Renormalons and N^3LO CORGI approach for $\hat{R}(s)_\tau$

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Outline

Chains & Bubbles

- Consider a QED 1-loop vacuum diagram
- Connects n of them....
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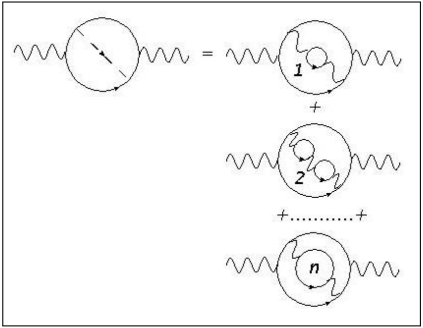

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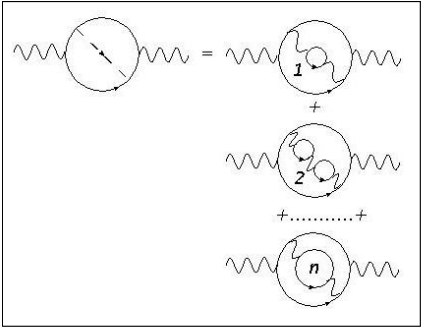

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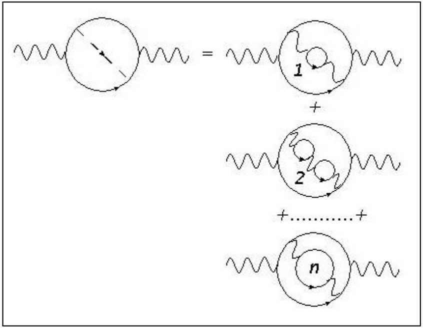

$$B^{\mu\nu}(k^2) = (\frac{-i}{k^2})[g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}]\frac{1}{1+\Pi_0} + (\frac{-i}{k^2})\frac{k^\mu k^\nu}{k^2}\xi$$

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- with z$=\pm z_n$ at where the singularities lie are called RENORMALONS.

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- and $X_3 = d_3 - d_1^3 - \frac{5c}{2} d_1^2 - (3X_2 - 2c2)d_1 - \frac{c3}{2}$

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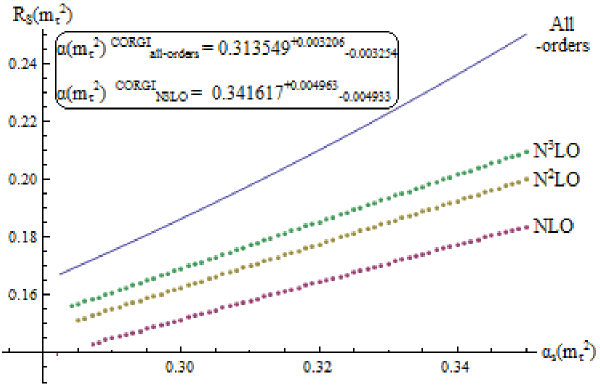
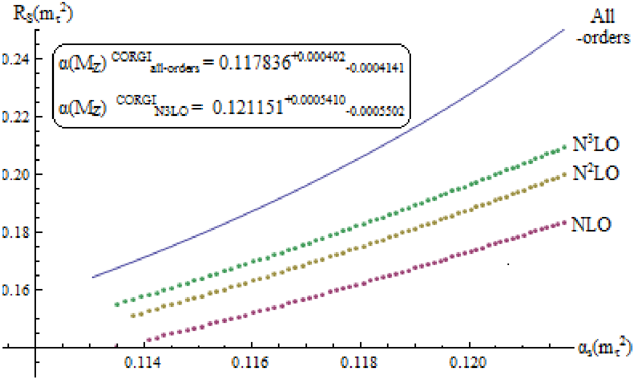
$\hat{R}(s)_\tau$ vs $\alpha_s(m_\tau^2)$



We then convert $\alpha_s(m_\tau^2)$ $\rightarrow$

$\hat{R}(s)_\tau$ vs $\alpha_s(m_Z)$

$\rightarrow$ $\alpha_s(m_Z)$ through flavour threshold...



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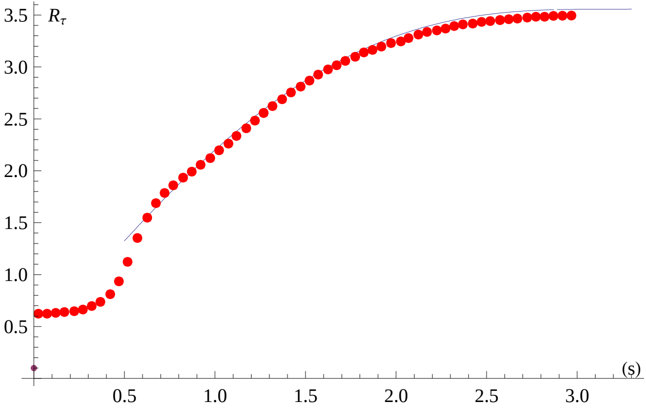
R_τ vs (s) - ALEPH



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- Prediction matches with ALEPH data for energy $s > 0.525$.

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- and any *Questions?*