Brunel University
Queen Mary, University of London
Royal Holloway, University of London
University College London

Intercollegiate post-graduate course in High Energy Physics

Paper 1: Standard Model I

Monday 21st January 2019 – 10:00

Time allowed for Examination: 3 hours

Answer any four questions

Books and notes may be consulted

Question 1 (20 marks)

(a) Show that for a four-momentum, p_μ, that the contraction $p_\mu p^\mu$ is Lorentz invariant by considering a Lorentz transformation along a spatial axis of your choice. **[4]**

(b) Draw the lowest-order Feynman diagram(s) for the processes $e^- \nu_e \to e^- \nu_e$ and $e^- \overline{\nu_e} \to e^- \overline{\nu_e}$. **[4]**

(c) Consider an electron anti-neutrino with energy, E_ν, scattering from a stationary electron. Show that the centre-of-mass energy is **[3]**

$$s = m_e \left(2E_\nu + m_e\right) .$$

(d) Draw a sketch of the total cross-section for the processes $e^- \nu_e \to e^- \nu_e$ and $e^- \overline{\nu_e} \to e^- \overline{\nu_e}$ between 100GeV and 10^9 GeV, identifying and explaining any interesting features. . **[4]**

(e) A recent CERN press-release discussed the Future Circular Collider. Discuss the advantages and disadvantages of the proposed electron-positron and proton-proton versions of this facility. **[5]**

Question 2 (20 marks)

(a) Consider the Møller scattering process, $e^-(P_1)e^-(P_2) \to e^-(P_3)e^-(P_4)$ with four-momenta as labelled. In the centre of mass frame the energy of both initial state electrons is E and the scattering angle of the outgoing electron (P_1 to P_3) is θ, in terms of these quantities determine the Mandelstam variables s, t and u. **[4]**

(b) Draw the lowest-order Feynman diagram(s) for Møller scattering and label the four-momenta transfer in terms of s, t or u. **[3]**

(c) In the high-energy limit the cross-section for Møller scattering is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8E^2}\left[\frac{1+\cos^4\theta/2}{\sin^4\theta/2} + \frac{2}{\sin^2\theta/2\cos^2\theta/2} + \frac{1+\sin^4\theta/2}{\cos^4\theta/2}\right].$$

Show that this can be simplified to

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2}\frac{\left(3+\cos^2\theta\right)^2}{\sin^4\theta}.$$

[7]

(d) Starting from the longer form of the cross-section above, write the cross-section in terms of the Mandelstam variables rather than the angle θ. **[2]**

(e) Use these results to write down, with a suitable explanation, the cross-sections in terms of Mandelstam variables for the processes $e^-\mu^- \to e^-\mu^-$ and $e^-e^+ \to \mu^-\mu^+$.

[4]

Question 3 (20 marks)

(a) The highest energy cosmic rays have $E \approx 10^{20}$eV. Assuming the cosmic ray collides with a proton in the atmosphere, determine how much higher in centre-of-mass energy such a collision is compared to that at the LHC. **[4]**

(b) Show that the sum of the three Mandelstam variables in the collision $AB \to CD$ is given by

$$s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2,$$

where m_i are the rest masses of the particles. **[3]**

(c) Taking $e^- e^+ \to e e^+$ to be the s-channel process, verify that

$$s = 4(k^2 + m^2)$$

$$t = 2k^2(1 \cos\theta)$$

$$u = 2k^2(1 + \cos\theta)$$

where θ is the centre-of-mass scattering angle and $k = |k_i| = |k_f|$ and k_i and k_f are, respectively, the momenta of the incident and scattered electrons in the centre-of-mass frame. **[6]**

(d) Considering combinations of the Pauli matrices,

$$\{\sigma_1, \sigma_2\}, \quad \{\sigma_1, \sigma_3\}, \quad \{\sigma_2, \sigma_3\}, \quad \{\sigma_1, \sigma_1\},$$

determine them explicitly to show that

$$\{\sigma_i, \sigma_j\} = 2\delta_{ij} I,$$

where I is the 2×2 identity matrix and δ_{ij} is the Kronecker delta. **[4]**

(e) Consider a momentum $\hat{p}$ in the direction specified by the polar coordinates θ and ϕ

$$\hat{p} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)\,.$$

Show that

$$\sigma \cdot \hat{p} = \begin{pmatrix} \cos\theta & \sin\theta e^{i\phi} \\ \sin\theta e^{i\phi} & \cos\theta \end{pmatrix}.$$

[3]

Question 4 (20 marks)

(a) Given the cross section for the scattering of an electron from a fixed Coulomb potential of point charge Ze:

$$\frac{d\sigma}{d\Omega} = \frac{2\left(Z\alpha\right)^2 m^2}{|\vec{q}|^4} Tr\left[\gamma^0 \frac{\not{p}_i + m}{2m} \gamma^0 \frac{\not{p}_f + m}{2m}\right],$$

where p_i and p_f are the initial and final momentum and $\vec{q} = \vec{p_f} - \vec{p_i}$, determine the Mott cross-section:

$$\frac{d\sigma}{d\Omega} = \frac{\left(Z\alpha\right)^2}{4\left(\gamma\beta^2\right)^2 \left(mc^2\right)^2 \sin^4(\theta/2)} \left(1 - \beta^2 \sin^2 \frac{\theta}{2}\right).$$

Trace theorems used should be explicitly stated. **[15]**

(b) Show that in the non-relativistic limit

$$\frac{d\sigma}{d\Omega} = \frac{\left(Z\alpha\right)^2}{16E^2 \sin^4(\theta/2)},$$

and in the extreme-relativistic limit

$$\frac{d\sigma}{d\Omega} = \frac{\left(Z\alpha\right)^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)}.$$

[5]

Question 5 (20 marks)

(a) Compton scattering is the two-body scattering of an electron and a photon, $e(P_1) + \gamma(P_2) \to e(P_3) + \gamma(P_4)$. The polarisation-averaged matrix-element squared for Compton scattering is given by

$$|\overline{M}|^2 = 2e^4 \left(\frac{P_1 \cdot P_2}{P_1 \cdot P_4} + \frac{P_1 \cdot P_4}{P_1 \cdot P_2} + 2m_e^2 \left(\frac{1}{P_1 \cdot P_2} - \frac{1}{P_1 \cdot P_4} \right) + m_e^4 \left(\frac{1}{P_1 \cdot P_2} - \frac{1}{P_1 \cdot P_4} \right)^2 \right).$$

Draw all lowest order Feynman diagrams for Compton scattering labelling the four-momenta as above. **[4]**

(b) In the high energy limit where mass can be neglected show that in the centre of mass frame that backwards scattering dominates and the matrix-element squared can be approximated by

$$|\overline{M}|^2 \approx \left(\frac{4e^4}{1 + \cos\theta} \right),$$

where θ is the scattering angle of the photon. **[6]**

(c) Starting from the matrix-element squared for Compton scattering, obtain an expression for the matrix-element squared for the the annihilation process

$$e^-(P_1)e^+(P_2) \to \gamma(P_3)\gamma(P_4).$$

In the centre of mass frame, show that in the relativistic limit ($E >> m$)

$$|\overline{M}|^2 \approx 4e^4 \left(\frac{1 + \cos^2\theta}{\sin^2\theta} \right),$$

where θ is the scattering angle of $\gamma(P_3)$ relative to $e^-(P_1)$. **[6]**

(d) Draw a sketch of an $e^-e^+ \to \gamma\gamma$ event and an $e^-e^+ \to \mu^+\mu^-$ event in a generic collider detector at LEP. Indicate which detector components are most important for identifying the two events. **[4]**

Question 6 (20 marks)

(a) One possible decay of a π^- is $\pi^- \to e^-\overline{\nu_e}$. Show that for a pion at rest

$$\frac{1}{2}\left(1-\frac{v_e}{c}\right) = \frac{m_e^2}{m_e^2+m_\pi^2},$$

where v_e is the velocity of the electron. **[5]**

(b) At the lowest order the partial decay rates for pions are given by

$$\frac{1}{\tau(\pi^- \to e^-\overline{\nu_e})} = \frac{\alpha_\pi^2}{4\pi}\left(1-\frac{v_e}{c}\right)p_e^2 E_e, \qquad \frac{1}{\tau(\pi^- \to \mu^-\overline{\nu_\mu})} = \frac{\alpha_\pi^2}{4\pi}\left(1-\frac{v_\mu}{c}\right)p_\mu^2 E_\mu,$$

where α_π is an effective coupling constant and E_e, E_μ and p_e, p_μ are the energy and momentum of the charged lepton. Show that **[5]**

$$\frac{\tau(\pi^- \to \mu^-\overline{\nu_\mu})}{\tau(\pi^- \to e^-\overline{\nu_e})} = \frac{m_e^2\left(m_\pi^2-m_e^2\right)^2}{m_\mu^2\left(m_\pi^2-m_\mu^2\right)^2}.$$

(c) Using the analogue of the above equation, estimate the following ratio for the K^- decay

$$\frac{\tau(\pi^- \to \mu^-\overline{\nu_\mu})}{\tau(\pi^- \to e^-\overline{\nu_e})}$$

and compare with the observed value of $(2.4 \pm 0.1) \times 10^-5$. **[3]**

(d) Given the lifetimes, $\tau(K^- \to \mu^-\overline{\nu_\mu}) = 1.948 \times 10^{-8}$s and $\tau(\pi^- \to \mu^-\overline{\nu_\mu}) = 2.603 \times 10^{-8}$s, estimate α_K/α_π. **[3]**

(e) Draw quark model diagrams for the decays $\pi^- \to \mu^-\overline{\nu_\mu}$ and $K^- \to \mu^-\overline{\nu_\mu}$, indicating the couplings at the vertices including CKM matrix terms. **[4]**