P_{g, \mathbb{P}=S}(z)=\frac{1}{9}(1-z)^{2}, \\
& z \Sigma^{\mathbb{P}=G S}\left(z, \mu^{2} ; \mu^{2}\right)=P_{q, \mathbb{P}=G S}(z)=\frac{2}{9} z^{2}(1-z), \\
& z g^{\mathbb{P}=G S}\left(z, \mu^{2} ; \mu^{2}\right)=P_{g, \mathbb{P}=G S}(z)=\frac{1}{4}(1+2 z)(1-z)^{2}
\end{aligned}
$$

Evolve these input Pomeron PDFs from $\mu^{2}$ up to $Q^{2}$ using NLO DGLAP evolution.

## Contribution to $F_{2}^{\mathrm{D}(3)}$ as a function of $\mu^{2}$

$$
\begin{aligned}
F_{2}^{\mathrm{D}(3)} & =\int_{\mu_{0}^{2}}^{Q^{2}} \frac{\mathrm{~d} \mu^{2}}{\mu^{2}} f_{\mathbb{P}}\left(x_{\mathbb{P}} ; \mu^{2}\right) F_{2}^{\mathbb{P}}\left(\beta, Q^{2} ; \mu^{2}\right) \\
f_{\mathbb{P}}\left(x_{\mathbb{P}} ; \mu^{2}\right) & =\frac{1}{x_{\mathbb{P}} B_{D}}\left[R_{g} \frac{\alpha_{S}\left(\mu^{2}\right)}{\mu} x_{\mathbb{P}} g\left(x_{\mathbb{P}}, \mu^{2}\right)\right]^{2}
\end{aligned}
$$

- Naïvely, $f_{\mathbb{P}}\left(x_{\mathbb{P}} ; \mu^{2}\right) \sim 1 / \mu^{2}$, so contributions from large $\mu^{2}$ are strongly suppressed.
- But $x_{\mathbb{P}} g\left(x_{\mathbb{P}}, \mu^{2}\right) \sim\left(\mu^{2}\right)^{\gamma}$, where $\gamma$ is the anomalous dimension. In BFKL limit $\gamma \simeq 0.5$, so $f_{\mathbb{P}}\left(x_{\mathbb{P}} ; \mu^{2}\right) \sim$ constant.
- HERA domain is in an intermediate region: $\gamma$ is not small, but is less than 0.5 .
- Plot integrand as a function of $\mu^{2}$ (using MRST2001 NLO PDFs) $\Rightarrow$ large contribution from large $\mu^{2}$.
- H1 (ZEUS) fits assume that $\mu^{2}<Q_{0}^{2}$, where
 $Q_{0}^{2}=3$ (2) $\mathrm{GeV}^{2}$ for H 1 (ZEUS) fits.


## Inhomogeneous evolution of DPDFs

$$
\begin{gathered}
F_{2}^{\mathrm{D}(3)}=\sum_{a=q, g} C_{2, a} \otimes a^{\mathrm{D}} \\
\text { where } a^{\mathrm{D}}\left(x_{\mathbb{P}}, z, Q^{2}\right)=\int_{\mu_{0}^{2}}^{Q^{2}} \frac{\mathrm{~d} \mu^{2}}{\mu^{2}} f_{\mathbb{P}}\left(x_{\mathbb{P}} ; \mu^{2}\right) a^{\mathbb{P}}\left(z, Q^{2} ; \mu^{2}\right)
\end{gathered}
$$



DGLAP term
Inhomogeneous cvolution of DPDFs is not a new idea:
"We introduce a diffractive dissociation structure function and show that it obeys the DGLAP evolution equation, but, with an additional
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= \\
\underbrace{\sum_{a^{\prime}=q, g} P_{a a^{\prime}} \otimes a^{\prime D}}_{\text {DGLAP term }}+\underbrace{f_{\mathbb{P}}\left(x_{\mathbb{P}} ; Q^{2}\right) P_{a \mathbb{P}}(z)}_{\text {Extra inhomogeneous term }}
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## Pomeron structure is analogous to photon structure Diffractive structure function

$$
\begin{aligned}
& F_{2}^{\mathrm{D}(3)}\left(x_{\mathbb{P}}, \beta, Q^{2}\right)=\underbrace{\sum_{a=q, g} C_{2, a} \otimes a^{\mathrm{D}}}_{\text {Resolved Pomeron }}+\underbrace{C_{2, \mathbb{P}}}_{\text {Direct Pomeron }} \\
& \frac{\partial a^{\mathrm{D}}\left(x_{\mathbb{P}}, z, Q^{2}\right)}{\partial \ln Q^{2}}=\underbrace{\sum_{a^{\prime}=q, g} P_{a a^{\prime}} \otimes a^{\prime \mathrm{D}}}_{\text {DGLAP }}+\underbrace{P_{a \mathbb{P}}(z) f_{\mathbb{P}}\left(x_{\mathbb{P}} ; Q^{2}\right)}_{\text {Inhomogeneous }}
\end{aligned}
$$

## Photon structure function

$$
F_{2}^{\gamma}\left(x_{B}, Q^{2}\right)=\underbrace{\sum_{a=q, g} C_{2, a} \otimes a^{\gamma}}_{\text {Resolved photon }}+\underbrace{C_{2, \gamma}}_{\text {Direct photon }}
$$

$$
\text { where } \frac{\partial a^{\gamma}\left(x, Q^{2}\right)}{\partial \ln Q^{2}}=\underbrace{\sum_{a^{\prime}=q, g} P_{a a^{\prime}} \otimes a^{\prime \gamma}}_{\text {DGLAP }}+\underbrace{P_{a \gamma}(x)}_{\text {Inhomogeneous }}
$$

## Dijets in diffractive photoproduction



- Effect of direct Pomeron coupling was considered by Kniehl-Kohrs-Kramer [Z. Phys. C 65 (1995) 657], but with Pomeron coupling to quarks like a photon: $\mathcal{L}_{\text {int }}=c \bar{q}(x) \gamma_{\mu} q(x) \phi^{\mu}(x)$.


## Need for NLO calculations

- NLO analysis of DDIS data is not yet possible.
- Need $C_{2, \mathbb{P}}$ and $P_{a \mathbb{P}}$ at NLO (help wanted!). Calculable with usual methods, e.g. LO diagrams are:


Dimensional regularisation: work in $4-2 \varepsilon$ dimensions, collinear singularity appears as $1 / \varepsilon$ pole multiplied by $P_{q \mathbb{P}}$, subtract in e.g. $\overline{M S}$ factorisation scheme to leave finite remainder $C_{2, \mathbb{P}}$.
$\Rightarrow$ Simplified analysis: take NLO $C_{2, a}$ and $P_{a a^{\prime}}\left(a, a^{\prime}=q, g\right)$, but LO $C_{2, \mathbb{P}}$ and $P_{\mathrm{ap}}$.

- Work in Fixed Flavour Number Scheme (no charm DPDF), with charm production via NLO $\gamma^{*} g^{\mathbb{P}} \rightarrow C \bar{C}$ [Riemersma et [Levin-Martin-Ryskin-Teubner,'97]. - For light quarks, include LO $q \bar{q}$ (higher-twist), but not $\gamma_{T}^{*} \mathbb{P} \rightarrow q \bar{q}$ (leading-twist).


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[Levin-Martin-Ryskin-Teubner,'97].
- For light quarks, include LO $\gamma_{L}^{*} \mathbb{P} \rightarrow q \bar{q}$ (higher-twist), but not $\gamma_{T}^{*} \mathbb{P} \rightarrow q \bar{q}$ (leading-twist).


## Description of DDIS data

- Take input quark singlet and gluon densities at $Q_{0}^{2}=3 \mathrm{GeV}^{2}$ in the form:

$$
\begin{aligned}
z \Sigma^{\mathrm{D}}\left(x_{\mathbb{P}}, z, Q_{0}^{2}\right) & =f_{\mathbb{P}}\left(x_{\mathbb{P}}\right) C_{q} z^{A_{q}}(1-z)^{B_{q}}, \\
z g^{\mathrm{D}}\left(x_{\mathbb{P}}, z, Q_{0}^{2}\right) & =f_{\mathbb{P}}\left(x_{\mathbb{P}}\right) C_{g} z^{A_{g}}(1-z)^{B_{g}},
\end{aligned}
$$

- Take $f_{\mathbb{P}}\left(x_{\mathbb{P}}\right)$ as in the H 12002 fit with $\alpha_{\mathbb{P}}(0), C_{a}, A_{a}$, and $B_{a}$ $(a=q, g)$ as free parameters.
- Treatment of secondary Reggeon as in H1 2002 fit.
- Fit H1 LRG and ZEUS $M_{X}$ data separately with cuts $M_{X}>2$ GeV and $Q^{2}>3 \mathrm{GeV}^{2}$. Allow overall normalisation factors of 1.10 and 1.43 respectively to account for proton dissociation.
- Statistical and systematic experimental errors added in quadrature.
- Two types of fits:
- "Regge" = 'Regge factorisation' approach (i.e. no $C_{2, \mathbb{P}}$ or $P_{a \mathrm{P}}$ ) as H1/ZEUS do.
- "pQCD" = 'perturbative QCD' approach with LO $C_{2, \mathbb{P}}$ and $P_{\mathrm{ap}}$.


## "pQCD" fits to H 1 and ZEUS $M_{x}$ data



## DPDFs from fits to H 1 and ZEUS $M_{X}$ data



- "pQCD" DPDFs are smaller at large $z$ due to inclusion of the higher-twist $\gamma_{L}^{*} \mathbb{P} \rightarrow q \bar{q}$.
- "pQCD" DPDFs have slightly more rapid evolution due to the inhomogeneous term.
- Difference between H 1 and ZEUS $M_{X}$ fits remains.


## $Q^{2}$ slope of H1 vs. ZEUS $M_{X}$ data

At LO, $\frac{\partial F_{2}^{\mathrm{D}(3)}}{\partial \ln Q^{2}}=\sum_{q} e_{q}^{2}\left(\sum_{a^{2}=q, g} P_{q a^{\prime}} \otimes{a^{\prime \mathrm{D}}}+P_{\mathrm{a} \mathbb{P}} f_{\mathrm{P}}\right)+\left(\gamma^{*} \mathbb{P} \rightarrow c \bar{c}\right)+\left(\gamma{ }_{\mathrm{L}}^{*} \mathbb{P} \rightarrow q \bar{q}\right)+\mathbb{R}$.


- Peak due to threshold for $\gamma^{*} \mathbb{P} \rightarrow c \bar{c}$ at $\beta=Q^{2} /\left(Q^{2}+4 m_{c}^{2}\right)$.
- Additional contributions to scaling violations apart from DGLAP contribution.
- All free parameters in 'DGLAP' part: ZEUS $M_{X}$ data have smaller scaling violations.


## $x_{\mathbb{P}}$ dependence of H 1 vs. ZEUS $M_{X}$ data

- Fit $\sigma_{r}^{\mathrm{D}(3)} \propto f_{\mathbb{P}}\left(X_{\mathbb{P}}\right)$ in each $\left(\beta, Q^{2}\right)$ bin with $\geq 4$ data points and $y<0.45$ (additional cut $x_{\mathbb{P}}<0.01$ for H 1 data):


- Inhomogeneous evolution can account for rise of $\alpha_{\mathbb{P}}(0)$ with $Q^{2}$.
- Inhomogeneous evolution breaks Regge factorisation.


## Predictions for diffractive charm production




- Data measured using LRG method.
- H1 DPDFs give good description, ZEUS $M_{X}$ DPDFs too small at low $\beta$.
- Direct Pomeron contribution significant at moderate $\beta$. These charm data points are included in the ZEUS LPS fit, but only $\gamma^{*} g^{\mathbb{P}} \rightarrow c \bar{c}$ contribution is included and not the $\gamma^{*} \mathbb{P} \rightarrow c \bar{c}$ contribution. Therefore, diffractive gluon from ZEUS LPS fit needs to be artificially large to fit the charm data.


## Non-linear evolution of inclusive PDFs

$$
\frac{\partial a\left(x, Q^{2}\right)}{\partial \ln Q^{2}}=\sum_{a^{\prime}=q, g} P_{a a^{\prime}} \otimes a^{\prime}-\int_{x}^{1} \mathrm{~d} x_{\mathbb{P}} P_{\mathbb{a} \mathbb{P}}\left(x / x_{\mathbb{P}}\right) f_{\mathbb{P}}\left(x_{\mathbb{P}} ; Q^{2}\right) .
$$



- Interesting application of DDIS formalism to calculate shadowing corrections to inclusive DIS via AGK cutting rules.
- Inhomogeneous evolution of DPDFs $\Rightarrow$ non-linear evolution of inclusive PDFs.
- More precise version of GLRMQ equation derived.
- Fit HERA $F_{2}$ data similar to MRST2001 NLO fit. Small- $x$ gluon enhanced at low scales.

For more details see Phys. Lett. B 627 (2005) 97.

## Summary

- Diffractive DIS is more complicated than inclusive DIS.
- Collinear factorisation holds, but need to account for direct Pomeron coupling:

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\begin{aligned}
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