MPHYGB17: Electrical Safety

First Lecture

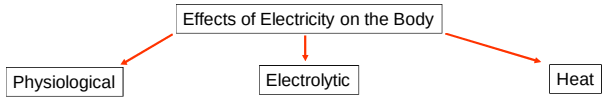
Part of ***Clinical Practice***

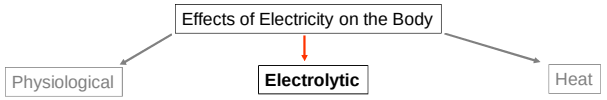
Nick Donaldson

Electrical Safety

Effects of Electricity on the Body

About 1000 people die in electrical accidents each year in the U.S.

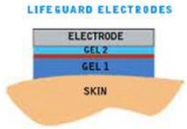




Oxygen evolution $2H_2O \rightarrow O_2\uparrow + 4H^+ + 4e^-$ Acid

Hydrogen evolution $2H_2O + 2e^- \rightarrow H_2\uparrow + 2OH^-$ Alkali

- Caused by direct current (DC)
- Regulations usually limit DC to mA
- Electrodes are sometimes not actually in contact with the skin



PALS™ Platinum Blue

Effects of Electricity on the Body

Physiological | Electrolytic | **Heat**

- Electrical heating occurs at all frequencies (c.f. microwave cooking)
- Harm is caused by rise in temperature in the tissue
- Regulations sometimes give limits on temperature rise rather that power absorption
- Temperature rises usually limited to 1 - 5 ºC (depending on application)
- SAR (Specific Absorption Rate) limit is 1.6W/kg from FCC.

Effects of Electricity on the Body

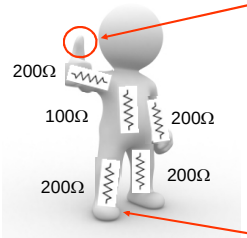
Physiological | Electrolytic | Heat

What is man?

Contact resistance

Skin resistivity:

15 kΩ.cm² to 1 MΩ.cm²

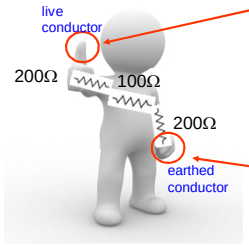


0 – 500pF

Limbs: 200Ω
Trunk: 100Ω

Capacitance to ground

Example: a small shock



Skin resistance:
2 cm²
1 MΩ.cm²
⇒ 500 kΩ

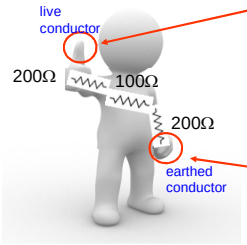
Skin resistance:
10 cm²
1 MΩ.cm²
⇒ 100 kΩ

Total resistance: 500,000 + 100,000 + 200 + 100 + 200 = 600,500 Ω

If source is 240V, current ≈ 240 / 600k = 0.4 mA

Example: a big shock (perhaps fatal)

Large contact areas, low skin resistivity



Skin resistance:
100 cm^2
15 kΩ.cm^2
⇒ **150 Ω**

Skin resistance:
100 cm^2
15 kΩ.cm^2
⇒ **150 Ω**

Total resistance: 150 + 150 + 200 + 100 + 200 = **800 Ω**

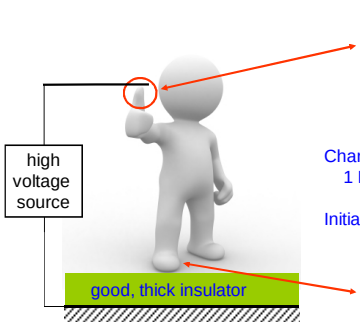
If source is 240V, current ≈ 240 / 800 = **300 mA**

Mains Shocks

The effect of the shock depends on the current.

The current depends on the contact area and the skin resistivity.

Contact resistance



Skin resistance:
1 cm^2
1 MΩ.cm^2
⇒ 1 MΩ

Charging time constant
1 MΩ ×50 pF = 50 μs

Initial current = $V_{source}/10^6$ in Amps

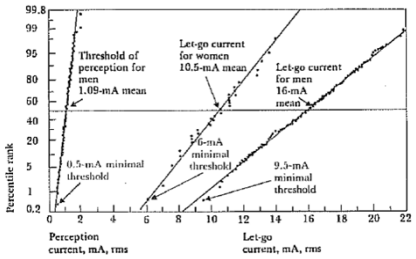
50 pF

Capacitance to ground

van der Graaf generator

Charging to high voltage is not painful and can be safe. The low resistance within the body (~100 Ω) means that the body is at a common potential,

Effects of mains current (50 or 60 Hz)



from Dalziel, 1973

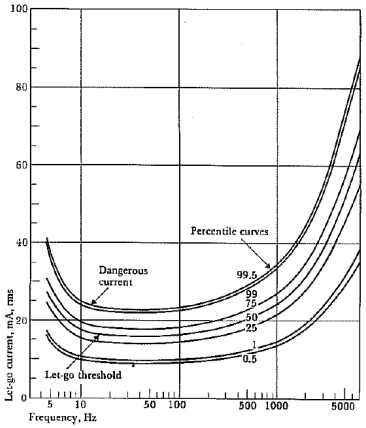
- Moistened hands holding 8-gauge copper wire
- Effect depends on contact area
- Gaussian distribution
- "Let-go" current is a poor term!
- Take-home message:
  - Threshold of perception ~1mA
  - Let-go current ~10mA

Why does the let-go current vary with frequency?

>1 kHz: exceeding the response time of the nerves

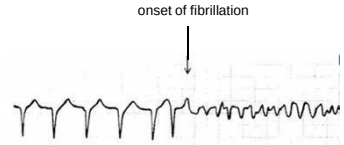
<10 Hz: muscles relax between contractions

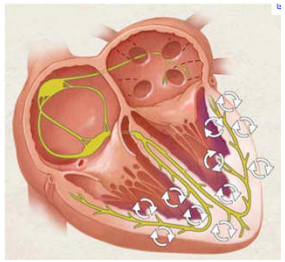
Take-home message: 50Hz is the most dangerous frequency. (The frequency was not chosen for safety!)



Ventricular Fibrillation

- Unco-ordinated contractions of the heart stop effective pumping
- Lack of blood causes rapid death
- Removal of the current may not stop the fibrillation
- A de-fibrillator must be used quickly ("Crash Cart")

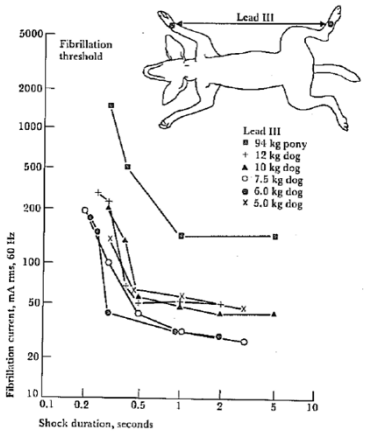




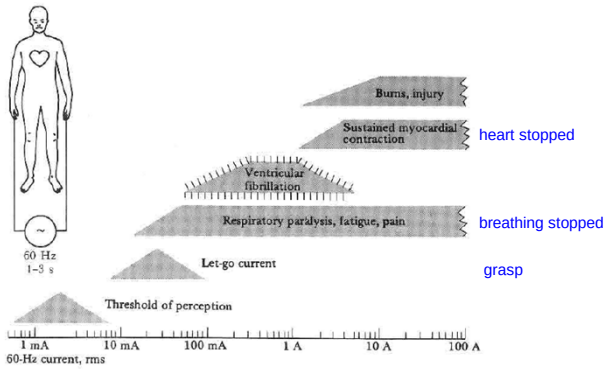
Strength-duration curves for ventricular fibrillation (macroshock)

Take-home message:

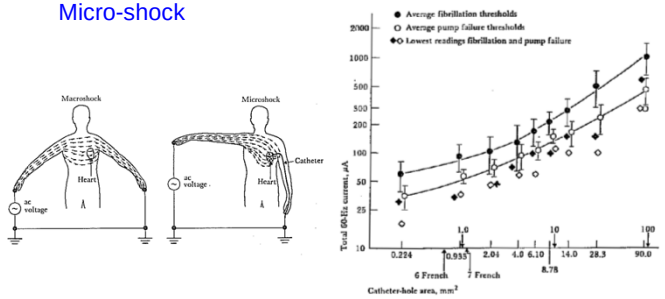
VF threshold likely to be at 50-100mA in man



Effects of AC and Macro-shock: summary



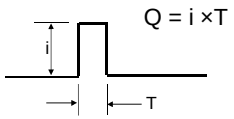
Micro-shock



Take-home message: micro-shock threshold for ventricular fibrillation ~10 μA

This threshold is >1000 times below the macro-shock threshold for VF:
dangerous because preventing such small currents is sometimes difficult.

Pulses of current applied to the chest



$Q = i \times T$

Pulse Function	Charge (μC)
TENS threshold	3
TENS maximum	7
Safe level	20
Hazard threshold	75
Heart pacing	100

T < 1 ms

Levels set by the US Food & Drugs Administration (FDA)

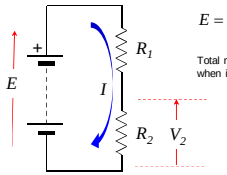
Alternating Current Theory

(without complex numbers)

Why study AC Theory in Electrical Safety?

The mains supply is the greatest hazard. It is *alternating* because it is generated in rotating generators and because the alternating voltage can be changed by transformers which lets the transmission losses (I^2R) be reduced. The voltage changes sinusoidally in time (very approximately). To be able to understand the risk, we have to be able to calculate the currents from the component properties and the drive voltages.

Ohm's Law for DC $V = I.R$



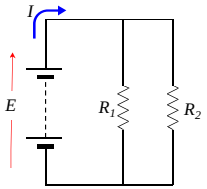
$E = IR_1 + IR_2 = IR_{total}$

Total resistance=sum of resistances when in series

Potential Divider:

$\frac{V_2}{E} = \frac{R_2}{R_1 + R_2}$

Series: the current is common to both resistors

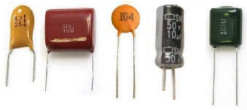


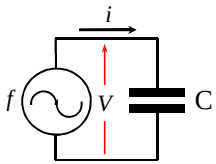
$I = \frac{E}{R_1} + \frac{E}{R_2} = E\left(\frac{R_1 + R_2}{R_1 R_2}\right)$

So $R_{total} = \frac{R_1 R_2}{R_1 + R_2}$

Parallel: the voltage is common to both resistors

How can we apply Ohm's Law in AC Situations?





Basic property of capacitors: $Q = C.V$

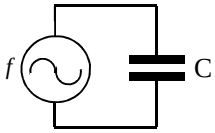
Charge ∝ Voltage across plates

Sinusoidal voltage is $v = v_p \sin \omega t$ $\omega = 2\pi.f$

Differentiating Q=CV with respect to time $\frac{dQ}{dt} = C\frac{dv}{dt}$ which is the current

But $\frac{dv}{dt} = \omega v_p \cos(\omega t)$

Therefore $i = \omega C v_p \cos(\omega t)$



What does this show?

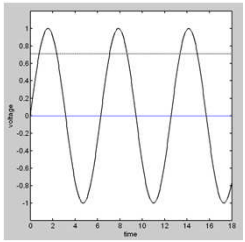
Applied voltage $v = v_p \sin \omega t$

Consequent current $i = \omega C v_p \cos(\omega t)$

- Amplitude of current is proportional to voltage
- Ratio of voltage amplitude / current amplitude is $\frac{v_p}{\omega C v_p} = \frac{1}{\omega C} = X_C$
- $\mathbf{X_C}$ is the *reactance* of the capacitor
- This is like Ohm's Law but reactance is frequency-dependent

- The current *leads* the voltage by 90°
- This is ***not*** like Ohm's Law applied with DC

A small diversion into the measurement of AC amplitudes



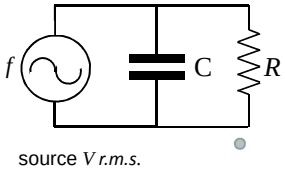
In AC circuits, the *root-mean-square* amplitude is used.

Why? Because it is the measure which has the same power as the equivalent steady voltage (i.e. DC).

For sine waves, the r.m.s. value is the peak divided by √2 (× 0.707)

The mains voltage is said to have an r.m.s. amplitude of about 230V, which is a peak amplitude of 380V.

Note that other waveforms do not have this relationship between peak and r.m.s amplitude. e.g. For a square wave (±V), the r.m.s. amplitude is also V.



source V *r.m.s.*

How do we deal with this?

A simple calculation: what is the current?

Angular frequency is $\omega = 2\pi . f$

Reactance is $X_C = \frac{1}{\omega C}$

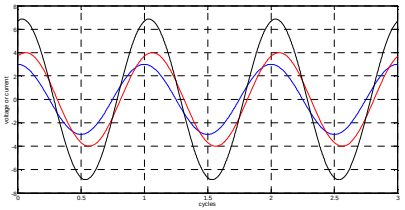
r.m.s. amplitude of current is $I = \frac{V}{X_C}$ (Ohm's Law)

And this current leads the voltage by 90°

Example
If C = 1nF (10^{-9} F) and *f* is 50Hz and V=240V, what is the current?

Answer: X = 3.18MΩ
I = 240/3.18 = 75μA

On the addition of sine waves

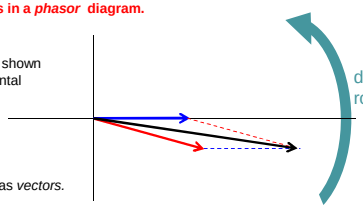


2 sine waves of same frequency but different phase and amplitude: add together point by point (try it in MATLAB!)

The sum is another sine wave.

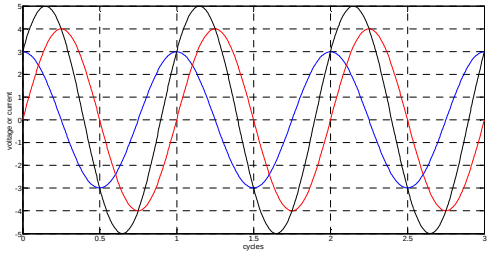
We can represent this in a *phasor* diagram.

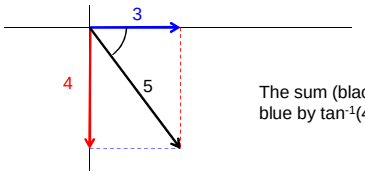
The instantaneous value, shown in the graph, is the horizontal length of the phasor.



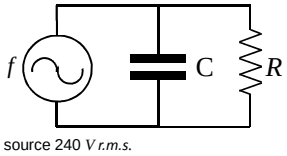
The phasors are added as *vectors*.

Special case: two waves in quadrature (90° apart)





The sum (black) lags the blue by $\tan^{-1}(4/3) = 53°$



source 240 *V r.m.s.*

Back to the parallel CR circuit

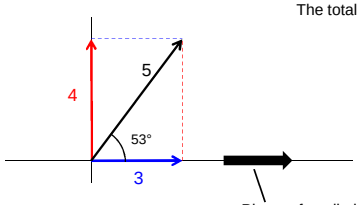
The voltage is common to the C and the R

Suppose *R* is 80Ω, resistor current =240/80 = 3A

f=50Hz, C=53μF so X=60Ω

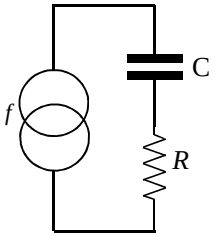
Therefore capacitor current is 240/60 = 4A

The total current is phasor sum, magnitude 5A



Note that the capacitor current (red) is *leading* the applied voltage

Phase of applied voltage (common)



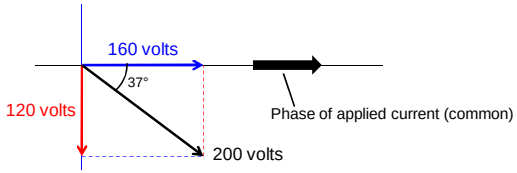
C and R in series (current source)

In this case the current is common to the C and the R.

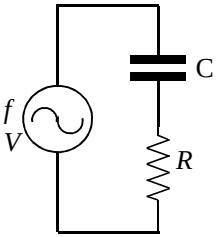
Therefore, make the current the reference phase.

Here the source is *constant-current* (not constant voltage)

If R = 80Ω and X = 60Ω, I_{source} = 2A



Note that the capacitor voltage *lags* the current

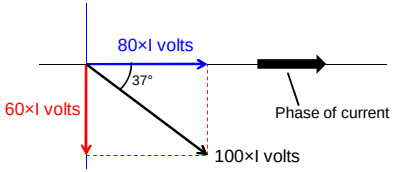


C and R in series (voltage source)

Now there is an applied voltage and we want to find the current. We know R, X and V. How can we do that?

The current is again common to the C and the R.

Suppose R = 80Ω, X = 60Ω and V = 240 volts
Say the current is I amps.



e.g. $80^2 + 60^2 = 100^2$

100×I is voltage across R and C in series.

This is 240 volts. Therefore 100×I=240 so I=2.4 amps.

C and R in series (voltage source) - more

How to do this quickly

You know R.

1. Given frequency and C, calculate the reactance X
   R and X are in series but are in quadrature (90° phase difference)
2. Add them as vectors to get *impedance* Z (magnitude and phase)

$$|Z| = \sqrt{R^2 + X^2} \qquad \angle Z = tan^{-1}\left(\frac{X}{R}\right)$$

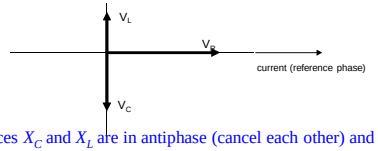
3. Magnitude of current is then applied voltage divided by |Z|
4. Phase of current is equal to phase of impedance but of opposite sign.

Inductors

For a capacitor	$X_C = \frac{1}{\omega C}$	Current leads voltage

For an inductor	$X_L = \omega L$	Current lags voltage

If a current is passed through a resistor, a capacitor and an inductor which are in **series**, the voltages across them appear on a phasor diagram like this.



The reactances X_C and X_L are in antiphase (cancel each other) and both are in quadrature with the resistance. Thus:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

If X_C and X_L are equal, cancelling each other, the circuit is at *resonance*.

Summary

- This has been an introduction to alternating current calculations
- It relates amplitudes (not instantaneous values) and phases
- The reactances of capacitors and inductors are frequencu-dependent
- Remember: if an alternating voltage is applied to a capacitors, the current leads by 90°, to an inductor, it lags by 90°.
- When drawing phasor diagrams, make the reference phase what is common to the components (current or voltage).
- The impedance is a phasor quantity that is the phasor sum of the resistive and reactive parts.

Try this

If a 1μF capacitor, a 1H inductor and a 5kΩ resistor are in series and connected across a 240volt 50Hz source, what is the magnitude of the current that flows?

$X_L = 2.\pi.50.1 = 314\Omega$

$X_C = 1/(2.\pi.50.10^6) = 3183\Omega$

$X = 3183 - 314 = 2869\Omega$

$|Z| = \sqrt{(5000^2 + 2869^2)} = 5760\Omega$

$|\text{current}| = 240/5760 = 41.7\text{mA}$