Heavy Flavour Hadroproduction

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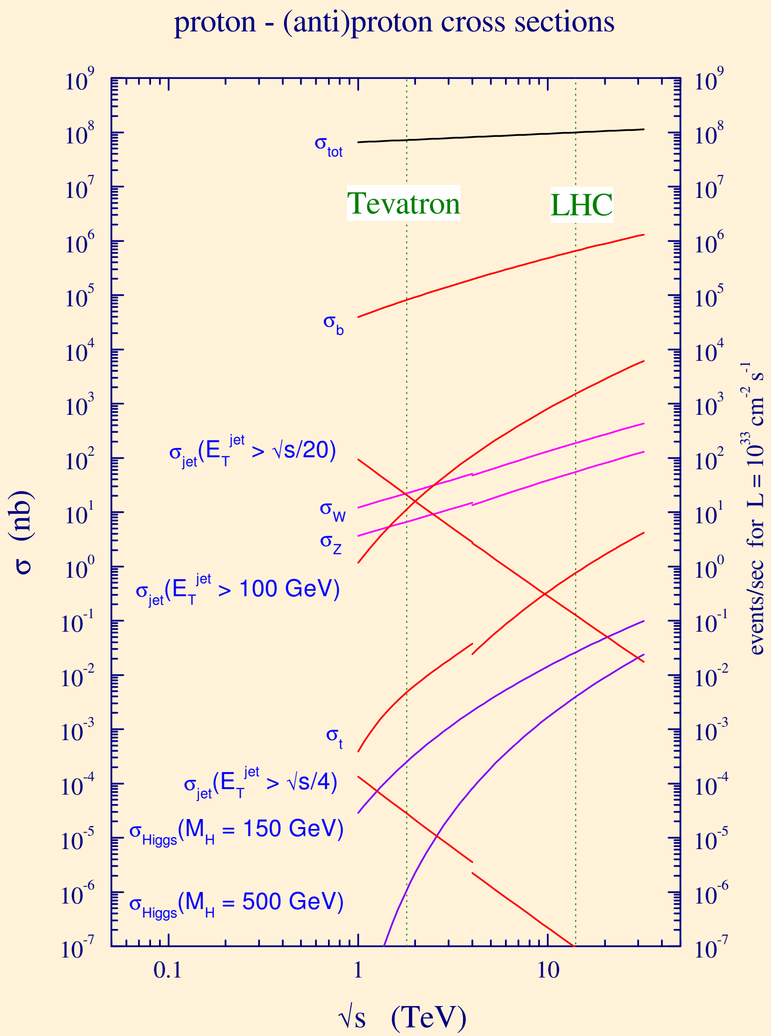
- Higher order corrections
- BFKL and high-energy resummation
- Threshold resummation
- Outlook

Heavy quark production

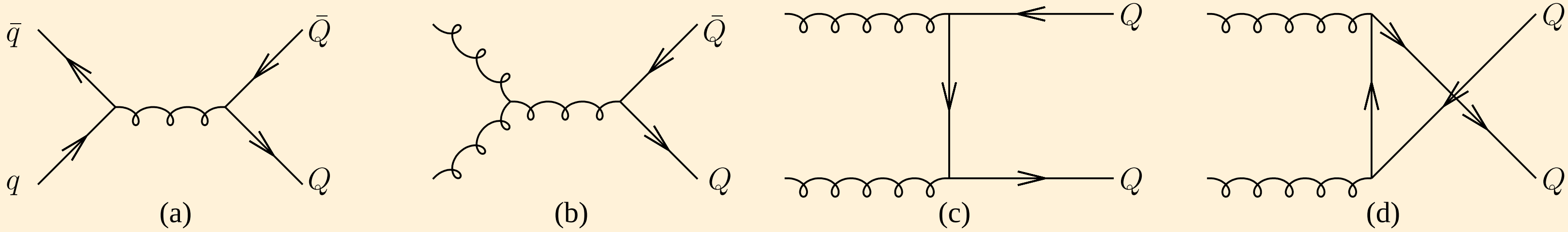


- The LHC will rapidly accumulate high statistics of $t\bar{t}$ and $b\bar{b}$ pairs.
- If SM predictions are sufficiently precise, comparison with experiment may reveal signals for new physics.
- The top-pair total cross section may be suitable as a 'standard candle' at the LHC, if theoretical (and experimental) uncertainties are further constrained.

(M. Mangano, at *HERA and the LHC*, CERN 2008, http://www.desy.de/~heralhc/)

(Figure from Campbell *et al*, *Rep. Prog. Phys.*, 70 (2007) 89)

Leading-order (LO) heavy flavour production



- Tree level processes:
  (a) $q\bar{q} \to Q\bar{Q}$.
  (b-d) $g\bar{g} \to Q\bar{Q}$ in s, t, u channels.

- The incoming quark and gluon legs are *partons*.

- At the LHC, $b\bar{b}$ production (in the central rapidity region) will occur at small x.

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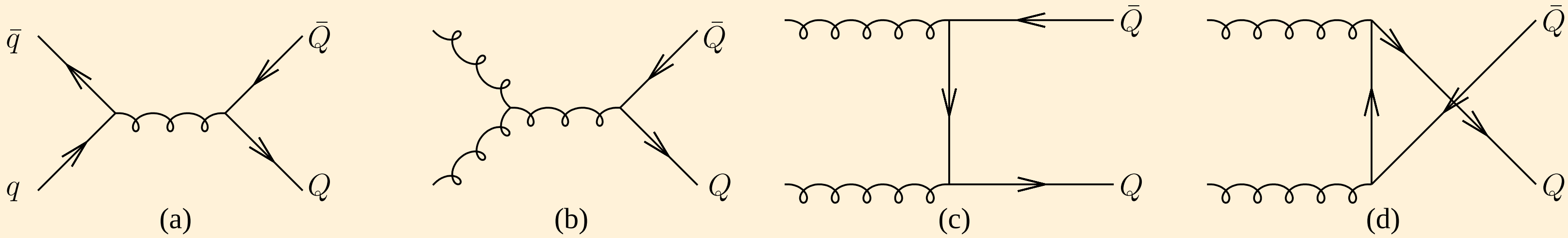

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- Matrix elements squared are computed using Feynman rules:

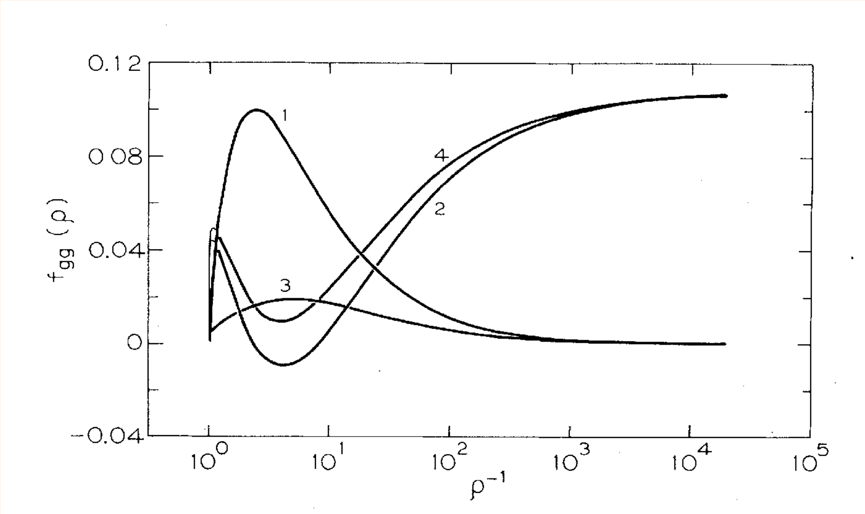
	$\overline{\sum}\lvert M_{ij}\rvert^2/g^4$
$q\bar{q} \to Q\bar{Q}$	$\frac{4}{9}\left(\tau_1^2 + \tau_2^2 + \frac{\rho}{2}\right)$
$gg \to Q\bar{Q}$	$\left(\frac{1}{6\tau_1\tau_2} - \frac{3}{8}\right)\left(\tau_1^2 + \tau_2^2 + \rho - \frac{\rho^2}{4\tau_1\tau_2}\right)$

$$\hat{s} = (p_1 + p_2)^2, \qquad \tau_1 = 2(p_1 p_3)/\hat{s}, \qquad \tau_2 = 2(p_2 p_3)/\hat{s}, \qquad \rho = 4M^2/\hat{s}.$$

Next-to-leading-order (NLO) heavy flavour production

- NLO: includes 1-loop diagrams and real and virtual corrections.
- Inclusive $gg \to Q + X$ cross section (as a function of $\rho^{-1} = \hat{s}/4M^2$):

  (Beenakker *et al*, *Phys. Rev.* D40 (1989) 54)



(1) Leading order (tree level)
(2-4) Various NLO contributions

$\rho \sim 1$: threshold region.
$\rho \gg 1$: high-energy limit.

Tevatron($t\bar{t}$): $1 \leq \rho^{-1} \lesssim 3$
LHC($t\bar{t}$): $1 \leq \rho^{-1} \lesssim 160$
LHC($b\bar{b}$): $1 \leq \rho^{-1} \lesssim 2 \times 10^5$

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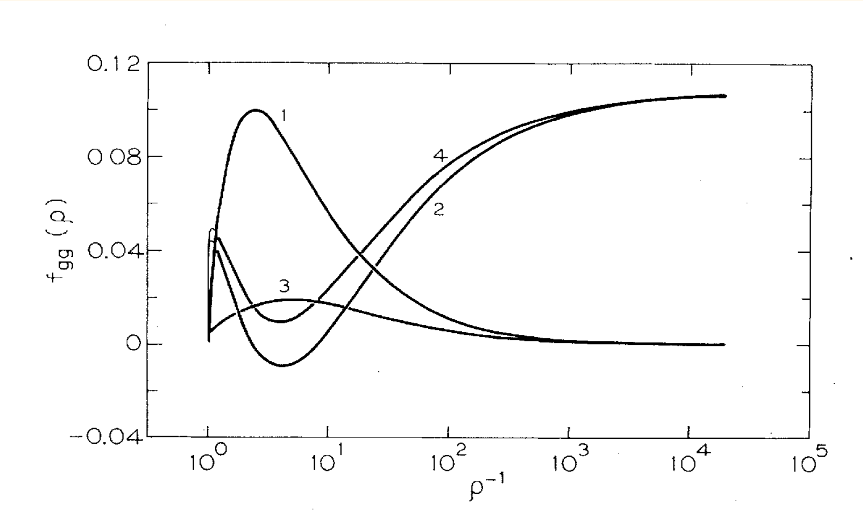

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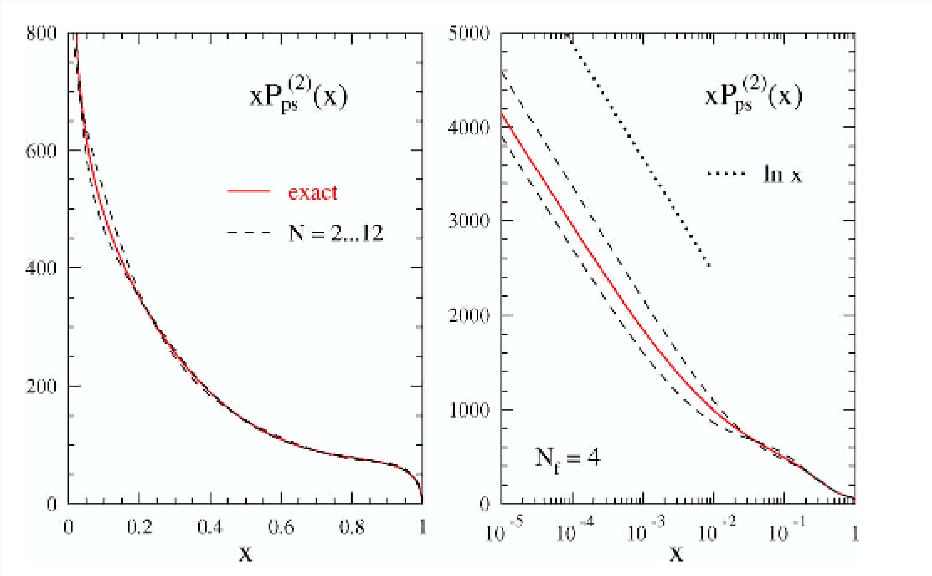
- NLO processes provide large corrections over the whole kinematic range, and dominate in both the threshold limit and the high-energy limit
- e.g. bottom quark hadroproduction: NLO prediction exceeds LO prediction by 100%.

Prospects for NNLO Heavy Flavour Production

- Only a few processes are known to NNLO (W/Z production, Higgs*). Recently: NNLO splitting functions. (Vogt *et al. Nucl. Phys.* B691 (2004) 129)



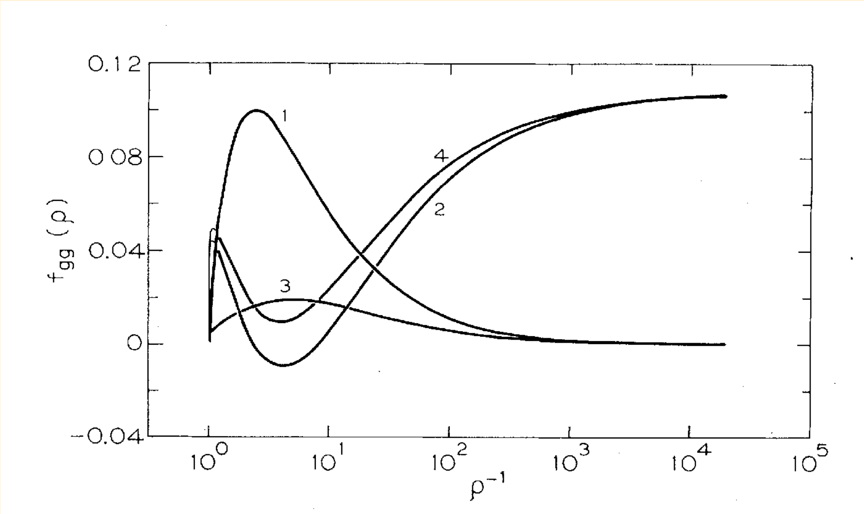
- NNLO pure singlet splitting function P_{ps}, exact result (solid line) compared to earlier estimates (dashed line) based on

  1. Integer moments (constraints at large x).
  2. Small-x logarithms.

- ...fairly good estimates predated the NNLO calculation.

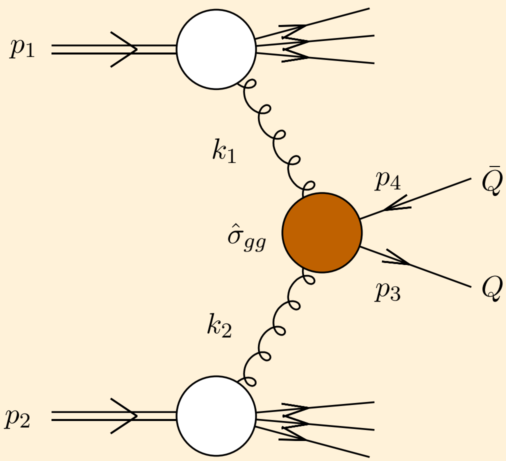
Resummation

- At least several years before NNLO heavy flavour hadroproduction is completed. However, we can resum leading logarithmic corrections in
  1. Threshold region, $\rho \sim 1$ Kidonakis *et al*, *Phys. Rev.*, D64 (2001) 114001
  2. High-energy limit, $\rho \gg 1$ Ball & Ellis, *JHEP*, 05:53 (2001)



- For $t\bar{t}$ at LHC or Tevatron, threshold corrections are important. High-energy resummation corrections may be less important.
- For $b\bar{b}$ at the LHC, both threshold and high-energy corrections are important.

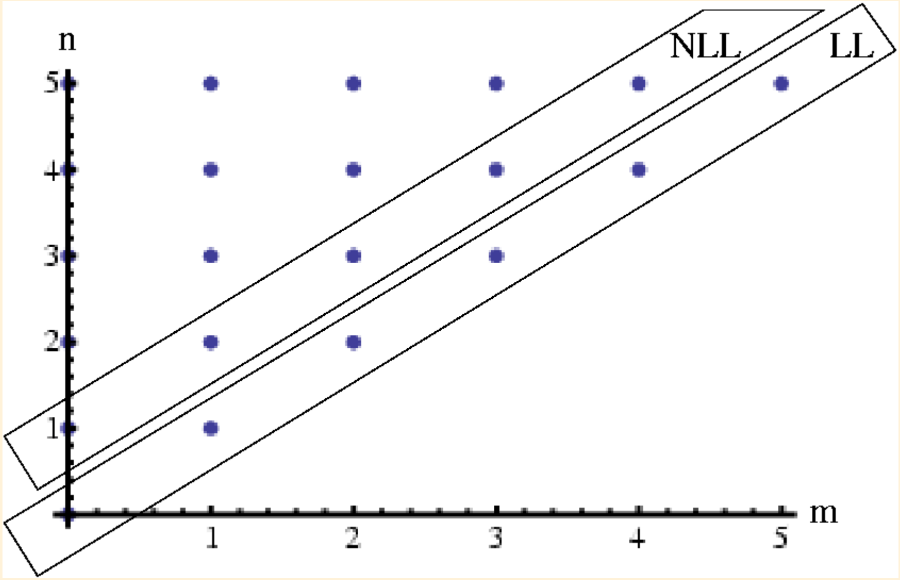
High-Energy Resummation



- Asymptotic freedom of QCD: α_S is small at the scale of the quark mass M.
- *Leading logarithmic terms* involve successive powers of $\alpha_S \ln S/M^2$.
- In the high energy limit ($S \gg M^2$), $\alpha_S \ln S/M^2 \sim 1$. NLO, NNLO and higher order terms may contribute large corrections.

High-Energy Resummation

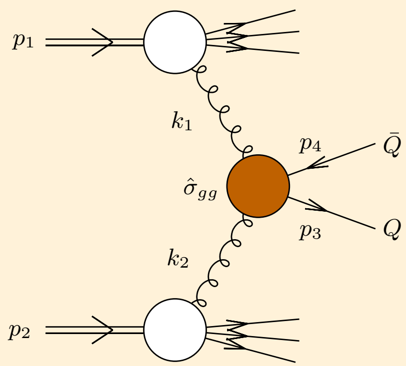
- In the high-S limit, QCD contributions to an amplitude are of the form $\alpha_S^m \ \ln^n S/M^2$.
- Balitsky-Fadin-Kuraev-Lipatov (BFKL): collect all the leading logarithms (LL)

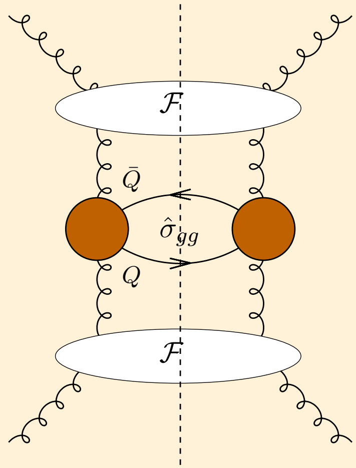


- Next-to-leading logarithms (NLL) are a subleading contribution to the amplitude.
- Ball & Ellis: resummation of the high-energy leading logarithms $\longrightarrow$ Provides an estimate on the high energy corrections to heavy quark production

  Ball & Ellis, *JHEP*, 05:53 (2001)

BFKL and High-Energy Resummation





- Factorisation of the total cross section σ. 'Higher twist corrections' are suppressed by powers of $\alpha_S(M^2)$

- A 'cut' diagram: uses the optical theorem to evaluate the inclusive cross section.

- Factorisation into the partonic cross section $\hat{\sigma}_{QQ}$ and the unintegrated gluon structure function $\mathcal{F}(z, k; Q_0^2)$

$$\sigma(\rho) = \frac{1}{4M^2} \int d^2\mathbf{k}_1 \int \frac{d\bar{z}_1}{\bar{z}_1} \int d^2\mathbf{k}_2 \int \frac{dz_2}{z_2}$$
$$\mathcal{F}(\bar{z}_1, \mathbf{k}_1)\ \mathcal{F}(z_2, \mathbf{k}_2) \hat{\sigma}_{QQ}(\rho/\bar{z}_1 z_2, \mathbf{k}_1/M, \mathbf{k}_2/M).$$

BFKL Evolution

- $\mathcal{F}$ obeys the BFKL evolution equation:

$$\mathcal{F}(x,\mathbf{k}) = \frac{1}{\pi}\delta(1-x)\delta(\mathbf{k}^2 - Q_0^2) \\ + \bar{\alpha_S}\int \frac{d^2\mathbf{q}}{\pi\mathbf{q}^2}\int \frac{dz}{z}\left(\mathcal{F}\left(\frac{x}{z},\mathbf{k}+\mathbf{q}\right) - \Theta(k-q)\mathcal{F}\left(\frac{x}{z},\mathbf{k}+\mathbf{q}\right)\right)$$

- The delta functions correspond to the lowest-order process (tree level).
- The integral term gives the effect of the exchanged gluon radiating further gluons.
- An explicit solution to the BFKL equation can be obtained by applying a Mellin transform in the variable x. Defined for $N \geq 0$ as

$$\mathcal{F}_N(\mathbf{k}) = \int_0^1 dx\; x^{N-1}\mathcal{F}(x,\mathbf{k})$$

- The Mellin transform 'undoes' the z-integral (a convolution). (Similar to a Laplace transform).

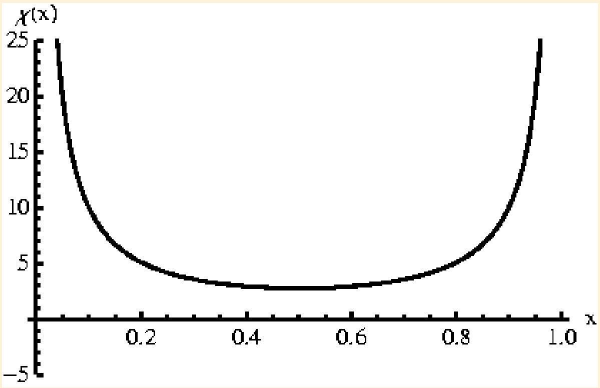
Lipatov characteristic function

- The solution is

$$\mathcal{F}_N(\mathbf{k}^2) = \frac{1}{\pi \mathbf{k}^2} \int_{1/2-i\infty}^{1/2+i\infty} \frac{d\gamma}{2\pi i} \left(1 - \frac{\bar{\alpha}_S}{N}\chi(\gamma)\right)^{-1} \left(\frac{\mathbf{k}^2}{Q_0^2}\right)^{\gamma} \tag{1}$$

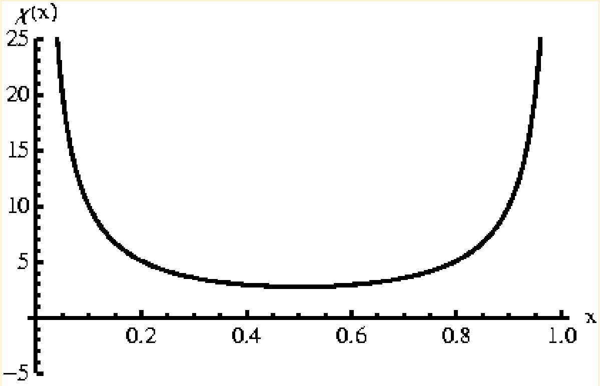
- $\chi(\gamma)$ is the *Lipatov characteristic function*:

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$



- Solve $\chi(\gamma_N) = \frac{N}{\bar{\alpha}_S}$ for γ_N (the *anomalous dimension*). This value dominates the integral in (1).

Lipatov characteristic function



- Ball & Ellis: convolve $\mathcal{F}$ with the partonic cross section $\hat{\sigma}_{QQ}$ to provide the resummed total cross section. Includes high-energy leading logarithms to all orders.
- For $N \gg \bar{\alpha_S}$,

$$\gamma_N = \frac{\bar{\alpha}_S}{N} + 2\zeta(3)\left(\frac{\bar{\alpha}_S}{N}\right)^4 + \ldots$$

  $\zeta(s)$ is the Riemann zeta-function.
- Provides an estimate on the high energy corrections to heavy quark production.

Ball & Ellis, *JHEP*, 05:53 (2001)

Threshold corrections

- Threshold large logarithms involve powers of $\alpha_S \ln(s - 4m_b^2)$.
- They are large in the limit that $(s - 4m_b^2)$ is small, that is, when s is just above threshold for production of the $b\bar{b}$ pair.
- These *threshold corrections* dominate the cross section near threshold and can be used to estimate the total cross section in this regime.
- Laenen *et al* developed a method of resumming these logarithms Laenen *et al Nucl. Phys.* B369 (1992) 543.
- Recent improvements by Kidonakis *et al* Kidonakis *et al Phys. Rev.* D64 (2001) 114001

Summary

- Recent result of Ball & Ellis for high-energy corrections.
- Recent result of Kidonakis *et al* for threshold corrections.
- Investigate whether both results can be incorporated into a single framework. This is an approach that has been successfully applied to parton splitting functions.