8. FISSION AND FUSION

8.1 Induced fission – fissile materials

We have discussed fission previously in Sec.2, where we saw that for a nucleus with $A \approx 240$, the Coulomb barrier, which inhibits spontaneous fission, is between 5 and 6 MeV. If a neutron with zero kinetic energy enters a nucleus to form a compound nucleus, the compound nucleus will have an excitation energy above its ground state equal to the neutron's binding energy in that ground state. For example, a zero-energy neutron entering a nucleus of ^{235}U forms a state of ^{236}U with excitation energy of 6.5 MeV. This energy is well above the fission barrier and the compound nucleus quickly undergoes fission, with decay products similar to those found in the spontaneous decay of ^{236}U. To induce fission in ^{238}U, on the other hand, requires a neutron with kinetic energy of at least 1.2 MeV. The binding energy of the last neutron in ^{239}U is only 4.8 MeV and an excitation energy of this size is below the fission threshold of ^{239}U.

The differences in the binding energies of the last neutron in even-*A* and odd-*A* nuclei are given by the pairing term in the semi-empirical mass formula. Examination of the value of this term leads to the explanation of why the odd-*A* nuclei

$$^{233}_{92}U\,,\ ^{235}_{92}U\,,\ ^{239}_{94}Pu\,,\ ^{241}_{94}Pu$$

are 'fissile' nuclei, i.e. nuclei whose fission may be induced by even zero-energy neutrons, whereas the even-*A* (even-Z/even-N) nuclei

$$^{232}_{90}Th\,,\ ^{238}_{92}U\,,\ ^{240}_{94}Pu\,,\ ^{242}_{94}Pu$$

require an energetic neutron to induce fission.

The most commonly used fuel in reactors is uranium, so we will focus on this element. Natural uranium consists of 99.3% ^{238}U and only 0.7% ^{235}U. The total and fission cross-sections, σ_{tot} and σ_f, respectively, for neutrons incident on ^{235}U are shown in Fig.8.1.

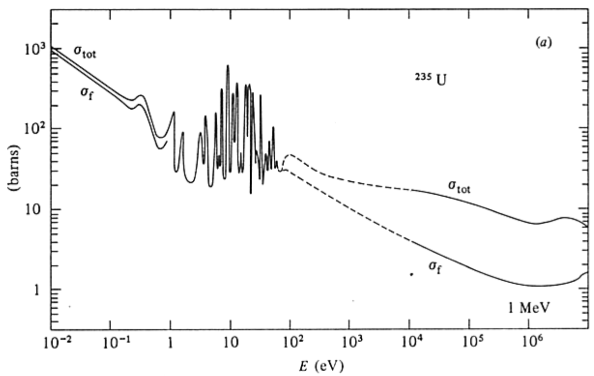

Fig.8.1 Total cross-section σ_{tot} and fission cross-section σ_f as a function of energy for neutrons incident on ^{235}U

The same cross-sections for neutrons incident on ^{238}U are shown in Fig.8.2.

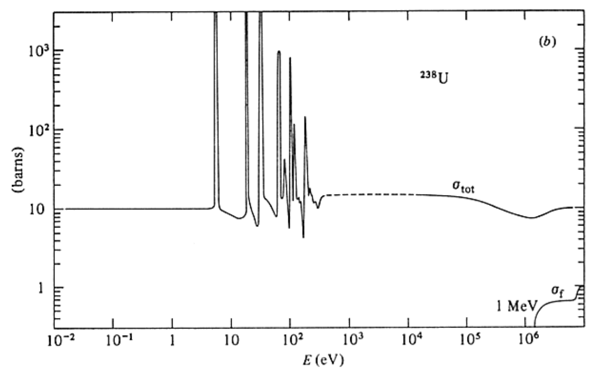

Fig.8.2 Total cross-section σ_{tot} and fission cross-section σ_f as a function of energy for neutrons incident on ^{238}U

The most important features of these figures are:

1. At energies below 0.1 eV, σ_{tot} for ^{235}U, is much larger than that for ^{238}U and the fission fraction is large (~ 84%). (The other 16% is mainly radiative capture with the formation of an excited state of ^{236}U plus one or more photons.)

2. In the region between 0.1 eV and 1 keV, the cross-sections for both isotopes shown prominent peaks corresponding to resonant capture of the neutron.

3. Above 1 keV, σ_f/σ_{tot} for ^{235}U is still significant, although smaller than at very low energies. In both isotopes σ_{tot} is mainly due to contributions from elastic scattering and inelastic excitation of the nucleus.

The measured widths of the low-energy resonances in the ^{235}U cross-section are $\sim 1\text{eV}$, and the compound nucleus formed at these resonances decay predominantly by fission. Therefore, we can deduce that fission takes place in a time of order

$$\tau_f = \hbar/\Gamma_f \approx 10^{-14}\,\text{s}$$

after neutron absorption. This is essentially instantaneous. The fission fragments are usually highly excited and quickly 'boil off' neutrons. The average number of these so-called *prompt* neutrons for ^{235}U is $n \approx 2.5$, with the value depending a little on the incident neutron energy. In addition, the decay products will decay by chains of β-decays and some of the resulting nuclei will themselves give off further neutrons. This component of the energy release is subject to a mean delay of about 13 s and may occur many years after this time. One of the consequences of this is that the *delayed* component may be emitted after the fuel has be used and removed from the reactor, leading to the biological hazard of radioactive waste.

8.2 Fission chain reactions

We have seen that in each fission reaction a large amount of energy is produced, which of course is what is needed for power production. However, just as important is the fact that in the fission decay products are other neutrons. For example, in the case of fission of ^{235}U on average $n = 2.5$ neutrons are produced. Since neutrons can induce fission, the potential exists for a sustained chain reaction, although a number of conditions have to be fulfilled for this to happen in practice. If we define

$$k \equiv \frac{\text{number of neutrons produced in the } (n+1)\text{th stage of fission}}{\text{number of neutrons produced in the } n\text{th stage of fission}}$$

then if $k = 1$ the process is said to be *critical* and a sustained reaction can occur. This is the ideal situation for the operation of a power plant based on nuclear fission. If $k < 1$, the process is said to be *subcritical* and the reaction will die out; if $k > 1$, the process is *supercritical* and the energy will grow very rapidly, leading to an uncontrollable explosion (nuclear fission bomb).

Again we will focus on uranium as the fissile material and consider the length and time scales for a chain reaction to occur. If we assume that the uranium is a mixture of the two isotopes ^{235}U and ^{238}U in the ratio $c:(1-c)$, then the average neutron total cross-section for this mixture is

$$\bar{\sigma}_{tot} = c\sigma_{tot}^{235} + (1-c)\sigma_{tot}^{238}$$

and the *mean free path,* i.e. the mean distance the neutron travels between interactions, is given by

$$\ell = 1/(\rho_{nucl}\,\bar{\sigma}_{tot})$$

where $\rho_{nucl} = 4.8\text{x}10^{28}$ nuclei /m^3 is the nuclei density of uranium metal. For example, the average energy of a prompt neutron from fission is 2 MeV and at this energy we can see from Figs.8.1 and 8.2 that $\sigma_{tot} \approx 7$ barns, so that $\ell \approx 3$cm. A 2MeV neutron will travel this distance in about $1.5\text{x}10^{-9}$s.

Consider firstly the case of the *explosive release* of energy in a nuclear bomb, using the highly enriched isotope ^{235}U (for simplicity we will take $c = 1$). From Fig.8.1, we see that a neutron with energy of 2 MeV has a probability of about 18% to induce fission in an interaction with a ^{235}U nucleus. Otherwise it will scatter and lose energy, so that the probability for a further interaction will be somewhat increased (because the cross-section increases with decreasing energy). If the neutron does not escape outside the target, the most probable number of collisions it will make before inducing fission is easily shown to be 6 so it will move a linear (net) distance of $\sqrt{6}\,\text{x}\,3\text{cm} \approx 7\text{cm}$ (the square root is because we are assuming that at each collision the direction changes randomly – *random walk*) in a time $t_p \approx 10^{-8}$s before inducing a further fission and being replaced on average by 2.5 new neutron with average energy of 2 MeV. [**Aside:** If the probability of inducing fission in a collision is p, the probability that a neutron has induced fission after n collisions is $p(1-p)^{n-1}$ and the mean number of collisions to induce fission will be $\bar{n} = \sum_{n=1}^{\infty} np(1-p)^{n-1}$. This can be estimated using the measured cross-sections.]

The above argument suggests that the critical mass of uranium ^{235}U, which will produce a nuclear explosion, is a sphere of radius about 7 cm. However, not all neutrons will be available to induce fission. Some will escape from the surface and some will undergo radiative capture. If the probability that a newly created neutron induces fission is q, then each neutron will on average lead to the creation of $(nq-1)$ additional neutrons in the time t_p. If there are $N(t)$ neutrons present at time t, then at time $t + \delta t$ there will be

$$N(t+\delta t) = N(t)\left[1 + (nq-1)(\delta t/t_p)\right]$$

In the limit as $\delta t \to 0$, this gives (you should check this)

$$\frac{dN}{dt} = \frac{(nq-1)}{t_p}N(t)$$

with solution

$$N(t) = N(0)\exp\left[(nq-1)t/t_p\right]$$

Thus the number increases or decreases exponentially, depending on whether $nq > 1$ or $nq < 1$. For ^{235}U, the number increases exponentially if $q > 1/n \approx 0.4$ (recall that $n \approx 2.5$). Clearly if the dimensions of the metal are substantially less than 7 cm, q will be small and the chain reaction will die out exponentially. However, a sufficiently large mass brought together at $t = 0$ will have $q > 0.4$. There will be neutrons present at $t = 0$ arising from spontaneous fission and since $t_p \approx 10^{-8}$ s, an explosion will occur in a microsecond. For a simple sphere of ^{235}U the critical radius at which $nq = 1$ is actually about 8.7 cm and the critical mass is 52 kg.

Despite the above, it is not easy to make a nuclear bomb! This is because the energy released as the assembly becomes critical will be sufficient to blow apart the fissile material unless special steps are made to prevent this. In the original "atom bombs", a subcritical mass was assembled and a small plug fired into a prepared hollow in the material so that the whole mass became supercritical. In later devices, the fissile material was a subcritical sphere of ^{239}Pu surrounded by chemical explosives. These are shaped so that when they explode the resulting shock wave *implodes* the plutonium to become supercritical.

8.3 Power from nuclear fission: nuclear reactors

The production of power in a controlled way for peaceful use is done in a *nuclear reactor* and is just as complex as producing a bomb. There are several distinct types of reactor available. A sketch of the main elements of one, known as a *thermal reactor*, is shown in Fig.8.3.

The most important part is the core, shown schematically in Fig.8.4. This consists of fissile material (fuel elements), control rods and the moderator. The roles of the control rods and the moderator will be explained presently. The most commonly used fuel is uranium and many thermal reactors use natural uranium, even though it has only 0.7% of ^{235}U. In this case a 2 MeV neutron has very little chance of inducing fission in a nucleus of ^{238}U. Instead it is much more likely to scatter inelastically, leaving the nucleus in an excited state and after a couple of such collisions the energy of the neutron will be below the threshold of 1.2 MeV for inducing fission in ^{238}U.

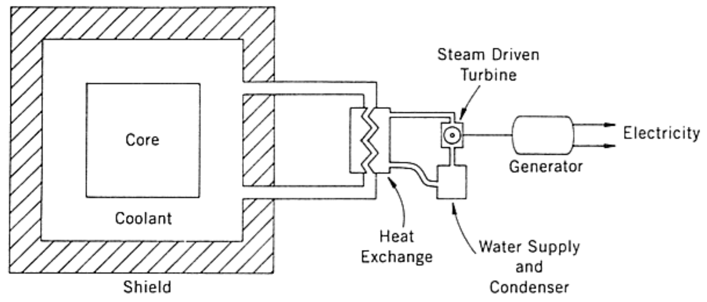

Fig.8.3 Sketch of the main elements of a nuclear power plant

A neutron with its energy so reduced will have to find a nucleus of ^{235}U, but its chances of doing this are very small unless its energy has been reduced to very low energies below 0.1 eV, where the cross-section is large. However, before that happens it is likely to have been captured into one of the ^{238}U resonances with the emission of photons. Thus, either the fuel must be enriched with a greater fraction of ^{235}U (2%-3% is common in commercial reactors), or if natural uranium is to be used, some method must be devised to overcome this problem.

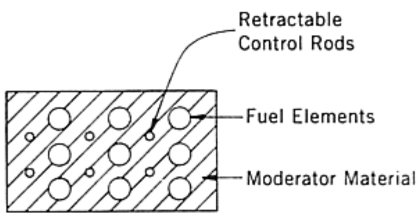

Fig.8.4 Sketch of the elements of the core of a reactor

This is where the moderator comes in. This surrounds the fuel elements and its volume is much greater than that of the elements. Its main purpose is to slow down fast neutrons produced in the fission process. Fast neutrons will escape from the fuel rods into the moderator and are reduced to very low energies by elastic collisions. In this way the absorption into resonances of ^{238}U is avoided. The moderator is a material with a negligible cross-section for absorption and ideally should also be inexpensive. In practice, heavy water (D_2O, where D is the deuteron, the bound state of a proton and a neutron) or graphite are the moderators of choice in many thermal reactors using natural uranium. For enriched reactors, ordinary water is used.

In the above we have ignored the delayed neutrons. In a nuclear weapon, the delayed neutrons are of no consequence because the explosion will have taken place long before they would have been emitted, but in a power reactor they are of crucial importance, because the fuel rods can remain in the reactors for several years. Taking account of delayed neutrons, each fission leads to $[(n+\delta n)q-1]$ additional neutrons, where we have defined δn as the number of delayed neutrons per fission. In the steady state operation, with constant energy

output, the neutron density must remain constant (i.e. $k=1$). Thus q must satisfy the critical condition

$$(n+\delta n)q-1=0$$

This is where the control rods play their part. They are usually made of cadmium, which has a high absorption cross-section for neutrons. Thus, by mechanically manipulating the control rods, i.e. by retracting or inserting them, the number of neutrons available to induce fission can be regulated. This mechanism is the key to maintaining a constant k value of unity and therefore a constant power output. The presence of the delayed neutrons is vital, because although the lifetime of a prompt neutron may be as much as 10^{-3}s in a thermal reactor, rather than 10^{-8}s we calculated for pure ^{235}U, this would still be a very short time to change q and hence avoid a nuclear catastrophe! The reactor design ensures that $nq-1<0$ always, so that the reactor can only become critical in the presence of delayed neutrons. Thus the time scale to manipulate the control rods becomes that of the delayed neutrons, which is adequate.

Returning to Fig.8.3, the core is surrounded by a coolant (often water), which removes the heat generated in the core from the energy deposited by the fission fragments. A thick concrete shield to prevent radiation leaks surrounds the entire setup. At startup, the value of k is set slightly higher than unity and is kept at that value until the desired power output is achieved and the operating temperature is reached, after which the value of k is lowered by adjusting the control rods. It is very important for thermal equilibrium that $dq/dT<0$, so that an increase in temperature leads to a fall in reaction rate. The rest of the plant is conventional engineering. Thus, the heated coolant gives up its heat in a heat exchanger and is used to boil water and drive a steam turbine, which in turn produces electricity.

It is worth calculating the efficiency with which one can expect to produce energy in a nuclear reactor. We can use the SEMF to calculate the energy released during fission, by finding the binding energies of the two fission products and comparing their sum to the binding energy of the decaying nucleus. For the fission of a single ^{235}U nucleus this is ~200 MeV or $3.2\text{x}10^{-11}$ joules. (About 180 MeV of this is in the form of 'prompt' energy.) We also know that 1 gram of any element contains A_0/A atoms, where A_0 is Avogardo's number. Thus one gram of ^{235}U has about $6\text{x}10^{23}/235\approx 3\text{x}10^{21}$ atoms and if fission were complete would yield a total energy of about 10^{11} joules, or 1 megawatt-day. This is about $3\text{x}10^{6}$ times greater than the yield obtained by burning (chemical combustion) 1 gram of coal. In practice only about 1% of the energy content of natural uranium can be extracted (overall efficiency is greatly reduced by the conventional the engineering required to produce electricity via stream turbines), but this can be greatly increased in another type of reactor, called a *fast breeder*.

In a *fast breeder reactor* there is no large volume of moderator and no large density of thermal neutrons is established. In such a reactor, the proportion of fissile material is increased to about 20% and fast neutrons are used to induce fission. The fuel used is ^{239}Pu rather than ^{235}U, the plutonium being obtained by chemical separation from the spent fuel rods of a thermal reactor. This is because some ^{238}U nuclei in the latter will have radiatively captured neutrons to produce ^{239}U, which subsequently decays as follows:

$$^{239}_{92}U \rightarrow ^{239}_{93}Np + e^- + \bar{\nu}_e$$
$$\downarrow \ (3.6\ \text{days})$$
$$^{239}_{94}Pu + e^- + \bar{\nu}_e$$

The value of n for ^{239}Pu is 2.96 and so it is very suitable for use in a fast reactor. In practice, the core is a mixture of 20% ^{239}Pu and 80% ^{238}U (*depleted* uranium, also obtained from spent fuel rods in thermal reactors) surrounded by a blanket of more ^{238}U where more plutonium is made. Such a reactor can produce more fissile ^{239}Pu than it consumes, hence the name 'breeder' and in principle can consume all the energy content of natural uranium, rather than the 1% used in thermal reactors.

Whatever type of reactor is used, a major problem is the generation of radioactive waste, including transuranic elements and long-lived fission fragments, which in some cases may have to be stored safely for hundreds of years. (In principle, there would be no such problem with fast breeder reactors, but in practice the ideal is not realised.) Much effort has been expended on this problem, but a totally satisfactory solution is not available. One ingenious idea is to "defuse" the long-lived fission fragments by using the resonance capture of neutrons to convert them to short-lived, or even to stable, nuclei. For example, ^{99}Tc (Technetium), which is a particularly unpleasant substance because it concentrates in several organs of the body and also in the blood, has a very long half-life. Fortunately it has a large resonant cross-section for neutron capture to a completely stable isotope ^{100}Ru (Ruthenium). The problems to be overcome are far from trivial. Firstly, the amount of radioactive waste is very large, so one problem is to find a source of neutrons capable of handling it. (Reactors themselves are one possible source!) Secondly, the neutron energy has to be matched to the particular waste material, which therefore has to be separated and prepared before being bombarded by the neutrons. All this would take energy and would increase the overall cost of energy production by nuclear power, which is already more expensive than conventional burning of fossil fuels.

8.4 Nuclear fusion: Coulomb barrier

We have seen that the plot of binding energy per nucleon has a maximum at $A \approx 60$ and slowly decreases for heavier nuclei. For lighter nuclei, the decrease is much quicker, so that with the exception of magic nuclei, lighter nuclei are less tightly bound than medium size nuclei. Thus, in principle, energy could be produced by a process that is the opposite of fission, whereby two light nuclei fuse to produce a heavier and more tightly bound nucleus. The energy released comes from the difference in the binding energies of the initial and final states. This process is called *nuclear fusion*. Since light nuclei contain fewer nucleons than heavier nuclei, the energy released per fusion is smaller than in fission. However, this is more than balanced by the far greater abundance of stable light nuclei in nature than very heavy nuclei. Thus fusion offer enormous potential for power generation, if the huge practical problems can be overcome.

The practical problems have their origin in the Coulomb repulsion, which inhibits two nuclei getting close enough together to fuse. This is

$$V_C = \frac{1}{4\pi\varepsilon_0}\frac{ZZ'e^2}{R+R'}$$

where Z and Z' are the atomic numbers of the two nuclei and R and R' are their radii. Recalling that $R = 1.2A^{1/3}$ fm, we have

$$V_C = \left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right)\frac{\hbar c\, Z Z'}{1.2\left[A^{1/3} + (A')^{1/3}\right]\text{fm}} = \frac{1}{137}\frac{197\,\text{MeV fm}}{1.2\,\text{fm}}\frac{Z Z'}{A^{1/3} + (A')^{1/3}}$$

and if we set $A \approx A' \approx 2Z \approx 2Z'$, then

$$V_C \approx \frac{1}{8}A^{5/3}\,\text{MeV}$$

Thus, with $A \approx 8$, $V_C \approx 4\,\text{MeV}$ and this energy has to be supplied to overcome the Coulomb barrier.

In principle, we could simply collide two accelerated beams of light nuclei, but in practice, nearly all the particles would be elastically scattered. The only practical way is to heat a mixture of the nuclei to supply enough thermal energy to overcome the barrier. The temperature necessary may be estimated from the relation $E = k_B T$, where k_B is Boltzmann's constant given by $k_B = 8.6\text{x}10^{-5}\,\text{eV}\,\text{K}^{-1}$. For an energy of 2 Mev, we find a typical temperature of $3\text{x}10^{10}\,\text{K}$. This is well above the typical temperature of $10^7\,\text{K}$ found in stellar interiors.

Fusion actually occurs at a lower temperature by a combination of the fact that the energy distribution is Maxwellian, with a high-energy tail, and tunnelling, which means that the full height of the Coulomb barrier does not have to be overcome. Recalling the work on α-decay in Section 2, the Gamow factor depends on the relative velocities and the charges of the reaction products. In practice, fusion takes place over a rather narrow range of energies. (See Fig.8.5)

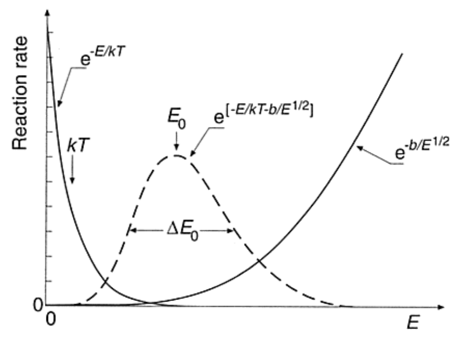

Fig.8.5 The right-hand curve is proportional to the Gamow factor and the left-hand curve is the exponential of the Maxwell distribution. The dotted curve is the combined effect and is proportional to the probability of fusion.

8.5 Stellar fusion

The energy of the Sun comes from nuclear fusion reactions, foremost of which is the so-called PPI cycle, which starts with the fusion of hydrogen nuclei to produce deuterium:

$$^1H + {}^1H \rightarrow {}^2H + e^+ + \nu_e + 0.42\,\text{MeV}$$

The deuterium then fuses with more hydrogen to produce ^3He:

$$^1H + {}^2H \rightarrow {}^3He + \gamma + 5.49\,\text{MeV}$$

and finally, two ^3He nuclei fuse to form ^4He:

$$^3He + {}^3He \rightarrow {}^4He + 2({}^1H) + 12.86\,\text{MeV}$$

The relatively large energy release in the last reaction is because ^4He is a doubly magic nucleus and so is very tightly bound. Overall, we have

$$4({}^1H) \rightarrow {}^4He + 2e^+ + 2\nu_e + 2\gamma + 24.68\,\text{MeV}$$

Because the temperature of the Sun is $\sim 10^7\,\text{K}$, all its material is fully ionised, i.e. it is a *plasma*. Thus, the positrons produced above will annihilate with electrons in the plasma to release a further 1.02 MeV of energy per positron and the total energy released is 26.72 MeV. However of this, each neutrino will carry off 0.26 MeV on average, which is lost into outer space. Thus on average, 6.55 MeV of electromagnetic energy is radiated from the Sun for every proton consumed in the PPI chain.

The PPI chain is not the only fusion cycle going on in the Sun, but it is the most important. Another interesting cycle, which plays an important role in the evolution of some stellar objects is the carbon, or CNO chain. This contributes only about 3% of the energy output of the Sun. In the presence of any of the nuclei $^{12}_{6}C$, $^{13}_{6}C$, $^{14}_{7}N$ or $^{15}_{7}N$, hydrogen will catalyse burning via the reactions

$$^{12}C + {}^1H \rightarrow {}^{13}N + \gamma + 1.95\text{ MeV}$$
$$^{13}N \rightarrow {}^{13}C + e^+ + \nu_e + 1.20\text{ MeV}$$

$$^{13}C + {}^1H \rightarrow {}^{14}N + \gamma + 7.55\text{ MeV}$$

$$^{14}N + {}^1H \rightarrow {}^{15}O + \gamma + 7.34\text{ MeV}$$
$$^{15}O \rightarrow {}^{15}N + e^+ + \nu_e + 1.68\text{ MeV}$$

and

$$^{15}N + {}^1H \rightarrow {}^{12}C + {}^4He + 4.96\text{ MeV}$$

(The ^{12}C, for example, could be present because the ^4He produced in the PPI cycle could fuse via the reaction $3({}^4He) \rightarrow {}^{12}C + 7.27\text{ MeV}$.) Thus, overall in the CNO cycle we have

$$4({}^1H) \rightarrow {}^4He + 2e^+ + 2\nu_e + 3\gamma + 24.68\text{ MeV}$$

These, and other, fusion chains all produce electron neutrinos as final state products and from detailed models of the Sun it can be calculated what should be the flux of such neutrinos at the surface of the Earth. The actual count rate is far lower than the theoretical expectation. This is the so-called *solar neutrino problem*. The solution to this problem is almost certainly neutrino oscillations, where some of the ν_e are converted to neutrinos of other flavours in their passage from the Sun to Earth. We have seen in Section 6, that this only possible if neutrinos have mass, so a definitive measurement of neutrino masses would be an important piece of evidence to finally resolve the solar neutrino problem. Our own high-energy physics group is actively involved in an experiment that will contribute to such measurements.

8.6 Fusion reactors

There is currently a large-scale effort to try to achieve controlled fusion in the laboratory, with the eventual aim to produce power. For this, the PP reactions are far too slow. However, Coulomb barriers for the deuteron ^2_1H are the same as for the proton and the exothermic reactions

$$^2_1H + {}^2_1H \rightarrow {}^3_2H + n + 3.27\text{ MeV}$$

$$^2_1H + {}^2_1H \rightarrow {}^3_1H + p + 4.03\text{ MeV}$$

suggest deuterium might be a suitable fuel for a fusion reactor. Deuterium is also present in huge quantities in sea water and is easy to separate at low cost. An even better reaction in terms of energy output is deuterium-tritium fusion:

$$^2_1H + {}^3_1H \rightarrow {}^4_2He + n + 17.62\text{ MeV}$$

This has two advantages: the heat of the reaction is greater; and the cross-section is much larger. The principal disadvantage is that tritium does not occur naturally (it has a mean life of only 17.7 years) and has to be manufactured, which increases the overall cost.

A working energy where the deuterium-tritium reaction has a reasonable cross-section is about 20 keV, i.e. $3\text{x}10^8\,\text{K}$. At these temperatures any material container will vapourise and so the central problem is how to contain the plasma for sufficiently long times for the reaction to take place. The two main methods are magnetic confinement, where the two ingredients are confined by electromagnetic fields; and inertial confinement, where small pellets of the 'fuel' are imploded by bursts of pulsed laser beams. Although the reactions have been observed, there is a very long way to go before a power plant could be built. A measure of how close to practicality is a particular design is provided by the *Lawson criterion* as follows.

To achieve a temperature T in a deuterium-tritium plasma, there has to be an input of energy $4\rho_d(3k_BT/2)$ per unit volume. Here ρ_d is the number density of deuterium ions and the factor of 4 comes about because ρ_d is equal to the number density of tritium ions and the electron density is twice this, giving $4\rho_d$ particles per unit volume. The reaction rate in the plasma is $\rho_d{}^2\Omega$, where Ω (measured in $\mathrm{m^3 s^{-1}}$) is related to the average cross-section for the fusion reaction. If the plasma is confined for time t_c, then, per unit volume of plasma,

$$L \equiv \frac{\text{energy output}}{\text{energy input}} = \frac{\rho_d{}^2\,\Omega\, t_c (17.6\ MeV)}{6\rho_d\, k_B\, T} \approx (10^{-19}\mathrm{m^3 s^{-1}})\,\rho_d\, t_c$$

where the experimental value $\Omega \approx 10^{-22}$ has been used. For a useful device, $L > 1$, which implies

$$\rho_d\, t_c > 10^{19}\,\mathrm{m^{-3} s}$$

This is the Lawson criterion. It implies that either a very high particle density or a long confinement time, or both, is required. No experimental rig has yet succeed in achieving this.